JITTER EFFECTS IN A MULTIPATH ENVIRONMENT

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ABSTRACT

This paper studies the effects of jitter in a multipath environment. The power spectral density of a process subjected to jitter and multipath is first derived. The problem of reconstruction of a time continuous process from observations subjected to jitter and multipath is then considered. Simulation and theoretical results are finally shown to be in good agreement.

1. INTRODUCTION

The problem addressed in this paper is the study of jitter in a multipath environment. A random process $Z(t)$ subjected to jitter and multipath can be written :

$$U(t) = \sum_{k=1}^{p} c_k Z(t - A_k(t)) + B(t) \quad t \in \mathbb{R} \quad (1)$$

where the jitter is modeled by a random process $\{A_k(t), t \in \mathbb{R}\}$, p represents the number of different paths, c_k is the complex amplitude associated to each path and $B(t)$ is an additive noise.

The random process $U(t)$ is observed at time instants $t = n \in \mathbb{Z}$. $Z(t)$ is assumed to be (wide sense) stationary, with a power spectral density (P.S.D.) denoted by $s_Z(\omega)$ such that [9]:

$$K_Z(\tau) = E[Z(t) Z^*(t-\tau)] = \int_{\mathbb{R}} s_Z(\omega) e^{i\omega\tau} d\omega \quad (2)$$

The additive noise $B(t)$, the jitters $A_k(t)_{k=1,\dots,p}$ and the original process $Z(t)$ are supposed to be independent.

The k^{th} jitter process $A_k(t)$ is supposed to be stationary in the sense that it is characterized by the two following characteristic functions [1] :

$$\begin{aligned} \Psi_k(\omega) &= E\left[e^{i\omega A_k(t)}\right] \\ \Phi_k(\tau,\omega) &= E\left[e^{i\omega(A_k(t)-A_k(t-\tau))}\right] \end{aligned} \quad (3)$$

In the following theoretical developments, the number of paths p and the coefficients c_k are assumed to be known. Note that the number of paths may be estimated using MDL criterion [2] and that MUSIC algorithm [3] [4] can be used to estimate the coefficients c_k and the delay means ($E[A_k(t)]$) [5], [6],[7],[8]. The P.S.D. $s_Z(\omega)$ of the transmitted random process $Z(t)$ is also assumed to be known : this is a realistic assumption when the type of coding (for example N.R.Z.) is known. The problem is to find the best approximation $\widehat{Z}(t)$ of $Z(t)$ (in the mean square sense) from the observation of $\{U(n)\}_{n\in\mathbb{Z}}$.

Section 2 derives the P.S.D. of $U(t)$ defined in Eq.(1) in terms of $s_Z(\omega)$, $s_B(\omega)$, $\Psi_k(\omega)$ and $\Phi_k(\tau,\omega)$. The recovery of $Z(t)$ is studied in Section 3. Simulation results and conclusion are given in Sections 4 and 5.

2. A DECOMPOSITION OF THE OBSERVED PROCESS

Let H_Z denotes the Hilbert space spanned by the random process $Z = \{Z(t), t \in \mathbb{R}\}$. In what follows, an isometry I_Z is used, defining a one-to-one correspondence between the random process $Z(t)$ and the function $e^{i\omega t}$:

$$Z(t) \overset{I_Z}{\longleftrightarrow} e^{i\omega t}$$

The main interest of this isometry lies in the transformation of any process distance into a complex exponential distance, as the corresponding inner products are equal :
If $\alpha(Z(t)) \overset{I_Z}{\longleftrightarrow} f(t,\omega)$ and $\beta(Z(t)) \overset{I_Z}{\longleftrightarrow} g(t,\omega)$, then

$$\begin{aligned} & E[\alpha(Z(t))\beta^*(Z(t))] = \langle \alpha(Z(t)), \beta(Z(t)) \rangle_{H_Z} \\ & = \langle f(t,\omega), g(t,\omega) \rangle_{L^2(s_Z)} = \int_{\mathbb{R}} f(t,\omega) g^*(t,\omega) s_Z(\omega) d\omega \end{aligned}$$

The jitter effect in a multipath environment can be analyzed as follows (see [10] for more explanations). The process $U(t)$ (Eq. (1)) can be decomposed into two parts :

$$U(t) = G(t) + W(t) \quad (4)$$

where

$$G(t) = \sum_{k=1}^{p} c_k G_k(t) \text{ and } W(t) = \sum_{k=1}^{p} c_k V_k(t) + B(t).$$

$G_k(t)$ (resp. $G(t)$)- corresponds to the orthogonal projection of $Z(t - A_k(t))$ (resp. $U(t)$) onto the Hilbert space H_Z as illustrated on figure 1.

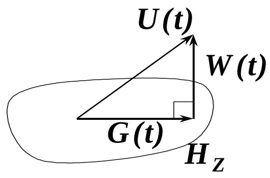

Figure 1: Projection of $U(t)$ onto the Hilbert space H_Z.

2.1. Projection onto the Hilbert space H_Z

$G_k(t)$ is such that

$$E\left[Z(t)\left(Z(t-A_k(t))-G_k(t)\right)^*\right]=0 \qquad (5)$$

Let

$$G_k(t) \overset{I_Z}{\longleftrightarrow} f(t,\omega)$$

Eq. (5) can be written :

$$\begin{aligned} E\left[Z(t)\,G_k(t)^*\right] &= \int_{\mathbb{R}} f(t,\omega)^*\, e^{i\omega t} s_Z(\omega)\, d\omega \\ &= E\left[Z(t)\,Z^*(t-A_k(t))\right] \end{aligned}$$

where

$$\begin{aligned} &E\left[Z(t)\,Z^*(t-A_k(t))\right] \\ &\quad = E\left\{E\left[Z(t)\,Z^*(t-A_k(t))\,/A_k(t)\right]\right\} \\ &\quad = E\left[K_Z(A_k(t))\right] = E\left[\int_{\mathbb{R}} e^{i\omega A_k(t)} s_Z(\omega)\, d\omega\right] \\ &\quad = \int_{\mathbb{R}} \Psi_k(\omega)\, s_Z(\omega)\, d\omega \end{aligned}$$

Consequently, $G_k(t)$ is defined in the isometry I_Z by

$$G_k(t) \overset{I_Z}{\longleftrightarrow} \Psi_k^*(\omega)\, e^{i\omega t} \qquad (6)$$

This means that $G(t)$ can be viewed as the output of a linear filter driven by $Z(t)$, with a frequency response $H_G(\omega)$ such that :

$$H_G(\omega) = \sum_{k=1}^{p} c_k \Psi_k^*(\omega) \qquad (7)$$

The P.S.D. of $G(t)$ is then obtained by Wiener-Lee relation:

$$s_G(\omega) = \left|\sum_{k=1}^{p} c_k \Psi_k^*(\omega)\right|^2 s_Z(\omega) \qquad (8)$$

2.2. P.S.D. of $U(t)$

Given Eq.(4), the P.S.D. of the random process $U(t)$ is :

$$\begin{aligned} s_U(\omega) &= s_G(\omega) + s_W(\omega) \qquad (9) \\ &= s_G(\omega) + \sum_{k=1}^{p} |c_k|^2 s_{V_k}(\omega) + s_B(\omega) \end{aligned}$$

where $s_G(\omega)$ is defined in Eq.(8) and $s_{V_k}(\omega)$ is such that :

$$\begin{aligned} K_{V_k}(\tau) &= E\left[V_k(t)\, V_k^*(t-\tau)\right] \\ &= \int_{\mathbb{R}} s_{V_k}(\omega)\, e^{i\omega\tau} d\omega \end{aligned}$$

The random processes $\{V_k(t)\}_{k=1,\ldots,p}$ are orthogonal to H_Z ($V_k(t) = Z(t-A_k(t)) - G_k(t)$) and uncorrelated (due to the independence of the $\{A_k(t)\}_{k=1,\ldots,p}$). Moreover, the autocorrelation function of $V_k(t)$ is of the form :

$$\begin{aligned} &K_{V_k}(\tau) = E\left[V_k(t)\, V_k^*(t-\tau)\right] \\ &= E\left[K_Z(\tau + A_k(t-\tau) - A_k(t))\right] \\ &\qquad - E\left[G_k(t)\, G_k^*(t-\tau)\right] \\ &= \int_{\mathbb{R}} \left(\Phi_k^*(\tau,\omega) - |\Psi_k(\omega)|^2\right) e^{i\omega\tau} s_Z(\omega)\, d\omega \end{aligned}$$

This allows to determine the P.S.D. of $V_k(t)$ such that

$$\begin{aligned} &\int_{\mathbb{R}} s_{V_k}(\omega)\, e^{i\omega\tau} d\omega \\ &\quad = \int_{\mathbb{R}} \left(\Phi_k^*(\tau,\omega) - |\Psi_k(\omega)|^2\right) e^{i\omega\tau} s_Z(\omega)\, d\omega \end{aligned}$$

and the expression of $s_U(\omega)$ in Eq.(9) can be derived. The random process $U(t)$ is sampled at $t = n \in \mathbb{Z}$. The P.S.D. of this sampled process denoted by $\widetilde{s}_U(\omega)$ and defined on $[-\pi,\pi]$ expresses as :

$$\widetilde{s}_U(\omega) = \sum_{m=-\infty}^{+\infty} s_U(\omega + 2\pi m)\,.$$

3. RECONSTRUCTION

Recall that $U(t)$ is observed at time instants $t = n \in \mathbb{Z}$. The best approximation $\widehat{Z}(t)$ (in a mean square sense) of $Z(t)$ belongs to the Hilbert space H_U spanned by $U = \{U(n), n \in \mathbb{Z}\}$ and is determined by the orthogonal projection of $Z(t)$ onto this space H_U :

$$E\left[\left(Z(t) - \widehat{Z}(t)\right) U^*(n)\right] = 0 \qquad \forall n \in \mathbb{Z} \qquad (10)$$

Using the isometry I_Z defined in Eq.(6), it can be shown that:

$$\begin{aligned} E\left[Z(t)\, U^*(n)\right] &= E\left[Z(t)\, G^*(n)\right] \qquad (11) \\ &= \int_{-\pi}^{+\pi} \alpha(t,\omega)\, e^{i\omega t} e^{-i\omega n} d\omega \end{aligned}$$

where

$$\alpha(t,\omega) = \sum_{k=1}^{p} \sum_{m=-\infty}^{+\infty} c_k^* e^{i2\pi m t} \Psi_k(\omega + 2\pi m)\, s_Z(\omega + 2\pi m)\,.$$

Let us define $\widehat{Z}(t)$ in the isometry I_U by

$$\begin{aligned} U(n) &\overset{I_U}{\longleftrightarrow} e^{i\omega n} \qquad (12) \\ \widehat{Z}(t) &\overset{I_U}{\longleftrightarrow} \mu(t,\omega) \end{aligned}$$

Using Eq.(11)and Eq.(12), Eq. (10) can be written, $\forall n \in \mathbb{Z}$:

$$\int_{-\pi}^{+\pi} \left(\alpha\left(t,\omega\right)e^{i\omega t} - \mu\left(t,\omega\right)\widetilde{s}_U\left(\omega\right)\right)e^{-in\omega}d\omega = 0 \quad (13)$$

Thus, the best approximation $\widehat{Z}(t)$ of $Z(t)$ from the observation of $U(n)$ is defined in the isometry I_U by

$$\mu\left(t,\omega\right) = \frac{\alpha\left(t,\omega\right)}{\widetilde{s}_U\left(\omega\right)}e^{i\omega t} \quad (14)$$

Consequently, $\widehat{Z}(t)$ can be viewed as the output of a filter, driven by $U(n)$, with a transfer function

$$R\left(t,\omega\right) = \frac{\alpha\left(t,\omega\right)}{\widetilde{s}_U\left(\omega\right)} \quad (15)$$

Straightforward computations provide the mean square error (MSE) between $Z(t)$ and $\widehat{Z}(t)$

$$\begin{aligned} \sigma_t^2 &= E\left[\left|Z\left(t\right) - \widehat{Z}\left(t\right)\right|^2\right] \quad (16) \\ &= K_Z\left(0\right) - \int_{-\pi}^{+\pi} \frac{\left|\alpha\left(t,\omega\right)\right|^2}{\widetilde{s}_U\left(\omega\right)}d\omega \end{aligned}$$

4. AN IMPORTANT EXAMPLE

Consider the example of a π-bandlimited process

$$s_Z\left(\omega\right) = 0 \quad for \quad \omega \notin [-\pi, +\pi]$$

In this case, the approximation filter depends no longer on the time instant t at which the reconstruction is made:

$$R\left(\omega\right) = \sum_{k=1}^{p} c_k^* \Psi_k\left(\omega\right)\frac{s_Z\left(\omega\right)}{\widetilde{s}_U\left(\omega\right)} \quad (17)$$

Thus, $\widehat{Z}\left(t\right)$ is obtained by linear filtering of $U\left(n\right)$ and the mean square error σ_t^2 can be expressed as :

$$\sigma_t^2 = \sigma^2 = K_Z\left(0\right) - \int_{-\pi}^{+\pi} \frac{\left|\sum_{k=1}^{p} c_k^* \Psi_k\left(\omega\right)s_Z\left(\omega\right)\right|^2}{\widetilde{s}_U\left(\omega\right)}d\omega \quad (18)$$

In order to validate these theoretical results, a N.R.Z. process $Z(t)$ subjected to Gaussian jitter in a multipath environment is considered. In this case, $\left\{A_k\left(t\right)\right\}_{k=1\ldots p}$ are supposed to be Gaussian with different means, denoted by τ_k and the same variance σ_A^2. The two characteristic functions defined in Eq. (3) are of the form

$$\Psi_k\left(\omega\right) = e^{i\tau_k\omega}e^{-\frac{1}{2}\sigma_A^2\omega^2} \qquad \Phi_k\left(\tau,\omega\right) = e^{-\sigma_A^2\omega^2\left(1-\rho_k(\tau)\right)}$$

where $\sigma_A^2\rho_k\left(\tau\right) = E\left[A_k(t)A_k\left(t-\tau\right)\right]$. In the following simulations, $\left\{A_k\left(t\right)\right\}_{k=1\ldots p}$ are uncorrelated.

First of all, it is important to illustrate the jitter effect on a NRZ process, taking into account only one path, without additive noise. Figure 2 displays the jitter effect on such process for different values of jitter standard deviation $\sigma_A = 3, 7, 15$ for a NRZ with 10 samples per bit.

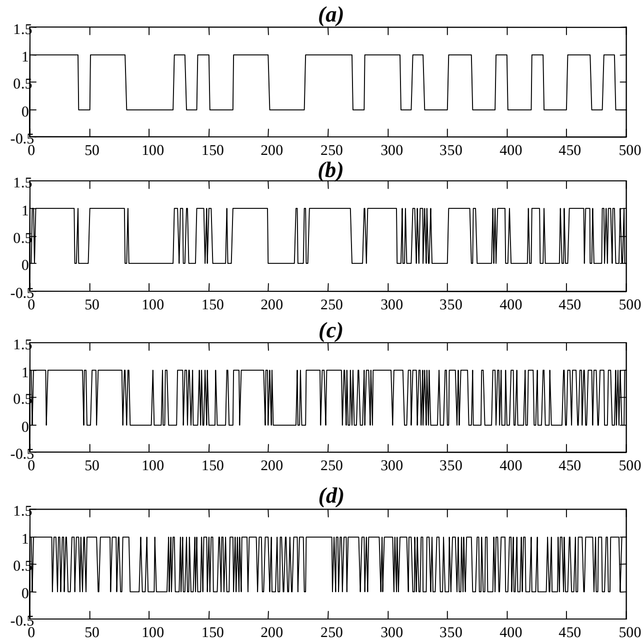

Figure 2: Effect of Gaussian jitter on a NRZ process $Z(t)$ such that $P[Z(t) = 1] = 0.5$, 10 samples per bit, (a) no jitter, (b) $\sigma_A = 3$, (c) $\sigma_A = 7$, (d) $\sigma_A = 15$.

In the frequency domain, the jitter tends to whiten the original process $Z(t)$, as shown on figure 3.

Figure 4(a) shows an example of a NRZ process and figure 4(b) of its distorted version subjected to 3 multipaths with SNR=10 dB and Gaussian jitter. The effects of the optimal linear filter are illustrated in figure 4(c) and figure 4(d) shows the effect of thresholding on this filtered process. It is worth noting that the optimal filter of Eq.(17) can be easily approximated by a F.I.R. filter.

It is also of interest to study the influence of the different parameters on the reconstruction. Two criteria are considered : the quadratic error (Eq.(18)) and the Bit Error Rate (BER) which is the reference in Communications. The quadratic error can be written as a Signal to Reconstruction Noise Ratio (SRNR) in dB. In the example of figure 4, the SRNR is approximately 44 dB and the BER $\approx$ 5%.

Figure 5 highlights the influence of jitter variance on the reconstruction. Both criteria behave similarly, despite they are related non-linearly.

Figure 6 shows that the reconstruction is not affected by additive noise, as soon as SNR $\geq$10 dB.

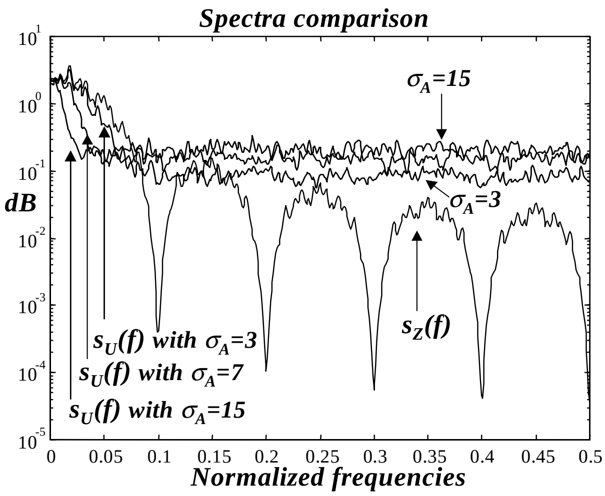

Figure 3: Jitter spectral effect.

5. CONCLUSION

This paper studies the effect of jitter in a multipath environment. The main focus of this paper is to derive an expression of an optimal filter in order to achieve the reconstruction of the continuous time original process. In the case of an original process under the Shannon condition, it is shown that this optimal estimation is the result of a linear filter independent of the time instant t at which the reconstruction is carried out. Simulations on a NRZ process subjected to Gaussian jitter and multipath are given, showing that this optimal filter can be approximated by a FIR one, leading to satisfying results.

6. REFERENCES

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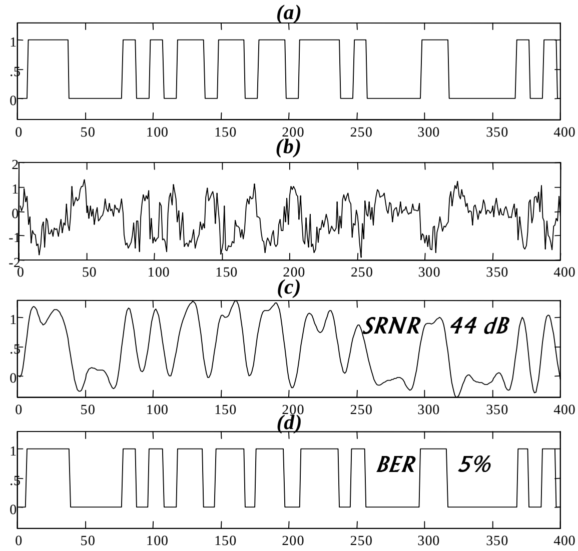

Figure 4: Reconstruction of a NRZ process: (a) NRZ process, (b) NRZ process subjected to 3 multipaths and Gaussian jitter ($\sigma_A = 3$, $\tau_1 = 12$, $\tau_2 = 5$, $\tau_3 = 0$), SNR=10dB, (c) filtered process, (d) filtered process after thresholding.

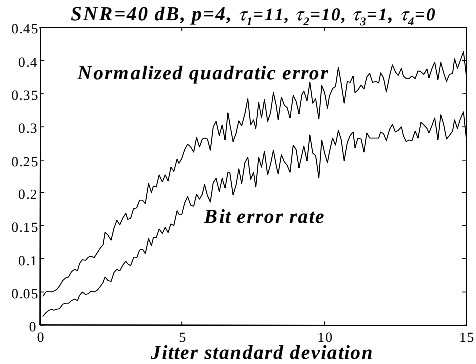

Figure 5: MSE and BER v.s. jitter standard deviation.

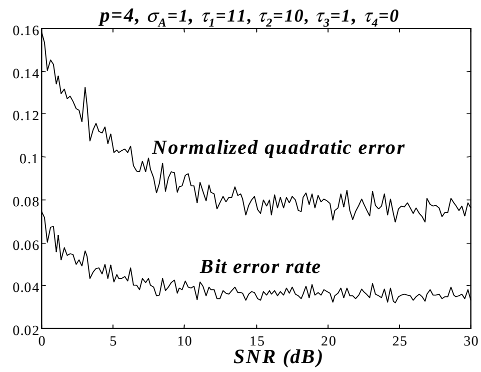

Figure 6: MSE and BER v.s. SNR.