

IMPROVED LOSSLESS COMPRESSION OF COLOR-MAPPED IMAGES BY AN APPROXIMATE SOLUTION OF THE TRAVELING SALESMAN PROBLEM

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ABSTRACT

A color-mapped image is composed of a palette and an image of indices. The color of each pixel is the palette color referenced by the pixel's index. The indices image is usually far from being smooth. This affects its compressibility by lossless image compression methods, due to their reliance on the natural smoothness of images. The indices images can be smoothed by re-ordering the palette, i.e., assigning the indices to the colors in a better way. In this paper we propose a new method for palette ordering based on an approximate solution of the Traveling Salesman Problem. The proposed method has low complexity dependent only on the size of the palette. It is found to significantly improve the compression ratios of color-mapped images coded with the JPEG-LS lossless compression standard. The resulting compression ratios are better than those achieved by the popular GIF image file format.

1. INTRODUCTION

The pixels of color-mapped images contain indices to palette colors instead of the colors themselves as in true color images. Color-mapped images are used for displaying images on color-limited displays or for storing bitmapped images.

The common file format for storing color-mapped images is the GIF file format [1]. This file format employs the LZW algorithm [2] to encode the indices image. One of the main drawbacks of GIF is that it encodes the image as a 1-dimensional stream obtained by a raster scan of the image. Except for the left neighbor of a pixel, the spatial correlation between the pixel and its neighbors isn't exploited.

The JPEG-LS [3] lossless image compression standard is intended for lossless and near-lossless compression of continuous tone images. This method combines a predictor and an encoder. The predictor predicts the value of the current pixel according to its neighbors that precede it in a raster scan of the image. The encoder encodes the prediction residuals using a context-based Golomb-Rice

coding. The encoder assumes a Laplacian distribution of the predictor residuals, which is a good approximation for natural continuous tone images.

The JPEG-LS encoder doesn't give good compression results for the indices image of a typical color-mapped image. This is due to the lack of smoothness of the indices image, which invalidates the Laplacian distribution assumption for the prediction residuals. The situation can be improved by a better assignment of the palette's indices that would result in a smoother indices image.

The need for color ordering of color-mapped images was first addressed for lossy compression of these images. Zaccarin et al. [4,6,8] used luminance-based ordering with a different ordering for different regions of the image. A similar ordering approach was employed by Chen et al. [5]. Waldemar and Ramstad [7] sorted the colors according to the number of times a color appears in the neighborhood of another color. Pei and Lo [9] used Self Organizing Maps to simultaneously color quantize an image and order the resulting colors according to their locations in the 3-dimensional color space.

Memon and Venkateswaran [10] proposed two heuristic color ordering algorithms for lossless color-mapped image compression. The ordering was done according to the number of times pixels having one color were used as predictors to pixels having another color. Memon and Rodila [11] used the Pairwise Merge (PM) from [10] to transcode color-mapped images from GIF to JPEG-LS. They also proposed an iterative version of the PM designed specifically for the Median Edge Predictor of JPEG-LS.

The paper is organized as follows: Section 2 elucidates on the need for color ordering and describes previously used ordering methods and the proposed method. The Traveling Salesman Problem and its solution by the Farthest Insertion Algorithm (FIA) appear in Section 3. In Section 4 we compare by simulations the performance of the proposed method to that of GIF and other ordering methods. Concluding remarks are made in Section 5.

2. COLOR ORDERING

Each color in the palette of a color-mapped image is assigned an index. We refer to this assignment of indices as color ordering. Changing the color ordering (color re-ordering) doesn't change the color-mapped image visually, but can affect its compressibility by lossless (and lossy) compression algorithms.

The compression performance of the GIF file format isn't affected by color ordering. All color orderings produce the exact same compression ratio for a given image. This is due to the fact that the LZW encoder employed by GIF regards different indices as different symbols and doesn't use their numerical values.

The situation is different for JPEG-LS. JPEG-LS uses a predictor to predict the value of the current pixel from the values of its neighbors that precede it in a raster scan. In Figure 1, "x" denotes the current pixel while pixels "a", "b" and "c" are used by JPEG-LS to predict the value of "x". Changing the indices by color re-ordering changes the outcome of the prediction and with it the achieved compression ratio. The change in the compression ratio may be substantial.

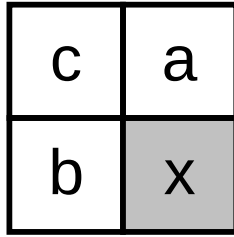


Fig. 1: The pixels used for the predictor of JPEG-LS

A color ordering scheme for lossless color-mapped image compression was presented by Memon et. al. [10,11]. This scheme is suitable for image compression methods that predict the value of the current pixel from the value of another pixel (the left pixel "b" or top pixel "a", for instance). In the following description of this scheme, it will be assumed that the left pixel "b" is used to predict the value of the current pixel "x".

The scheme works as follows. In the first phase, the image is scanned and the number of times each color appears as the left neighbor of any of the other colors is recorded. Every such instance is named by the authors a "connection" between the two neighboring colors. In the second phase the colors are ordered in such a way that colors with many connections between them will have close indices in the ordering.

The ordering is done by two approaches: a Simulated Annealing (SA) approach and a heuristic approach named the Pairwise Merge (PM) heuristic. The SA approach is

much more complex computationally than the PM heuristic and doesn't give significantly better results. It is shown in [10] that this color ordering lowers the entropy of the prediction residuals of the compressed indices images.

In [11] the PM heuristic is adapted to JPEG-LS. An adaptation is necessary because JPEG-LS uses three pixels to predict the value of the current pixel and not only one as is assumed in the PM heuristic. Further more, iterative re-ordering of the indices image is suggested for JPEG-LS. Significant improvements over the compression ratio achieved by GIF are reported. The iterative re-ordering is reported to improve these results only slightly.

The method proposed here is based on the assumption that images contain objects and that objects are constructed from pixels with similar colors. The conclusion from this assumption is that the colors should be ordered in the palette according to the distances between them in 3-dimensional color space. The result of this ordering would be that neighboring pixels from the same object will have close indices which should result in better prediction.

Various methods for ordering the colors in such a way that similar colors would have close indices were tested. Among these were methods based on the Binary Switching Algorithm (BSA) [12], Multidimensional Scaling (MDS) [13], the space filling Hilbert scan [14] and others. The best results were obtained by a method based on an approximate solution of the Traveling Salesman Problem (TSP). This method is directed at generating a minimal length tour passing only once through each of the palette colors residing in 3-dimensional color space. The index of a color is determined according to its position in the tour. The motivation is that such an ordering is expected to have the desired qualities and obtain a better compression ratio. A detailed description of this method appears in the next section.

It is important to note, that if the color-mapped image is generated by color quantizing a true color image, a good color ordering can be generated by using the LBG [15] vector quantization algorithm. The splitting procedure used by this algorithm tends to group similar palette colors together and generate a good color ordering.

3. COLOR ORDERING BY AN APPROXIMATE SOLUTION OF THE TRAVELING SALESMAN PROBLEM

The TSP is the problem of finding the shortest closed tour passing only once through each one of N points in a K-dimensional space [16]. This is an NP-complete problem for which extensive research has yielded many approximate solutions that give very good results with relatively low complexity.

One of the simplest solutions is the Farthest Insertion Algorithm (FIA) [16]. This algorithm works for TSPs with a distance measure that satisfies the triangle inequality. The FIA's steps are:

- Step 1: Start with a partial tour consisting of a single point i .
 - Step 2: If the current partial tour T does not include all the points, find the point k not on T with the maximum distance from T , i.e., with the maximum distance from the closest point j on T (Figure 2a).
 - Step 3: Add k to the partial tour T by inserting it between j and one of the two tour neighbors of j . The neighbor is selected as to minimize the added tour length (Figure 2b).
- Note: In the first two iterations j has less than two neighbors. In these iterations add k to T anywhere.
- Step 4: Repeat steps 1-3 N times, till all the points are added to the tour T .
 - Step 5: Repeat steps 1-4 N times, each time beginning with a different point i . From the N generated tours select the shortest one.

The FIA has complexity of $O(N^3)$. It is a heuristic approach that doesn't have analytic bounds on its performance. There have been specially generated instances of the TSP for which the FIA tour was nearly 50% longer than the optimal one, but for typical instances it is within 5% of the optimal tour [16].

A crude but useful method for evaluating the quality of the FIA tour, for a given set of N points, is comparing its length to the length of the Minimum Spanning Tree (MST) of the points. A MST is a connected graph passing through all the points and having a minimum sum of the arcs' lengths. The length of the MST is a lower bound on the length of the optimal tour for the TSP, while twice its length is an upper bound on the length of the optimal TSP tour. The MST can be found in $O(N^2)$ [16].

The FIA results could be improved by applying another heuristic approach: using the dual Nearest Insertion Algorithm (NIA) [16], and selecting the best tour from the results of both algorithms. The average performance of the NIA for typical instances of the TSP is similar to that of the FIA. The complexity of the NIA is also $O(N^3)$. Using both algorithms can give a somewhat better tour in many instances at the expense of doubling the complexity.

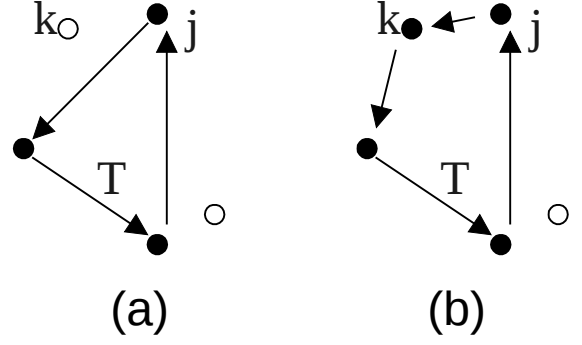


Fig 2: (a) Selecting the point to be added to the tour. (b) Adding the selected point to the tour.

4. SIMULATION RESULTS

This section compares the proposed TSP-based method with other color ordering methods. These methods were used to order the palettes of various color-mapped images (with 256 colors) before using the HP Labs JPEG-LS software [17] to compress their indices images. Grayscale versions of the images appear in Figure 3. The default parameters values of the software were used, without any attempt to better adjust them to the special nature of indices images.

Table 1 expresses the resulting compression ratios in terms of the number of bits per pixel (BPP) of the indices images for the various orderings. The first column on the left gives the results of the proposed TSP based method. The second column contains the results of the ordering generated by color quantizing with the LBG algorithm. Column three gives the best results from three versions of the PM heuristic. The versions include prediction with the left pixel (denoted by "L" in column three), prediction with the top pixel (denoted by "T") and the iterative version adapted to the JPEG-LS predictor (never the best). Column four contains the average results of five random color orderings. The last column gives for comparison the results in BPP obtained using the GIF file format. It should be noted, that the GIF file contains also the palette and image header, but their sizes are insignificant relative to the sizes of the indices images. For the sizes of the images tested (512x512 pixels) the palette and header add less than 0.03 BPP to the GIF file.

The results demonstrate that a random color ordering is bad on average and generates a compression ratio worse than that of the GIF file format. The PM heuristic improves the ordering and sometimes outperforms GIF. The best results are obtained by the proposed TSP-based method. If the color-mapped image is generated by color quantizing with the LBG algorithm, a good ordering can be generated. The performance of this ordering is similar to that of the best version of the PM heuristic.



Fig. 3: Grayscale versions of the tested images

Image	TSP	LBG	PM	Rand	GIF
Lena	6.17	7.05	6.64(L)	8.05	7.02
Peppers	4.05	4.95	4.20(T)	5.99	4.14
Monkey	6.76	7.06	6.80(T)	8.31	7.72
Yarn	4.56	4.57	5.34(T)	6.88	4.69
Flower	3.39	3.75	4.59(L)	5.67	4.05

Table 1: Compression ratios (BPP) of various orderings

Another advantage of the proposed method over the PM heuristic is that it doesn't require scanning the image to compute the number of "connections" between every pair of palette colors. This results in a shorter total run time of the proposed method over the PM heuristic and especially over the iterative version adapted for JPEG-LS.

5. CONCLUSION

The proposed TSP-based color ordering is a simple and fast algorithm that substantially improves the compression ratio of color-mapped images. This algorithm outperforms previously suggested methods in terms of compression ratio and computational complexity.

It is of interest to note that the TSP-based index ordering method was also found useful for assigning indices to code vectors of vector quantizers (VQ), to combat channel errors [18].

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