VOLTERRA FILTERS USING MULTIRATE SIGNAL PROCESSING AND THEIR APPLICATION TO LOUDSPEAKER SYSTEMS

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ABSTRACT

In this paper, we propose two methods for reducing the computational complexity of Volterra filters. First, a method reducing the computational complexity of Volterra filters is proposed. This method can be realized by incorporating multirate signal processing into the Volterra filters. Hence, it is possible to operate the band-limited Volterra filter at a low sampling rate and with a short system length. Second, we also propose a method to replace the conventional Volterra filter with one including many zero coefficients by using multirate signal processing. The conventional Volterra filter is band-limited in order to avoid aliasing so that waste arithmetic is done. In contrast, the Volterra filter including many zero coefficients derived by the proposed method can eliminate such waste arithmetic.We demonstrate the effectiveness in their application to loudspeaker systems whose nonlinear distortions generally concentrate in the lower frequency band. Even though the processed frequency band is limited, the proposed method has about 0.03 times as many computational complexities as the conventional method.

1. INTRODUCTION

Loudspeaker systems have very complex structure. Hence, the radiated sound has various distortions, that is, linear and nonlinear distortions. These distortions make the sound quality of loudspeaker systems poor. To solve this problem, we have already proposed a nonlinear inverse system [1] for removing the distortions. This system requires a nonlinear model because loudspeaker systems have nonlinearity. The Volterra series expansion [2][3] satisfies this requirement, that is, if we identify the Volterra kernels of the series by using adaptive Volterra filters [4], we could obtain an exact nonlinear model. The computational complexity of Volterra filters, however, becomes huge as the order of the series or the system order of Volterra filters becomes high. Hence, we have to reduce the computational complexity. To achieve this purpose, we have directed our attention to a characteristic of the nonlinear distortion in loudspeaker systems. The nonlinear distortion occurs mainly in the lower frequency band. Hence, if we remove only the nonlinear distortion in the lower frequency band, we could reduce the computational complexity. The realization could be achieved by applying multirate signal processing to Volterra filters. However, we cannot simply apply the principles of multirate signal processing for linear filters to nonlinear filters because it is generally impossible to change the arrangement of Volterra filters and the front or the back system. Volterra filters may also cause aliasing even if the maximum frequency of the input signals is limited to half the sampling rate. Considering these issues, we need to derive a new theory on multirate signal processing for Volterra filters. Hence, this paper proposes a method applying multirate signal processing to Volterra filters. The proposed method limits the input signals to the band as not causing aliasing distortion. The limitation makes application of multirate signal processing to Volterra filters possible and then the computational complexity of the Volterra filters is reduced because the proposed method can operate at a low sampling rate and with a low system order. This paper also introduces a system form realizing the above method. Simulation results on reducing the nonlinear distortion of a loudspeaker system demonstrate that the proposed method can reduce the nonlinear distortion to the same level as the conventional method while the proposed method has about 0.03 times as many computational complexities as the conventional one.

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2. VOLTERRA SERIES EXPANSION

2.1. Discrete Volterra Series

Loudspeaker systems can be modeled by using the Volterra series expansion [2]. We assume that Volterra kernels have a finite memory length N and do not treat the third or more terms in order to make the discussion easy. The input-output relation of the systems is represented by

$$
\begin{aligned}
y(n) &= \sum_{k_1=0}^{N-1} h_1(k_1)x(n-k_1) \\
&+ \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} h_2(k_1,k_2)x(n-k_1)x(n-k_2), \quad (1)
\end{aligned}
$$

where $x(n)$ and $y(n)$ are the sampled input and output signals, respectively; $h_1(k_1)$ and $h_2(k_1,k_2)$ are the first- and second-order discrete Volterra kernels, respectively.

2.2. Discrete Fourier Transform of Volterra Series

The Discrete Fourier transform (DFT) of equation (1) is given by

$$Y(m) = H_1(m)X(m) + A_1[H_2(m_1, m_2)X(m_1)X(m_2)], \quad (2)$$

where $X(m)$ and $Y(m)$ are the N-point DFTs of $x(n)$ and $y(n)$, respectively; $H_1(m)$ and $H_2(m_1, m_2)$, which are called the fist- and second-order Volterra Frequency Response (VFR) are the N-point DFTs of $h_1(k_1)$ and $h_2(k_1, k_2)$, respectively. A_1 is called the first-order reduction operator and has the role of mapping a function with multi-dimensional dependent variables to one with one-dimensional variables. The first-order reduction operator is given by

$$\begin{aligned} Y_2(m) &= A_1[Y_2'(m_1, m_2)] \\ &= \frac{1}{N} \sum_{m_1+m_2=(m+rN)} Y_2'(m_1, m_2) \;\; r = 0, 1. \quad (3) \end{aligned}$$

3. VOLTERRA FILTER USING MULTIRATE SIGNAL PROCESSNING

3.1. Volterra Filter Operating at Low Sampling Rate and with Short System Length

First, let us derive the relation between the Volterra filter operating at a low sampling rate and with a short system length H_{2A} and the conventional Volterra filter H_2. That is, the relation between H_2 and H_{2A} is derived so that the output signal of the upper system in Fig.1, $y_1(n)$, equals that of the lower system in Fig.1, $y_2(n)$. The DFT of $y_1(n)$ is represented as

$$\begin{aligned} Y_1(m) &= \frac{1}{2}X_1'(m^{\frac{1}{2}}) + \frac{1}{2}X_1'(-m^{\frac{1}{2}}) \\ &= A_1[\frac{1}{4}X_1'(m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}}) + \frac{1}{4}X_1'(m_1^{\frac{1}{2}}, -m_2^{\frac{1}{2}}) \\ &+ \frac{1}{4}X_1'(-m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}}) + \frac{1}{4}X_1'(-m_1^{\frac{1}{2}}, -m_2^{\frac{1}{2}})] \\ &= A_1[\frac{1}{4}H_2(m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}})X(m_1^{\frac{1}{2}})X(m_2^{\frac{1}{2}}) \\ &+ \frac{1}{4}H_2(m_1^{\frac{1}{2}}, -m_2^{\frac{1}{2}})X(m_1^{\frac{1}{2}})X(-m_2^{\frac{1}{2}}) \\ &+ \frac{1}{4}H_2(-m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}})X(-m_1^{\frac{1}{2}})X(m_2^{\frac{1}{2}}) \\ &+ \frac{1}{4}H_2(-m_1^{\frac{1}{2}}, -m_2^{\frac{1}{2}})X(-m_1^{\frac{1}{2}})X(-m_2^{\frac{1}{2}})], \quad (4) \end{aligned}$$

where

$X_1'(m_1, m_2) = H_2(m_1, m_2)X(m_1)X(m_2)$.

Next, the DFT of $y_2(n)$ is also represented as

$$\begin{aligned} Y_2(m) &= A_1\left[H_{2A}(m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}})X_2'(m_1^{\frac{1}{2}})X_2'(m_2^{\frac{1}{2}})\right] \\ &= A_1[\frac{1}{2}H_{2A}(m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}})\left\{\frac{1}{2}X(m_1^{\frac{1}{2}}) + \frac{1}{2}X(-m_1^{\frac{1}{2}})\right\} \\ &\times \left\{\frac{1}{2}X(m_2^{\frac{1}{2}}) + \frac{1}{2}X(-m_2^{\frac{1}{2}})\right\}] \\ &= A_1[\frac{1}{4}H_{2A}(m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}})X(m_1^{\frac{1}{2}})X(m_2^{\frac{1}{2}}) \\ &+ \frac{1}{4}H_{2A}(m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}})X(m_1^{\frac{1}{2}})X(-m_2^{\frac{1}{2}}) \\ &+ \frac{1}{4}H_{2A}(m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}})X(-m_1^{\frac{1}{2}})X(m_2^{\frac{1}{2}}) \\ &+ \frac{1}{4}H_{2A}(m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}})X(-m_1^{\frac{1}{2}})X(-m_2^{\frac{1}{2}})], \quad (5) \end{aligned}$$

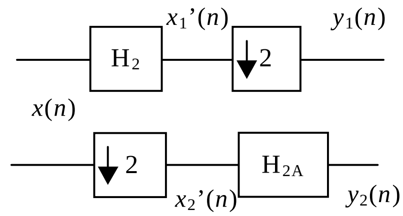

Fig. 1. The relation of H_2 and H_{2A}.

where

$X_2'(m) = \frac{1}{2}X(m^{\frac{1}{2}}) + \frac{1}{2}X(-m^{\frac{1}{2}})$.

We assume the following equation as Eq.(4) equals Eq.(5).

$$\begin{aligned} H_{2A}(m_1, m_2) &= H_2(m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}}) + H_2(m_1^{\frac{1}{2}}, -m_2^{\frac{1}{2}}) \\ &+ H_2(-m_1^{\frac{1}{2}}, m_2^{\frac{1}{2}}) + H_2(-m_1^{\frac{1}{2}}, -m_2^{\frac{1}{2}}) (6) \end{aligned}$$

It can be seen from Eq.(6) that Eq.(4) would equal Eq.(5) if H_2 and the input signal are limited to $fs/4$ (fs: sampling frequency). Hence, we can implement the Volterra filters at a low rate and with a short system length. Note that the system forms shown in Fig.1 cause aliasing because the frequency band of those output signals equals twice frequency band of the input signal in the case of the second-order Volterra filter. Hence, we need to limit the input signals and H_2 to $fs/8$.

3.2. Application to Nonlinear Inverse System

In this section, we apply the principle derived in the previous section to the nonlinear inverse system, which can reduce the nonlinear distortion of loudspeaker systems. As a result, we could obtain a system form with a computational advantage. Let us explain the conventional nonlinear inverse system. Fig.2 shows the system form, which can reduce the second-order nonlinear distortion of a loudspeaker system. In Fig.2, D_1 and D_2 are the linear and the second-order nonlinear elements of the loudspeaker system, respectively. If $H_2 = D_2$ and $D_1 H_1 = z^{-\Delta}$ (H_1 is the linear inverse filter of D_1), then the second-order nonlinear distortion of the loudspeaker system can be reduced because two second-order nonlinear signals output through two different signal paths cancel each other. However, the computational complexity of H_2 is so huge. We therefore apply the principle derived in the previous section to H_2. First, inserting the low pass filter LPF8 at the front of H_2 and the decimator and interpolator between H_1 and H_2, then the system form shown in Fig.3 is obtained. In Fig.3, LPF4 and LPF8 are low pass filters whose cutoff frequencies are $fs/4$ and $fs/8$, respectively. Next, changing the arrangement of H_1 and the interpolator in Fig.3 and applying the relation shown in Fig.1 (the arrangement of H_2 and the decimator is changed), we can obtain the final system

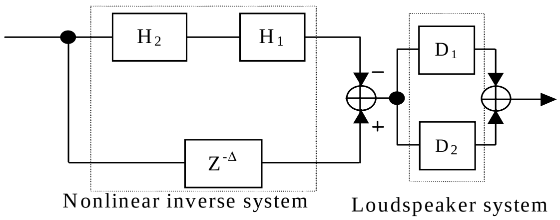

Fig. 2. Conventional nonlinear inverse system.

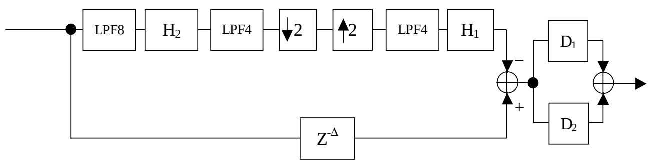

Fig. 3. Band-limited nonlinear inverse system.

form shown in Fig.4. In Fig.4, H_{1A} and H_{2A} are the down-sampled version of H_1 and H_2, respectively. Hence, the system form shown in Fig.4 can operate at the lower sampling rate. The system form has some advantages. First, we can reduce the computational complexity per a sample time because the sampling rate is decreased to 1/2. Second, the computational complexity of the Volterra filter is reduced by down-sampling because of realizing the same frequency resolution with half system length. We can also identify H_{1A} and H_{2A} by adaptive Volterra filters at a low sampling rate.

4. VOLTERRA FILTER WITH MANY ZERO COEFFICIENTS

4.1. Principles of Volterra Filter with Many Zero Coefficients

In this section, we show that the computational complexity of the second-order Volterra filter could be reduced if the filter or the input signal is limited to $fs/4$ (fs : sampling frequency). The second-order Volterra filter must be generally limited to $fs/4$ in order to avoid the aliasing involved in the output signal. In this case, the second-order Volterra filter can be represented by another second-order Volterra filter down-sampled to 1/2. The coefficients of the down-sampled Volterra filter equal the original filter coefficients where both of k_1 and k_2 are even numbers. However, the down-sampled Volterra filter causes aliasing distortion because the frequency band of the output signal equals twice frequency band of the input signal in the case of the second-order Volterra filter. Hence, we need to up-sample the down-sampled Volterra filter twice, that is, we need to make the coefficients, whose numbers k_1 and/or k_2 are odd numbers, zero. The Volterra filter obtained by this procedure is defined as

$$h_2'(k_1,k_2) = \begin{cases} h_2(k_1,k_2) & k_1,k_2 = 0,2,4\cdots \\ 0 & Otherwise. \end{cases} \quad (7)$$

It can be seen from Eq.(7) that the coefficients of $h_2'(k_1,k_2)$ are nonzero only if both of k_1 and k_2 are even numbers. This

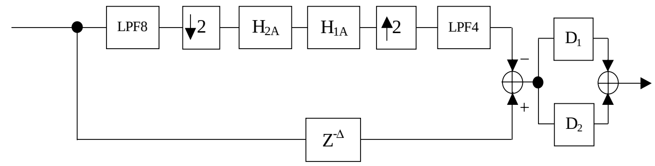

Fig. 4. Equivalent block diagram of Fig.3.

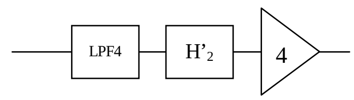

Fig. 5. The form of the Volterra filter including zero value.

fact is very essential to reduce the computational complexity. Next, the DFT of Eq.(7) is represented in the form

$$\begin{aligned} H_2'(m_1,m_2) &= \frac{H_2(m_1,m_2)}{4} + \frac{H_2(m_1,-m_2)}{4} \\ &+ \frac{H_2(-m_1,m_2)}{4} + \frac{H_2(-m_1,-m_2)}{4}. \quad (8) \end{aligned}$$

It can be seen from Eq.(8) that the system form shown in Fig.5 equals the original second-order Volterra filter H_2. That is, if the input signal of the modified Volterra filter $h_2'(k_1,k_2)$ is limited to $fs/4$ and the output signal is multiplied by 4, $h_2(k_1,k_2) = h_2'(k_1,k_2)$. The system form shown in Fig.5 can reduce the computational complexity because $3N/4$ coefficients of $h_2'(k_1,k_2)$ are zero. Hence, we can derive a new system form not to calculate these coefficients. Fig.6 shows the system form. In Fig.6, H_{2A} denotes the second-order Volterra filter that operates at a low sampling rate and has a short system length. In the arithmetic of H_2', only the coefficients whose value is nonzero are multiplied by the corresponding input signals. In this case, the multiplied input signals are classified into two groups according to whether sample time is odd or even number. Hence, multirate signal processing is applied to the Volterra filter H_2' in order to realize the above arithmetic. That is, the down-sampler and delay unit in Fig.6 can classify input signals into two groups, and the up-sampler and delay unit in Fig.6 can synthesize these two signal paths. Hence, the system form shown in Fig.6 can reduce the computational complexity of the second-order Volterra filter to 1/4.

4.2. Application to Nonlinear Inverse System

In this section, the Volterra filter with many zero coefficients is applied to the nonlinear inverse system shown in Fig.4 in order to reduce the computational complexity. H_{2A} in Fig.4 can be replaced with the system form shown in Fig.6. Hence, the system form shown in Fig.7 is obtained. In Fig.7, H_{2A}' denotes the Volterra filter that operates at a low sampling rate and has a short system length. The system form shown in Fig.7 can reduce the nonlinear distortion for the input signal limited to $fs/8$.

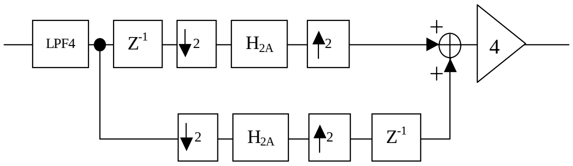

Fig. 6. The form of the Volterra filter using multirate signal processing.

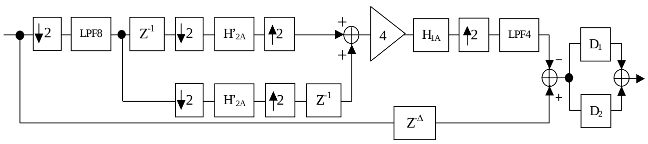

Fig. 7. Block diagram of the proposed nonlinear inverse system.

5. EXPERIMENTAL RESULTS

We input two sinusoidal signals to a loudspeaker system, and compare the output spectra before and after reducing the nonlinear distortion in order to demonstrate the effectiveness of the system form shown in Fig.7. Table 1 shows the simulation condition. Table 2 show the reduction results. In Table 2, the reduction effect means the difference between before and after reduction. HM and IM also mean the harmonic and the intermodulation distortions, respectively. It can be seen from Table 2 that the proposed system form has the same reduction ability as the conventional nonlinear inverse system for any frequency. Next, we compare the computational complexity of the proposed system form with that of the conventional one. The number of multipliers in the conventional system form is represented as

$$2 \times (N_{v2})^2 + N_{v1}. \quad (9)$$

The number of adders in the conventional one is represented as

$$N_{v2}^2 - 1 + N_{v1} - 1. \quad (10)$$

The number of multipliers in the proposed one is represented as

$$\frac{1}{2} \times \left\{ 2 \times \left(\frac{N_{v2}}{4} \right)^2 + \frac{N_{v1}}{2} \right\} + 2 \times N_L. \quad (11)$$

The number of adders in the proposed one is represented as

$$\frac{1}{2} \times \left\{ \left(\frac{N_{v2}}{4} \right)^2 - 1 + \frac{N_{v1}}{2} - 1 \right\} + 2 \times (N_L - 1), \quad (12)$$

where N_{v1} and N_{v2} are the system lengths of the linear inverse filter H_1 and the second-order Volterra filter H_2, respectively. N_L also denotes the system length of the low pass filter LPF4. If the system length of the second-order Volterra filter H_2 is long, then the computational complexity would be determined only by N_{v2}. Thus the proposed system form has about 0.03 times as many computational complexities as the conventional one while having the same reduction performance.

Table 1. Simulation condition

Tap length of unknown system	256
Input voltage	6.0[V]
Tap length of unknown nonlinear system	256×256
Tap length of LPF4,LPF8	256
Tap length of H_{1A}	512
Tap length of H_{2A}	128×128
Sampling frequency	44.1[kHz]

Table 2. Effectiveness of reducing the second order distortion.(f_1=861.33 [Hz], f_2=344.53 [Hz])

	Frequency	Reduction Effect	
		Conventional [dB]	Proposed [dB]
HM	$2f_1$	10.74	11.02
	$2f_2$	29.83	16.80
IM	$f_1 - f_2$	41.72	52.89
	$f_1 + f_2$	33.03	27.84

6. CONCLUSION

In this paper, we have proposed Volterra filters using multirate signal processing, and reduced the computational complexity of the nonlinear inverse system by applying them. By assuming that the second-order Volterra filter and the input signal are limited to $fs/4$ to avoid aliasing in the output signal, we have derived the method for reducing the computational complexity of the second-order Volterra filter. Experimental results have demonstrated the effectiveness of the proposed method. We will investigate the effectiveness in real loudspeaker systems and extend the proposed method to the third-order Volterra filters.

7. REFERENCES

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