VARYING-RADIX NUMERATION SYSTEM AND ITS APPLICATIONS

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ABSTRACT

At present, there are no effective methods for modeling on representing sequence containing position information in some special problems. A novel varying-radix numeration system is proposed to solve this kind of problems. This varying-radix numeration system differs from common fixed radixes numeration systems. The applications of varying-radix numeration system in a First-In-Last-Out (FILO) stack problem and Multi-Pulse Excited Linear Prediction (MPELP) vocoder have illustrated the availability and benefit of this specific numeration system.

1. INTRODUCTION

With the progress of human history, some numeration systems have been developed in order to meet different applications, such as fixed-radix, mixed-radix and mixed-base numeration systems.
In most mathematical and engineering theories researches and applications, fixed-radix numeration systems such as decimal system and binary system are used to represent numbers and calculations. While for historical or conventional reasons, some mixed-radix numeration systems are still in use. For example, when measuring time in hours: tens of minutes: minutes format, people are indeed using different radices for the different digit places: 10 and 6 respectively. Mixed-base numeration system was a more general numeration system in which there are no necessarily integral ratios between the bases of all terms. Definitions of existing numeration systems are as follows [1] (To be short, fraction parts isn't included in discussion.):

- Fixed-radix system: A radix numeration system in which all digit places have the same constant radix. In fixed-radix systems the weights of successive digit places are successive integral power of a single radix, each multiplied by the same factor. The widely used decimal, binary and hexadecimal systems are typical.
- Mixed-radix system: A radix numeration system in which all radices of each digit place are constant but do not all necessarily be the same. Hours: ten of minutes: minutes and Minutes: tens of seconds: seconds were typical.
- Mixed-base system: A numeration system in which a number is represented as the sum of a series of terms. Each of them consists of a mantissa and a base, the base of a given term is constant for a given application, but the bases is such that there are no necessarily integral ratios between the bases of all the terms. Days: tens of hours: hours and Years: tens of month: month are typical. For Days: tens of hours: hours, the bases of three digit places are 24:10:1 when choosing hour as the unit, in which 24/10 isn't an integer.

The above classical numeration systems might be good as a representation method for number. Based on the thinking of radix and base for numeration systems, here we have proposed a new varying-radix numeration system in order to model or represent some special sequence problems related with positions information. The varying-radix numeration system is not aimed at as another representation method for numbers, but as a new model to solve some special problems. A varying-radix system can be defined as a numeration system in which all radixes of digit places are varying according to the expressed number, the construction of varying-radix system and its computational rules are described in detail at second section. The procedure of solving problems with this theory is described in the third section by two applications, which are FILO stock process and locative information quantization in MPELP vocoder

2. RULES OF VARYING-RADIX NUMERATION SYSTEM

In a positional numeration system, a number is represented by an ordered set of characters stream, e.g.: 3221. It is in such a way that the value contributed by a character depends upon its position as well as its value. The contribution to add up value is named as base for certain digit place and usually the bases of a number are in descending magnitudes.
A varying-radix number also can be described as a series of characters: $v(k)v(k-1)v(k-2)...v(i)...v(1)$. The so-called varying-radix means the radixes of each position in the above stream are not fixed. Instead they are depended on specific rules for special application. For the *i*th position of a varying-radix number, we define *v(i)* as the value and *o(i)* as the radix. All the *o(i)*s and *v(i)*s are integers.
In order to define a varying-radix numeration system exactly in different applications, we have tried to find an optimum representation for radix numbers. Here we have proposed a universal computation rule for varying radix, as following equation. One can also refer to those application specific representations shown in the next section.

$$o(i) = p(i) + 1 - \sum_{j=i+1}^{k} v(j) \qquad i = 1 \ldots k \tag{1}$$

Where p(i) is an integer number which is decided according to special application. For example, if there are six position of a varying-radix number and p(i) is a constant number 24, then equation (1) could be rewritten as:

$$o(i) = 25 - \sum_{j=i+1}^{6} v(j) \qquad i = 1 \ldots 6 \tag{2}$$

In decimal numeration system, number is counted continuously 1,2,3… , varying-radix numeration system can be explained in the same way, as shown in Fig.1. When counting varying-radix numbers, keep Equation (2) in mind to calculate correct radixes of different digit positions. A carry is caused when the value of a position reaches its radix.

Decimal	Varying-Radix number						Radixes of different positions					
Index	v(6)	v(5)	v(4)	v(3)	v(2)	v(1)	o(6)	o(5)	o(4)	o(3)	o(2)	o(1)
0	0	0	0	0	0	0	25	25	25	25	25	25
1	0	0	0	0	0	1	25	25	25	25	25	25
2	0	0	0	0	0	2	25	25	25	25	25	25
.		.					.					
.		.					.					
.		.					.					
24	0	0	0	0	0	24	25	25	25	25	25	25
25	0	0	0	0	1	0	25	25	25	25	25	24
26	0	0	0	0	1	1	25	25	25	25	25	24
.		.					.					
.		.					.					
.		.					.					
48	0	0	0	0	1	23	25	25	25	25	25	24
49	0	0	0	0	2	0	25	25	25	25	25	23
.		.					.					
.		.					.					

Fig.1. Varying-radix Number Lists

It seems that this varying- radix numeration system has never been heard of before and is strange, however it is really an effective method to solve some problems, as can be seen below.

3. TWO APPLICATIONS OF USING VARYING-RADIX NUMERATION SYSTEMS

In this section, the proposed varying-radix numeration system reveals its computational effectiveness and excellence in special field. Two typical application of varying-radix numeration system are shown and discussed in details.

3.1 The model of random FILO stack

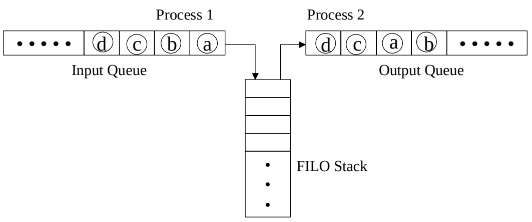

Fig.2. Random FILO stack

Consider a simple random stack model , as shown in Fig.2: A queue of variables are waiting for being pushed into Fist-In-Last-Out(FILO)[2] Stack by process 1 with random time. Another process, process 2 is to pop up the variables from FILO stack at random time. The question is how to model the FILO system and give all status for output queue.

Usually, one would solve it by try-and-error simulation, which is time-consuming. Here we take the following steps. At first, this model is described by binary digits series. "1" stands for pushing a variable into the stack; "0" stands for popping up a variable from the stack. Therefore there has one-to-one correspondence between stack status and a series of 0-1 digits. For example, the series of digits in Equation (3) stands for a output queue status of (a) (c) (b) (d)….

$$C = 1011001011\ \ 10\cdots \tag{3}$$

Obviously the first digit of C is 1(no push, no pop). The push-pop of stack is described in an operable way. Take a four-variables case; there all totally four "1", while the numbers of "0" between two"1" is vary. The series of binary can also be described as following way:

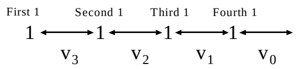

Fig.3. Series Representation of series of 0-1 digits

In which v_i is the number of "0" between two adjacent "1". The sequence of v_i is corresponding to the switching process. And we can get a set of equations and inequations as follows:

$$\begin{aligned} &\sum_{i=0}^{3} v_i = 4 \\ &v_3 \le 1 \\ &v_2 \le 2 - v_3 \\ &v_1 \le 3 - v_2 - v_3 \\ &v_0 = 4 - v_1 - v_2 - v_3 \end{aligned} \tag{4}$$

As v_0 can be calculated from the rest, we can use a three-digit varying-radix number $v_3v_2v_1$ to represent this sequence. The radixes of different positions are o(3) = 2 | o(2) = 3-v_3 | o(1) = 4-v_2-v_3. Refer to equation (1), p(i)=4–i and v(i)=v_i.

We can solve this random FILO problem by counting a varying-radix number from 0 to the possible maximum. When counting numbers one by one, we will get desired sequences and maximum number of output sequence. the following Fig.4 show there are altogether 14 output queue status exist.

Decimal	Varying-radix number			Radixes of different positions		
Index	v(3)	v(2)	v(1)	o(3)	o(2)	o(1)
0	0	0	0	2	3	4
1	0	0	1	2	3	4
2	0	0	2	2	3	4
3	0	0	3	2	3	4
4	0	1	0	2	3	3
5	0	1	1	2	3	3
6	0	1	2	2	3	3
7	0	2	0	2	3	2
8	0	2	1	2	3	2
9	1	0	0	2	2	3
10	1	0	1	2	2	3
11	1	0	2	2	2	3
12	1	1	0	2	3	2
13	1	1	1	2	2	2

Fig.4. Random FILO Model Series List

As shown, the proposed varying-radix numeration system gives a simple solution for the status of random FILO problem with little computational and memory costs.

3.2 Positional Information Representation of MPELP Speech Codec

In the Multi-Pulse Excited Linear Prediction (MPELP) speech codec, an excitation sequence, which consists of multiple non-uniformly spaced pulses, is transmitted as residual signal. In the vocoder both the amplitude and position of the pulses are determined sequentially one pulse at a time during analysis. The MPELP algorithm typically uses 4-6 pulses every 5ms. Excitation coding in the MPELP algorithm is more expensive than in the classical linear predictive vocoders, because MPELP codes both the amplitudes and the position of the pulses. The proposed varying-radix numeration system can represent positional information effectively other than table looking-up. We use high rate excitation Multipulse Maximum Likelihood Quantization (MP-MLQ), which is part of ITU G.723 [3] as an example.

In G.723 MP-MLQ vocoder, the residual signal $\{r[n]\}_{n=0..59}$ is transferred as a new vector to the MP-MLQ block. This block performs the quantization of this vector. The quantization process is approximating the target vector $r[n]$ by $r'[n]$:

$$r'[n] = \sum_{j=0}^{n} h[j] \cdot v[n-j], \quad 0 \le n \le 59 \tag{5}$$

Where $v[n]$ is the excitation to the combined filter $S[z]$ with impulse response $h[n]$ and define as:

$$v[n] = G \sum_{k=0}^{M-1} \alpha_k d[n - m_k], \quad 0 \le n \le 59 \tag{6}$$

Where G is the gain factor, $\{\alpha_k\}_{k=0..M-1}$ and $\{m_k\}_{k=0..M-1}$ are the signs (± 1) and the positions of Dirac functions respectively and M is the number of pulses, which is 6 for even subframes and is 5 for odd subframes. There is a restriction on pulse positions. The positions can be either all odd or all even.

To represent $\{m_k\}_{k=0..M-1}$ in a compressed and easy-to-use way, the optimal pulse positional information can be obtained and transmitted. It is obvious that there are altogether $\binom{30}{M}$ different positional distributions; here M may be 5 or 6. Take M as 6, $2^{19} \le \binom{30}{6} \le 2^{20}$ indicates that 20 bits is enough to represent position information, so using 30 bits without compression will waste 10 bits every frame. In this case a compact varying-radix number $v(M)v(M-1)...v(1)$ can be used to denote an excitation sequence $\underset{v(0)\ v(1)\ ....\ v(M)}{\overset{00100001......1000}{\uparrow \quad \uparrow \quad ...... \quad \uparrow}}$. In which $v(i)$ is the number of "0" bits between "1" (pulse) bits. Because of $\sum_{i=0}^{M} v(i) = N - M$ (N=30 is the total number of points in the subframe and M is number of Direc functions in a subframe), clearly, $v(0) = N - M - \sum_{i=1}^{M} v(i)$, and $v(M)v(M-1)...v(1)$ can represent location information exactly.

It is the constraint condition $0 \le \sum_{i=1}^{1\sim M} v(i) \le N - M$ that enlightens us to use varying-radix numeration system with radixes as $o(i) = N - M + 1 - \sum_{j=i+1}^{M} v(j)$, thus the desired position quantization can be realized. Next it is necessary to find out an effective way to transform $v(M)v(M-1)...v(1)$ sequence to decimal sequence, which can be easily transferred to binary stream and transmitted by computers. That is to find a function $f(o,b,v)$ as the corresponding decimal number of the varying-radix number with radix o, digit position b and value v in that position.

For above example, we can state the conversion from varying-radix numbers to decimal numbers as:

$$v(6)v(5)v(4)v(3)v(2)v(1) = f(25,6,v(6)) + f(25 - v(6),5,v(5)) + f(25 - v(6) - v(5),4,v(4)) + ... + f(25 - v(6) - v(5) - v(4) - v(3) - v(2),1,v(1)) \tag{7}$$

A general expression is:

$$v(M)v(M-1)v(M-2)......v(1) = \sum_{i=i}^{M} f(N - M + 1 - \sum_{j=i+1}^{M} v(j), i, v(i)) \tag{8}$$

Equation (8) gives a computational transformation model from varying-radix number to decimal number. A computable conversion processing is derived as follows.

Consider a varying-radix number $\underset{b \quad b-1 \; b-2 \qquad 2 \; 1}{\overset{v-1 \vdots o-v \vdots 0 \vdots 0 \vdots .... \vdots 0 \vdots 0}{\uparrow \quad \uparrow \quad \uparrow \qquad \uparrow \uparrow}}$, in which o is the radix of the bth position. So the radix for the (b-1)th position is o-v+1. Adding this number with 1, we get:

$$f(o,b,v) = f(o,b,v-1) + f(o-v+1,b-1,o-v) + 1 \tag{9}$$

Performing successive recursion, we have found a simple and useful equation:

$$f(o,b,v) = v + \sum_{p=0}^{v-1} f(o-p,b-1,o-p-1) = \sum_{p=0}^{v-1} \binom{o-p+b-2}{b-1} \tag{10}$$

Finally we can get the conversion method from varying-radix number to decimal number as:

$$v(M)v(M-1)v(M-2)......v(2)v(1) = \sum_{i=1}^{M} f(N - M + 1 - \sum_{j=i+1}^{M} v(j), i, v(i)) = \sum_{i=1}^{M} \sum_{p=0}^{v(i)-1} \binom{N - M + 1 - \sum_{j=i+1}^{k} v(j) - p + i - 2}{i-1} \tag{11}$$

Using (11), we can calculate decimal number from the corresponding varying-radix number directly. Of course during implementation, binary representation should be transformed from this decimal number. The conversion method from decimal number to varying-radix number is with a similar process.

The above varying-radix numeration system gives a compact and easy-to-use expression of pulse positional information for MPELP speech coding. Similar applications of varying-radix numeration system are also effective in other data compression.

4. CONCLUSION

In this paper, a novel varying-radix numeration system has been proposed for modeling and solving some problems with positional information. The radixes at different digit positions of a number are varied in different conditions. According to the proposed positional varying-radix rules, effective models and algorithms are developed for two special examples, which proved its uniqueness and availability. This new numeration system is expected to be helpful for other similar scientific and engineering applications.

5. REFERENCE

[1] Standard Terminology, ISO/IEC 2382.

[2] William Ford and William Topp, Data Structure with C++, Prentice Hall Inc. 1996.

[3] ITU-U Draft Recommendation of G.723-Dual Rate Speech Coder for Multimedia Communication Transmission at 5.3 & 6.3 kbits/s.