COMPUTATIONALLY EFFICIENT DOA ESTIMATION BASED ON LINEAR PREDICTION WITH CAPON METHOD

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ABSTRACT

Of the several methods for estimating the direction of multiple signals with an array antenna, superresolution direction-of-arrival (DOA) estimation techniques, such as MUSIC and ESPRIT, have been in the spotlight. Although the performance of these techniques is reliable, their computational costs are considerable.

We propose a new DOA estimation technique using the linear prediction (LP) method in conjunction with the Capon method. In our proposed technique, the LP method is used to estimate the true and spurious DOAs, and the true DOAs can be selected by evaluating the relative signal powers obtained by Capon method. To estimate the number of true DOAs, we apply the values of Capon's array output power to the decision criterion, such as minimum description length (MDL). Simulation results showed that the proposed technique gives a maximum of about eighty-percent in computational cost reduction compared with MUSIC and that the technique accurately estimated the DOAs.

1. INTRODUCTION

Wireless communication systems having intelligent base stations equipped with array antennas have been studied in recent years to find ways to make efficient use of the frequency band. It has been found that array signal processing has a lot of merits, such as interference canceling, adaptive beamforming, direction-of-arrival (DOA) estimation, time-of-arrival (TOA) estimation, and space diversity[1]-[3]. DOA estimation is a particularly important technique that can be used to determine the signal angle profile and to form the most suitable antenna directivity in a multipath environment.

The superresolution DOA estimation schemes based on the eigenvalue decomposition, such as MUSIC and ESPRIT, have been proposed up to now[4, 5], have significantly high computational costs. Thus, the construction of a digital signal processor (DSP) unit in an intelligent base station is by no means an economical matter. Future wireless communication systems will require computationally efficient DOA estimation schemes in order to reduce the costs of hardware.

We propose a new DOA estimation technique that does not require the eigenvalue decomposition. The proposed technique uses the merits of both the LP and Capon methods[6, 7]. After the problem statement presented in Section 2, we briefly explain the DOA estimation procedure used in the LP method in Section 3. In Section 4, we explain how to detect only the true DOAs from the estimated DOAs using the Capon method. We also present in this section another estimation procedure used when the antenna elements of the array are uniformly spaced. The simulation results of detection probability of the number of signals and root mean square error (RMSE) of a DOA estimate are presented in Section 5. In addition, we also present a comparison of computational cost for our proposed technique and MUSIC.

2. PROBLEM STATEMENT

We consider a linear array of M isotropic antenna elements, as shown in Fig. 1, and N $(N < M)$ narrowband signals $\{s_i(t)\ ;\ i = 1, \cdots, N\}$, which are zero mean and mutually uncorreated, impinging from distinct unknown directions $\{\theta_i\}$. Then, the array observation vector $\boldsymbol{y}(t)$ can be expressed as

$$\begin{aligned} \boldsymbol{y}(t) &= \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \\ &= [y_1(t), \cdots, y_M(t)]^T, \end{aligned} \tag{1}$$

where

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_N)] \tag{2}$$

$$\boldsymbol{s}(t) = [s_1(t), \cdots, s_N(t)]^T \tag{3}$$

$$\boldsymbol{n}(t) = [n_1(t), \cdots, n_M(t)]^T. \tag{4}$$

In the above equations, $\boldsymbol{A}$ is a matrix of the steering vectors, $\boldsymbol{s}(t)$ is a signal vector, $\boldsymbol{n}(t)$ is an additive white noise vector which is uncorrelated with $\boldsymbol{s}(t)$, and $\boldsymbol{a}(\theta_i)$ is a steering

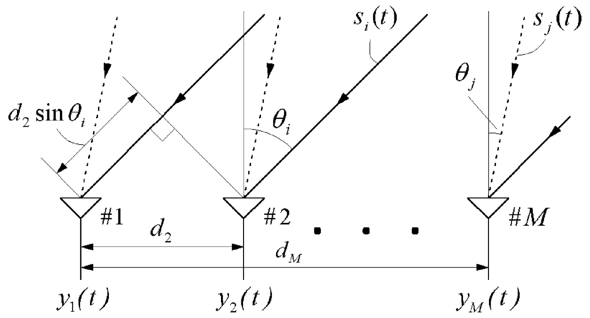

Fig. 1. Array antenna geometry.

vector in the direction θ_i expressed as

$$\boldsymbol{a}(\theta_i) = \left[1, e^{jkd_2 \sin\theta_i}, \cdots, e^{jkd_M \sin\theta_i}\right]^T, \tag{5}$$

where $k = 2\pi/\lambda$ is the phase constant, λ is the signal wavelength, d_i is the distance between the first antenna element and the i-th antenna element, and the superscript T denotes the transpose.

3. DOA ESTIMATION

The merits of the LP method are its high-resolution spatial spectrum and low computational cost. Here, we briefly describe the estimation procedure.

For the array observation vector, we can form a covariance matrix

$$\begin{aligned} \boldsymbol{R} &= E\left[\boldsymbol{y}(t)\boldsymbol{y}^H(t)\right] \\ &= \boldsymbol{A}E\left[\boldsymbol{s}(t)\boldsymbol{s}^H(t)\right]\boldsymbol{A}^H + E\left[\boldsymbol{n}(t)\boldsymbol{n}^H(t)\right] \\ &= \boldsymbol{ASA}^H + \sigma^2\boldsymbol{I}_M, \end{aligned} \tag{6}$$

where $E[\cdot]$ denotes the expectation operator, superscript H denotes the Hermitian transpose, $\boldsymbol{S}$ is a signal covariance matrix, σ^2 is the noise variance, and $\boldsymbol{I}_M$ is the $M \times M$ identity matrix. In practice, $\boldsymbol{R}$ is unknown and an estimate $\hat{\boldsymbol{R}}$ is obtained from L snapshots of $\boldsymbol{y}(t)$ and given by

$$\hat{\boldsymbol{R}} = \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{y}(l)\boldsymbol{y}^H(l). \tag{7}$$

Let $\boldsymbol{w}_{LP}$ denote the weight vector in the LP method: it can be written as

$$\boldsymbol{w}_{LP} = \frac{\boldsymbol{R}^{-1}\boldsymbol{T}}{\boldsymbol{T}^H\boldsymbol{R}^{-1}\boldsymbol{T}} = [1, w_2, \cdots, w_M]^T, \tag{8}$$

where

$$\boldsymbol{T} = [1, 0, \cdots, 0]^T \tag{9}$$

is the $M \times 1$ constant vector. Using $\boldsymbol{w}_{LP}$ and the mode vector $\boldsymbol{a}(\theta)$, the spatial spectrum $P_{LP}(\theta)$ is defined by

$$P_{LP}(\theta) = \frac{1}{\left|\boldsymbol{w}_{LP}^H\, \boldsymbol{a}(\theta)\right|^2}. \tag{10}$$

Note that the $P_{LP}(\theta)$ has K $(N < K \leq M-1)$ spectrum peak directions $\{\theta_i\ ;\ i = 1, \cdots, K\}$ that include N true signal directions and $K - N$ spurious directions. Estimating the number of signals is important in practice.

4. ESTIMATION OF THE NUMBER OF SIGNALS

We estimate the number of signals using the Capon method, which is based on the minimum variance technique. Capon's spatial spectrum $C(\theta)$ is given by

$$C(\theta) = \frac{1}{\boldsymbol{a}^H(\theta)\boldsymbol{R}^{-1}\boldsymbol{a}(\theta)}. \tag{11}$$

Although the resolution of $C(\theta)$ is not so high and the computational cost is high, $C(\theta)$ is in proportion to the array output power and its dynamic range is nearly the product of the maximum signal-to-noise ratio (SNR) of the impinging signals and M. Thus, $C(\theta_i)$ $(\equiv p_i)$ is considered to be the relative power of a signal impinging from θ_i. For K spectrum peak directions of $P_{LP}(\theta)$, $\{p_i\}$ is given by

$$p_i = C(\theta_i) = \frac{1}{\boldsymbol{a}^H(\theta_i)\boldsymbol{R}^{-1}\boldsymbol{a}(\theta_i)}, \quad \text{for} \quad i = 1, \cdots, K. \tag{12}$$

When the number of signals is N, $\{p_i\}$ have the following relation:

$$p_1 \geq \cdots \geq p_N > p_{N+1} = \cdots = p_K = \sigma^2. \tag{13}$$

From (13) , we can estimate N immediately, however, it is unadvisable to determine N by comparing $\{p_i\}$ to a threshold value when the SNR is low. Generally, an objective judgement criterion, such as Akaike information criteria (AIC) or minimum description length (MDL), is used for the estimation of N[8]. Since rank$(\boldsymbol{ASA}^H)$ is N, the eigenvalues $\{\lambda_i\ ;\ i = 1, \cdots, M\}$ of $\boldsymbol{R}$ have the following relation:

$$\lambda_1 \geq \cdots \geq \lambda_N > \lambda_{N+1} = \cdots = \lambda_K = \cdots = \lambda_M = \sigma^2. \tag{14}$$

Here, we describe the relation between $\{p_i\}$ and $\{\lambda_i\}$. $\boldsymbol{E}_S = [\boldsymbol{e}_1, \cdots, \boldsymbol{e}_N]$ denotes the signal subspace matrix and $\{\boldsymbol{e}_i\ ;\ i = 1, \cdots, N\}$ are the signal eigenvector. Since $\boldsymbol{E}_s$ spans the same signal subspace as $\boldsymbol{A}$, there must exist a unique nonsingular $N \times N$ matrix $\boldsymbol{Q}$, that is

$$\boldsymbol{A} = \boldsymbol{E}_S\boldsymbol{Q} \tag{15}$$

or

$$\boldsymbol{a}(\theta_i) = \sum_{n=1}^{N} q_{ni}\boldsymbol{e}_n \ , \tag{16}$$

where $\{q_{ij} \ ; \ i,j = 1,\cdots,N\}$ are elements of $\boldsymbol{Q}$. Using $\{\lambda_i\}$ and $\{\boldsymbol{e}_i\}$, $\boldsymbol{R}^{-1}$ can be rewritten as

$$\boldsymbol{R}^{-1} = \sum_{m=1}^{M} \frac{1}{\lambda_m}\boldsymbol{e}_m\boldsymbol{e}_m^H. \tag{17}$$

Therefore, from (12), (16) and (17)

$$\begin{aligned}
\frac{1}{p_i} &= \boldsymbol{a}^H(\theta_i)\boldsymbol{R}^{-1}\boldsymbol{a}(\theta_i) \\
&= \left(\sum_{n=1}^{N} q_{ni}^*\boldsymbol{e}_n^H\right)\left(\sum_{m=1}^{M} \frac{1}{\lambda_m}\boldsymbol{e}_m\boldsymbol{e}_m^H\right)\left(\sum_{n=1}^{N} q_{ni}\boldsymbol{e}_n\right) \\
&= \sum_{n=1}^{N} \frac{|q_{ni}|^2}{\lambda_n} \ , \qquad (18)
\end{aligned}$$

that is

$$\boldsymbol{p}^{-} = (\boldsymbol{Q}^* \odot \boldsymbol{Q})^T \boldsymbol{\lambda}^{-}, \tag{19}$$

where superscript * denotes the complex conjugate, $\odot$ denotes the Hadamard product, and

$$\begin{aligned}
\boldsymbol{p}^{-} &= [1/p_1,\cdots,1/p_N]^T \qquad (20) \\
\boldsymbol{\lambda}^{-} &= [1/\lambda_1,\cdots,1/\lambda_N]^T. \qquad (21)
\end{aligned}$$

In the derivation of (18) , we used

$$\boldsymbol{e}_i^H\boldsymbol{e}_j = \begin{cases} 1 & \text{if} \ \ i=j \\ 0 & \text{otherwise} \end{cases} . \tag{22}$$

Although (18) presents the exact relation between $\{p_i\}$ and $\{\lambda_i\}$, from (13) and (14) we assume that $\{p_i\}$ and $\{\lambda_i\}$ have approximately the same distribution. If we select MDL as the decision criterion, it is given by using $\{p_i\}$ instead of $\{\lambda_i\}$

$$\begin{aligned}
\mathrm{MDL}(\hat{N}) &= -2\ln\left(\frac{\prod_{i=\hat{N}+1}^{K} p_i^{\frac{1}{K-\hat{N}}}}{\frac{1}{K-\hat{N}}\sum_{i=\hat{N}+1}^{K} p_i}\right)^{L(K-\hat{N})} \\
&+ \hat{N}(2K-\hat{N})\ln L \\
&\quad \text{for} \quad \hat{N} = 1,\cdots,K-1 \ , \qquad (23)
\end{aligned}$$

where $\hat{N}$ is the possible number of signals. The number of signals is determined as the value of $\hat{N}$ that minimizes the MDL. From the relation between (12) and (13) , the estimates of the true DOAs $\{\hat{\theta}_i \ ; \ i = 1,\cdots,\hat{N}\}$ are obtained.

As a special case, we consider a uniformly spaced linear array antenna where the distance between two adjacent elements is d. Then, the mode vector expressed in (5) can be rewritten as

$$\begin{aligned}
\boldsymbol{a}(\theta) &= \left[1, e^{jkd\sin\theta}, \cdots, e^{j(M-1)kd\sin\theta}\right]^T \\
&= \left[1, z, \cdots, z^{M-1}\right]^T \equiv \boldsymbol{f}(z) \ , \qquad (24)
\end{aligned}$$

where $z = e^{jkd\sin\theta}$. From (10) and (24) , the direction search computation can be replaced with the computation of polynomial root of the $F(z)$:

$$\begin{aligned}
F(z) &= \boldsymbol{w}_{LP}^H \, \boldsymbol{f}(z) \ \left(\equiv \boldsymbol{w}_{LP}^H \, \boldsymbol{a}(\theta)\right) \\
&= 1 + w_2^* z + \cdots + w_M^* z^{M-1} \\
&= 0. \qquad (25)
\end{aligned}$$

Let $\{z_i \ ; \ i = 1,\cdots,M-1\}$ denote the roots of $F(z)$, the spectrum peak directions, which includes N signal diretions and $M-1-N$ spurious directions, are obtained from

$$\hat{\theta}_i = \sin^{-1}\left\{\frac{\arg(z_i)}{kd}\right\} \quad \text{for} \quad i = 1,\cdots,M-1 \ , \tag{26}$$

where $\arg(z_i)$ denotes the argument of complex number z_i. Then, we can get $\hat{N}$ and $\{\hat{\theta}_i\}$ using (12) and (23) . Note that we must replace K in (12) and (23) with $M-1$.

5. SIMULATION RESULTS

In the simulation, a uniformly spaced linear array antenna with eight elements separated by half a wavelength ($d = 0.5\lambda$) was used. Three signals with DOAs $\theta_1 = -60°$, $\theta_2 = 10°$, and $\theta_3 = 30°$ were present. Each signal was modulated using 16QAM, and the number of snapshots was 100 ($L = 100$). The angle step of the direction search computation was 0.1 degrees. Simulation results were obtained after averaging over 10000 trials.

Figure 2 shows the detection probability of the exact number of signals, when the SNR of $s_3(t)$ varied from 14 to -4 dB and MDL (23) was used as the decision criterion. Through the simulation, SNR of $s_1(t)$ and $s_2(t)$ were 17 and 20 dB, respectively. In this figure, 'Proposed' and 'Eigenvalue' in the legend correspond to the values of MDL when $\{p_i\}$ and $\{\lambda_i\}$ was used, respectively. This result indicates that the signal detection ability was perfect on the condition that the SNR was larger than -2 dB.

Figure 3 shows the RMSE of estimate $\hat{\theta}_3$. The RMSE obtained by the proposed technique was almost the same as that of MUSIC.

Figure 4 shows the results of an investigation of the number of computations using the same parameters as the preceding example except that the number of signals varied from 1 to 7. The main feature of the proposed technique is that the number of computations is constant even if the number of signals varies. In MUSIC, computational cost

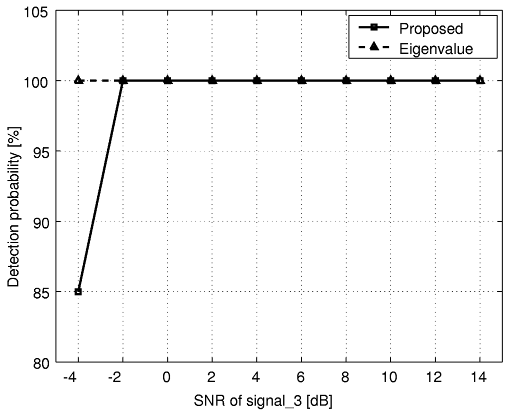

Fig. 2. Comparison of detection probability of N.

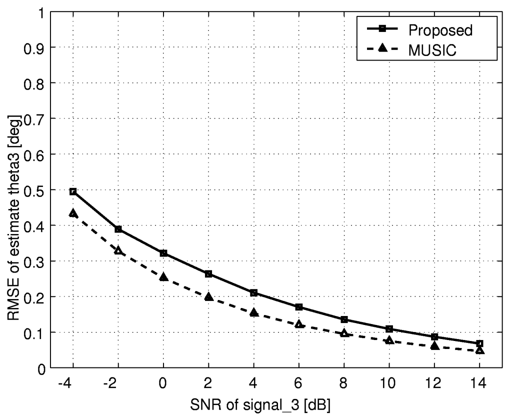

Fig. 3. Comparison of RMSE of estimate $\hat{\theta}_3$.

depends on the number of signals which affects the direction search computation. The computational cost of our proposed technique is significantly low when compared with that of MUSIC. The eigenvalue decomposition and the matrix computation in the direction search cause the difference of computational costs between both methods.

6. CONCLUSION

We have proposed a computationally efficient DOA estimation technique that is based on using the LP method with the Capon method. Since our proposed technique combines the high-resolution spatial spectrum of the LP method with the reliable array output power of the Capon method, it can estimate not only DOAs but also the number of signals. The estimation accuracy of our technique is almost the same as that of MUSIC. Therefore, it would be useful for the miniaturization and cost reduction of the DSP unit in the base station because of its significantly low computational

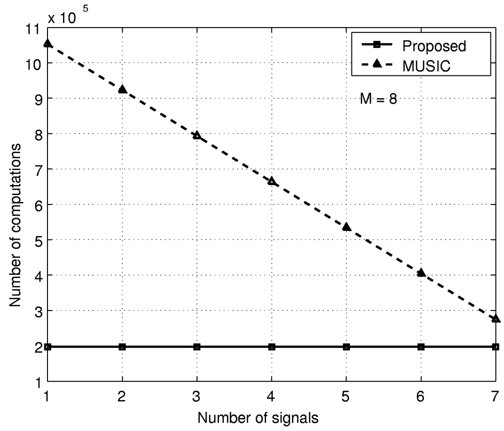

Fig. 4. Comparison of the number of computations.

cost. Now, we are planning to extend this technique into the tracking of mobile terminals for the new generation of mobile communication.

7. REFERENCES

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