A ROBUST KALMAN FILTER DESIGN FOR IMAGE RESTORATION

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ABSTRACT

In image deconvolution or restoration using Kalman filter, the image and blur models are required to be known for the restoration process. Generally, the accuracy of the restoration depends on the accuracy of the given models. Unfortunately, the image and blur models are normally unknown in practice. To solve the problem, an identification stage is employed to estimate the image and blur models. However, the estimated models are seldom accurate especially with the presence of noise in the image. This paper presents a robust Kalman filter design for image deconvolution that can accommodate the inaccuracy in the estimated image and blur models. If the inaccuracy can be modelled as addictive white Gaussian noise with a known variance, it can be stochastically account for in the robust filter design. In the simulation tests performed, the robust design achieved improved accuracy in the image restoration even though inaccurate image and blur models were used.

1. INTRODUCTION

In the process of blind image deconvolution or restoration, the true image is being estimated from a degraded observation only. Blind image deconvolution techniques can be broadly categorized into 2 groups. The first group involved estimating the blur and true image concurrently. Recent techniques belonging to the first group include the expectation maximization (EM) algorithm [1], the iterative blind deconvolution (IBD) algorithm [2], McCallum's simulated annealing (SA) algorithm [3] and the non-negativity and support constraints recursive inverse filtering (NAS-RIF) [4]. Most of them employ some form of optimisation to estimate the true image among various unknown parameters of the image.

In comparison to the first group, the second group performs the image parameters identification and true image estimation in two separate processes. The estimation (or restoration) stage generally employs a classical deconvolution method such as Kalman [5-8], Wiener or least-squares filtering. These classical deconvolution methods require a priori information on the image, like noise statistics, image and blur models, which are to be first identified in an identification stage. Recent identification methods make use of the maximum likelihood (ML) [9], generalized cross-validation (GCV) [10] or residual spectral matching technique [11]. Accurate image and blur models are essential for accurate image restorations. However, making accurate estimations on the models are difficult especially with the presence of noise in the image. To counter this problem, this paper presents a robust Kalman filtering design that can minimize the effects from the inaccurate image and blur models by factoring the inaccuracies in the models. The inaccuracies or errors in the models are modelled as addictive white Gaussian noise.

The spatial correlation between neighbouring pixels in true image can be represented by the Gauss-Markov model. The image process is modelled as an autoregressive (AR) process driven by a white Gaussian noise process w(i,j) with variance σ_w^2 described by:

$$s(i,j) = \sum_{m,n \in Ra} a(m,n)s(i-m, j-n) + w(i,j) \qquad (1)$$

where s(i,j) indicates the pixel intensity level at column i and row j and Ra represents the non-symmetric half-plane (NSHP) support model. The NSHP support specifies the extent of pixel correlation within a region. An illustration on the NSHP support can be found in Figure 1. a(m,n) are the MSE image model coefficients which minimize $E\{w(i,j)\}^2$. Alternatively, this image system can be seen as a recursive model with a modelling uncertainty defined by σ_w^2. The blurred and noisy pixel, r(i,j), is defined by the linear degradation formation system:

$$r(i,j) = \sum_{m,n \in Rh} h(m,n)s(i-m, j-n) + v(i,j) \qquad (2)$$

where h(m,n) is the space-invariant PSF with support Rh and v(i,j) represents the observation noise with variance σ_v^2. In the case of inaccurate image and blur models, Equation (1) and (2) become (3) and (4) respectively, as shown:

$$s(i,j) = \sum_{m,n \in Ra} [a(m,n) + \zeta(m,n)]s(i-m, j-n) + w(i,j) \qquad (3)$$

$$r(i,j) = \sum_{m,n \in Rh} [h(m,n) + \eta(m,n)]s(i-m, j-n) + v(i,j) \qquad (4)$$

where w(i,j), ζ(i,j), ν(i,j) and η(i,j) are mutually independent white noise .

This paper starts with the formulation of the Reduced Order Model Kalman Filter (ROMKF) for the image deconvolution problem in Section 2. Section 3 covers the development and formulation of the proposed robust Kalman filter design. Finally, details on experiments and restoration results are given in Section 4.

2. REDUCED ORDER MODEL KALMAN FILTER (ROMKF)

In the state representation, the size of the state vector is directly proportional to the width of the image. Hence, a large image will result in a very large state vector that is a computational burden. The reduced update Kalman filter (RUKF) is devised to reduce the heavy computational load posed by a large state vector [6]. In the RUKF, only the local states are updated due to the local extent of the correlation exhibited by most images. Hence, the gains outside the local states are assumed to be zero.

In comparison to the RUKF, the reduced order model Kalman filter (ROMKF) only includes the local states in its state vector. Thus, the ROMKF has a much smaller and manageable state vector. In term of performance, the both types of filters are found to be on par with each other [7]. The state and observation equations of the ROMKF are:

$$x(i, j) = Ax(i-1, j) + Bw(i, j) + Eu(i, j) \tag{5}$$

$$y(i, j) = Hx(i, j) + v(i, j) \tag{6}$$

where A is the transition matrix, H is the blur matrix and B and E are the selection matrices. u(i,j) is the input that includes the most recent estimates of states, which are not found in the last state vector. The states are missing due to the ROM approximation.

3. PROBLEM FORMULATION FOR ROBUST KALMAN FILTER

In the deconvolution using Kalman filter, A (image model) and H (blur model) matrices are to be estimated beforehand. The inaccuracies in the estimates are detrimental to the deconvolution process. However, if the statistics of the uncertainties in the model estimates are known, the effects of the uncertainties can be minimized by employing a robust filtering design. In the robust design, the random uncertainties in the image and blur models are represented by $\Delta A\zeta(i,j)$ and $\Delta H\eta(i,j)$ respectively in the state and observation equations as shown below:

$$x(i, j) = [A + \Delta A\zeta(i, j)]x(i-1, j) + Eu(i, j) + Bw(i, j) \tag{7}$$

$$y(i, j) = [H + \Delta H\eta(i, j)]x(i, j) + v(i, j) \tag{8}$$

where w(i,j), ζ(i,j), ν(i,j) and η(i,j) are mutually independent white noise with covariance matrixes Qw, $Q\zeta$, $Q\nu$ and $Q\eta$ respectively. ζ (i,j) and η(i,j) are assumed to be scalars.

By rewriting (7), we have,

$$\begin{aligned} x(i, j) &= Ax(i-1, j) + Eu(i, j) + \\ &\quad \Delta A\zeta(i, j)x(i-1, j) + Bw(i, j) \\ &= Ax(i-1, j) + Eu(i, j) + w_a(i, j) \end{aligned} \tag{9}$$

where

$$w_a(i, j) = \Delta A\zeta(i, j)x(i-1, j) + Bw(i, j) \tag{10}$$

Since ζ (i,j) is uncorrelated to w(i,j), $w_a(i,j)$ is also white noise. The covariance matrix Q_{wa} of $w_a(i,j)$ can be calculated as

$$\begin{aligned} Q_{wa} &= \overline{E}\{w_a(i, j)w_a^T(i, j)\} \\ &= \overline{E}\{[\Delta A\zeta(i, j)x(i-1, j) + Bw(i, j)]. \\ &\quad [\Delta A\zeta(i, j)x(i-1, j) + Bw(i, j)]^T\} \\ &= \Delta A\overline{E}\{x(i-1, j)x^T(i-1, j)\}\Delta A^T Q_\zeta + BQ_w B^T \\ &= \Delta AX(i-1, j)\Delta A^T Q_\zeta + BQ_w B^T \end{aligned} \tag{11}$$

where $\overline{E}\{.\}$ denotes mathematical expectation and

$$X(i-1, j) = \overline{E}\{x(i-1, j)x^T(i-1, j)\} \tag{12}$$

To find X(i-1,j), we have to first derive X(i,j) from (9), i.e.

$$\begin{aligned} X(i, j) &= \overline{E}\{[Ax(i-1, j) + Eu(i, j) + w_a(i, j)]. \\ &\quad [Ax(i-1, j) + Eu(i, j) + w_a(i, j)]^T\} \\ &= AX(i-1, j)A^T + Eu(i, j)u^T(i, j)E^T + Q_{wa} \\ &= AX(i-1, j)A^T + Eu(i, j)u^T(i, j)E^T + \\ &\quad \Delta AX(i-1, j)\Delta A^T Q_\zeta + BQ_w B^T \end{aligned} \tag{13}$$

Hence, by recursion,

$$\begin{aligned} X(i-1, j) &= AX(i-2, j)A^T + \\ &\quad Eu(i-1, j)u^T(i-1, j)E^T + \\ &\quad \Delta AX(i-2, j)\Delta A^T Q_\zeta + BQ_w B^T \end{aligned} \tag{14}$$

Following the same steps for the state equation, we first rewrite the observation equation in (8) as

$$\begin{aligned} &y(i, j) = Hx(i, j) + v_a(i, j) \\ &\textit{where} \;\; v_a(i, j) = \Delta H\eta(i, j)x(i, j) + v(i, j) \end{aligned} \tag{15}$$

and $v_a(i,j)$ is white noise with a covariance matrix

$$\begin{aligned} Q_{va} &= \overline{E}\{v_a(i, j)v_a^T(i, j)\} \\ &= \Delta HX(i, j)\Delta H^T Q_\eta + Q_V \end{aligned} \tag{16}$$

By using the results in (11), (13), (14) and (16), the robust Kalman filter can be implemented with the following Kalman equations:

$$\hat{x}_b(i, j) = A\hat{x}_a(i-1, j) + Eu(i, j) \tag{17}$$

$$P_b(i, j) = AP_a(i-1, j)A^T + Q_{wa} \tag{18}$$

$$K(i, j) = P_b(i, j)H^T[HP_b(i, j)H^T + Q_{va}]^{-1} \tag{19}$$

$$\hat{x}_a(i, j) = \hat{x}_b(i, j) + K(i, j)[y(i, j) - H\hat{x}_b(i, j)] \tag{20}$$

$$P_a(i, j) = [I - K(i, j)H]P_b(i, j) \tag{21}$$

where subscript a represents update and b denotes prediction.

4. SIMULATIONS

4.1 Experiment setup

The simulations were performed with the test configuration as shown in Table 1.

Image	Lenna	Synthetic
Description	portrait of a lady 256x256	synthetically constructed, 256x256
Modelling error, σ_w^2	0.0014	0.0001
Boundary conditions	initial $\hat{s}$(i,j) = g(i,j) initial X(i,j) = 0	initial $\hat{s}$(i,j) = g(i,j) initial X(i,j) = 0

Table 1. Test configuration

The image model coefficients are shown in Figure 1. The model correlates pixel s(i,j) to 4 of its neighbouring pixels.

	a_{11} =-0.267 s(i-1,j-1)	a_{01} =0.538 s(i,j-1)	a_{-11} =0.261 s(i+1,j-1)	s(i+2,j-1)
s(i-2,j)	a_{10} =0.452 s(i-1,j)	s(i,j)		

Figure 1. Image support

4.2 Experimental results

The effect on MSE due to uncertainties in the image model

σ_a	0	0.01	0.02	0.03
a_{11}	-0.267	-0.257	-0.247	-0.237
a_{01}	0.538	0.548	0.558	0.568
a_{-11}	0.261	0.271	0.281	0.291
a_{10}	0.452	0.462	0.472	0.482
σ_w^2 ($\times 10^{-4}$)	1.00	2.36	4.49	8.04
MSE (%)	0.0346	1.114	14.42	901.2

Table 2. The effect on MSE due to uncertainties in the image model of Synthetic image

In Table 2, the effect of uncertainties in the image model was tested on a synthetically constructed image blurred by 1x3 linear motion blur at a SNR of 10dB. The reasons for using Synthetic image (synthetically constructed) instead of Lenna image (real) are twofold. Firstly, the 1x1x1 image model assumes a highly localized pixel correlation and hence, it is unable to represent both dynamic edges and textures in the real image accurately. This can be observed from the large variance of the modelling uncertainty. Tests have shown that there is little or no change in the MSE's of the restored Lenna image, despite the deviations introduced in the image model. Secondly, the test requires the filtering process to have more dependence on the state predictions, which are based on the image model, and this can be brought about by either reducing the image model uncertainty or increasing the noise level in the observation (so that it is less reliable). Since Lenna image has a large pixel intensity variance, an even larger noise variance is required to degrade the image to a SNR of 10dB. At this noise level, the image is too noisy to be processed. Due to the reasons, the synthetically constructed image was used in place of a real image, as a test on the latter will not give meaningful result.

σ_a is the standard deviation introduced into the image coefficients to simulate the inaccuracy in the model. The test in Table 2 shows that the MSE increases (deteriorates) with the increase in deviations in the image model. This happens because the image model becomes less reliable (increasing σ_w^2) with the increase in deviations in the image model coefficients.

The effect on MSE due to uncertainties in the blur model

σ_n	0	0.0707	0.1414	0.2121
Coefficients,				
h_0	0.3333	0.2833	0.2333	0.1833
h_{-1}	0.3333	0.4333	0.5333	0.6333
h_{-2}	0.3333	0.2833	0.2333	0.1833
MSE (%)	0.0343	0.0723	0.0966	0.1039

Table 3. The effect on MSE due to uncertainties in the blur model

The test in Table 3 was performed on Lenna image blurred by 1x3 motion blur at a SNR of 35.1dB. σ_n is the standard deviation introduced into the blur model to simulate the inaccuracy in the model. To preserve the conservative property of the blur system, the deviations in the model are chosen such that the sum of all its coefficients is equal to one. The test shows that the MSE increases (decreasing accuracy in restorations) with the level of uncertainty in the blur model.

Robust deconvolution for image model with uncertainties

In this section, the test on uncertainties in the image model was performed using the proposed robust filter design. The result in Table 4 shows marked improvements in MSE's of the restored images when the robust design was used.

σ_a	0.01	0.02	0.03
$\sigma_a^2=Q_\zeta$	1.0×10^{-4}	4.0×10^{-4}	9.0×10^{-4}
ROMKF, MSE (%)	1.114	14.42	901.2
Robust ROMKF, MSE (%)	0.422	0.596	0.567
Improvement (dB)	4.2	13.8	32.0

Table 4. Results for robust filtering with uncertain image model

Robust deconvolution for blur model with uncertainties

The test for uncertain blur models in Table 3 was carried out again in this section, but with the proposed robust filter design.

σ_n	0.0353 (11%)	0.0707 (21%)	0.1414 (42%)
$\sigma_n^2=Q\eta$	1.25×10^{-3}	2.5×10^{-3}	0.01
ROMKF, MSE (%)	0.0441	0.0723	0.0966
Robust ROMKF, MSE (%)	0.0431	0.0593	0.0813
Improvement (%)	2.3	18.0	15.8

Table 5. Results for robust filtering with uncertain blur model

Table 5 compares the performance of the robust and conventional filter design. When the robust ROMKF is used, improvements in the MSE's are observed for the various degrees of inaccuracy, in the form of deviations, in the blur.

Modelling a blur with a smaller support and uncertainty

This test demonstrates how a deconvolution filter with a maximum blur support size of 3x3 can handle a bigger 5x5 blur. The Lenna image is blurred by 5x5 uniform motion blur as defined in (22) and has white Gaussian noise added it to give a SNR of 40 dB. The MSE of the degraded image is 0.214%. When a support size of 3x3 is assumed, the actual 5x5 blur will be truncated as shown in (23).

$$h = \begin{bmatrix} 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \end{bmatrix} \quad (22)$$

$$h_{truncated} = \begin{bmatrix} 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 \end{bmatrix} \quad (23)$$

$$\hat{h} = \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} + \eta(x, y) \quad (24)$$

$$\begin{aligned} Q_\eta &= (\frac{1}{9} - 0.04)^2 \\ &= (0.0711)^2 \end{aligned} \quad (25)$$

To maintain the conservative property and 2D profile of the blur, the actual 5x5 blur is being modelled as a robust 3x3 blur $\hat{h}$ consisting an uniform 3x3 blur with an uncertainty representing the difference in coefficient values as shown in (24). The variance of the uncertainty term $Q\eta$ is defined in (25). Table 6 shows that the robust blur model offers a much better restoration compared to the truncated blur. Hence, the test shows that the 5x5 blur can be modelled as a smaller order blur of 3x3 size.

Blur models	h(x,y)	$h_{truncated}$(x,y)	$\hat{h}$(x,y)
MSE (%)	0.189	17.6	0.245

Table 6. Results for modelling of blur using a smaller support

5. CONCLUSION

A new robust filter design for image deconvolution or restoration is proposed and verified in simulation tests to be a feasible idea. In the design, any uncertainties in the image and blur models are being taken into account in the filtering process. In the test on uncertain image model, a synthetically constructed image, which can be represented by the 1x1x1 image model accurately, is used. In the simulation tests employing the robust design, a marked improvement in MSE of up to 32 dB can be achieved for a deviation of 0.03 in the image model. Simulation tests have also shown that the robust ROMKF can achieve improvements of 2.3-18% in the MSE's over the conventional ROMKF for 11-42% of deviations in the blur model. The robust blur modelling also allows a filter with a smaller support size to restore images that are blurred by larger support blur models. This is done by using a robust blur model, which consists a smaller-support blur with a similar 2D profile as the actual blur and an uncertainty term to account for the difference in the blur coefficients.

If an iterative identification method is used, we can first establish a relationship between the number of iterations used and the accuracy in the estimated image and blur models. By using the known relationship, inaccuracy in image and blur estimates can be compensated using the robust ROMKF. Thus, the identification time can be shortened and the dependence on the accuracy of the models can also be reduced.

6. REFERENCES

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