COMPENSATION OF AMPLIFIER NONLINEARITIES ON WAVELET PACKET DIVISION MULTIPLEXING

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ABSTRACT

Wavelet packet division multiplexing (WPDM) is a high-capacity, flexible and robust orthogonal multiplexing scheme in which the message signals are waveform coded onto wavelet packet basis functions for transmission. However, WPDM suffers from severe performance degradation in the presence of high-power amplifier (HPA) nonlinearities. In this paper, data predistortion using the *p*th-order Volterra inverse is proposed to combat the amplifier nonlinearities in a WPDM system. A 5th-order Volterra inverse with truncated memory length is designed based on the Volterra series channel model. Computer simulations are presented to demonstrate the capability of the proposed technique in compensating amplifier nonlinearities even under system parameter discrepancy. Guidelines are also proposed for designing wavelet filter which leads to better predistortion with the truncated Volterra inverse.

1. INTRODUCTION

Wavelet packet division multiplexing (WPDM) [1] is an orthogonal multiplexing scheme using wavelet packet basis functions as coding waveforms. The orthogonality properties of the wavelet packets provide a substantial increase in channel capacity and added robustness against many adverse channel environments [1]−[3]. However, in the presence of nonlinear high-power amplifier (HPA), the performance of WPDM degrades significantly due to its large signal dynamics. The effects of amplifier nonlinearities on WPDM has been analyzed and a nonlinear channel model of WPDM has also been derived using Volterra series [4]. In this paper, a 5th-order Volterra inverse with truncated memory length is designed based on the Volterra series channel model. Data predistortion is performed on the equivalent sequence at the root of the WPDM tree to alleviate the distortion due to amplifier nonlinearities. The merit of this method is its simple implementation and reduced computational complexity. Structure and complexity of the inverse will be discussed. Computer simulations are presented to verify the ability and robustness of the inverse. Guidelines are also proposed in designing wavelet filter for better predistortion.

First, let us briefly review WDPM [1]. Let $g_0[n]$, $g_1[n]$ be a unit-energy real causal FIR conjugate quadrature mirror filter (QMF) pair of length N which are orthogonal to their even translates. An iterative algorithm [5] may be used to find the function $\phi_{01}(t) = \sqrt{2}\sum_n g_0[n]\phi_{01}(2t - nT_0)$ for a given T_0. Subsequently, we can define a family of functions $\phi_{lm}(t), l \geq 0, 1 \leq m \leq 2^l$, in a binary tree structure, with the subscripts denoting the "level" of a node in the tree and its position within the level, respectively. The functions at the *terminals* of the tree form a *wavelet packet* [5]. They are self and mutually orthogonal at integer multiples of $T_l = 2^l T_0$ and have a finite duration $(N-1)T_l$. The message symbols $\sigma_{lm}[n]$ are waveform coded by pulse amplitude modulation of $\phi_{lm}(t - nT_l)$ and are then added together to form the composite signal $s(t)$. By exploiting the wavelet packet tree structure, WPDM can be implemented using a transmultiplexer and a single modulator,

$$s(t) = \sum_k \sigma_{01}[k]\phi_{01}(t - kT_0) \tag{1}$$

where $\sigma_{01}[k] = \sum_{(l,m)\in\mathcal{T}}\sum_n f_{lm}[k - 2^l n]\sigma_{lm}[n]$ is the equivalent data sequence, with $\mathcal{T}$ being the set of terminal index pairs and $f_{lm}[k]$ the equivalent filter from the (l,m)th terminal to the root of the tree. The original messages can be recovered from $\sigma_{01}[k]$ using $\sigma_{lm}[n] = \sum_k f_{lm}[k - 2^l n]\sigma_{01}[k]$.

Using the discrete-time $(2N+1)$th-order Volterra model, the nonlinear WPDM channel model can be expressed as [4]

$$\hat{\sigma}_{01}[n] = \sigma_{01}[n] + \underbrace{\sum_{k=1}^{N} H_{2k+1}\{\sigma_{01}[n]\}}_{\substack{\text{interference due to}\\ \text{HPA nonlinearities}}} \tag{2}$$

where

$$H_{2k+1}\{\sigma_{01}[n]\} = \sum_{m_1}\cdots\sum_{m_{2k+1}} h_{2k+1}[m_1, \cdots, m_{2k+1}] \cdot \prod_{r=1}^{k+1}\sigma_{01}[n - m_r]\prod_{s=k+2}^{2k+1}\sigma_{01}^*[n - m_s] \tag{3}$$

and

$$h_{2k+1}[m_1, \cdots, m_{2k+1}] \triangleq \gamma_{2k+1}\int \phi_{01}(t)\prod_{i=1}^{2k+1}\phi_{01}(t + m_i T_0)dt \tag{4}$$

are the $(2k+1)$th-order Volterra operator and kernel, respectively. γ_{2k+1} denotes the $(2k+1)$th-order complex Taylor coefficient. Notice that only odd order terms appear here due to the bandpass nature of the nonlinearity.

2. DATA PREDISTORTION

Consider the system shown in Figure 1 where a nonlinear system represented by G with Volterra kernels g is preceded by a *p*th-order Volterra inverse $F_{(p)}$ with Volterra kernels f. Denote the

This work was partially supported by a research grant awarded by the Hong Kong Research Grant Council.

nonlinear system resulting from the cascade of $F_{(p)}$ and G by H with Volterra kernels h. The 1st-, 3rd- and 5th-order h-kernels are then given by

$$h^{(1)}_{n;a} = g^{(1)}_{n;v} f^{(1)}_{v;a} \tag{5}$$

$$h^{(3)}_{n;a,b,c} = g^{(1)}_{n;v} f^{(3)}_{v;a,b,c} + g^{(3)}_{n;v,w,x} f^{(1)}_{v;a} f^{(1)}_{w;b} f^{(1)*}_{x;c} \tag{6}$$

and

$$\begin{aligned} h^{(5)}_{n;a,b,c,d,e} =& g^{(1)}_{n;v} f^{(5)}_{v;a,b,c,d,e} \\ &+ g^{(3)}_{n;v,w,x} f^{(1)}_{v;a} f^{(1)}_{w;b} f^{(3)*}_{x;c,d,e} \\ &+ g^{(3)}_{n;v,w,x} f^{(1)}_{v;a} f^{(3)}_{w;b,c,d} f^{(1)*}_{x;e} \\ &+ g^{(3)}_{n;v,w,x} f^{(3)}_{v;a,b,c} f^{(1)}_{w;c} f^{(1)*}_{x;e} \\ &+ g^{(5)}_{n;v,w,x,y,z} f^{(1)}_{v;a} f^{(1)}_{w;b} f^{(1)}_{x;c} f^{(1)*}_{y;d} f^{(1)*}_{z;e} \end{aligned} \tag{7}$$

For simplicity, we are using the tensor notation which implies that any term in which a given index appears twice must be summed over the appropriate range of this index.

Now, let us consider the 5th-order compensation. Under the assumption that the linear part of system G, that is, the 1st-order g-kernel denoted by $g^{(1)}$, is invertible, it is possible to find a system F such that its cascade with G gives a system with no linear distortion, that is,

$$\begin{aligned} f^{(1)}_{n;v} g^{(1)}_{v;a} &= g^{(1)}_{n;v} f^{(1)}_{v;a} \\ &= \delta_{n;a} = \begin{cases} 1, & n = a \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{8}$$

This choice provides the 1st-order compensator. The 3rd-order compensator is then obtained by choosing $f^{(3)}$ such that $h^{(3)} = 0$. The 3rd-order f-kernel can therefore be expressed as

$$f^{(3)}_{n;a,b,c} = -f^{(1)}_{n;v} g^{(3)}_{v;w,x,y} f^{(1)}_{w;a} f^{(1)}_{x;b} f^{(1)*}_{y;c} \tag{9}$$

Similarly, the 5th-order f-kernel can be written as

$$\begin{aligned} f^{(5)}_{n;a,b,c,d,e} = -f^{(1)}_{n;u} \Big(& g^{(3)}_{u;v,w,x} f^{(1)}_{v;a} f^{(1)}_{w;b} f^{(3)*}_{x;c,d,e} \\ &+ g^{(3)}_{u;v,w,x} f^{(1)}_{v;a} f^{(3)}_{w;b,c,d} f^{(1)*}_{x;e} \\ &+ g^{(3)}_{u;v,w,x} f^{(3)}_{v;a,b,c} f^{(1)}_{w;d} f^{(1)*}_{x;e} \\ &+ g^{(5)}_{u;v,w,x,y,z} f^{(1)}_{v;a} f^{(1)}_{w;b} f^{(1)}_{x;c} f^{(1)*}_{y;d} f^{(1)*}_{z;e} \Big) \end{aligned} \tag{10}$$

From the derived Volterra channel model [4], we have $g^{(1)}_{n;v} = \delta_{n;v}$, therefore, the corresponding 1st-order Volterra kernel for the compensator is

$$f^{(1)}_{n;v} = \delta_{n;v} \tag{11}$$

By substituting (11) into (9) and (10), we obtain the 3rd- and 5th-order f-kernels as

$$f^{(3)}_{n;a,b,c} = -g^{(3)}_{n;a,b,c} \tag{12}$$

and

$$\begin{aligned} f^{(5)}_{n;a,b,c,d,e} &= -(g^{(3)}_{n;a,b,x} f^{(3)*}_{x;c,d,e} + g^{(3)}_{n;a,w,e} f^{(3)}_{w;b,c,d} \\ &\quad + g^{(3)}_{n;v,d,e} f^{(3)}_{v;a,b,c} + g^{(5)}_{n;a,b,c,d,e}) \\ &= g^{(3)}_{n;a,b,x} g^{(3)*}_{x;c,d,e} + 2 g^{(3)}_{n;a,w,e} g^{(3)}_{w;b,c,d} + g^{(5)}_{n;a,b,c,d,e} \end{aligned} \tag{13}$$

With the Volterra kernels, the block diagram of a 5th-order Volterra inverse is shown in Figure 2.

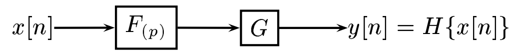

Fig. 1. Nonlinear system preceded by Volterra inverse

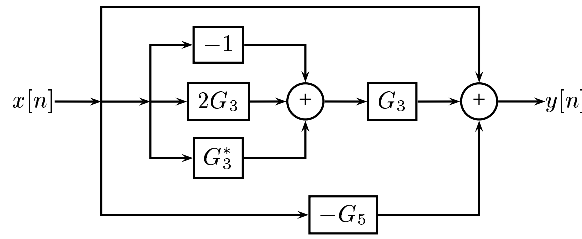

Fig. 2. Block diagram of 5th-order Volterra inverse

3. COMPUTATIONAL COMPLEXITY

For symmetric Volterra kernels, the total number of multiplications required for a discrete-time $(2N+1)$th-order Volterra series with memory length M_k for the kth-order Volterra operator gives a measure of the computational complexity, denoted by $\mathbf{C}_{2N+1}$,

$$\mathbf{C}_{2N+1} = \sum_{n=0}^{N} \binom{M_{2n+1} + n}{n+1} \binom{M_{2n+1} + n - 1}{n} (2n+1) \tag{14}$$

where $\binom{n}{k}$ is the binomial coefficient. For both Daubechies and Symlets filters [5] with length N,

$$M_k = \begin{cases} 1, & k = 1 \\ 2N-3, & \text{otherwise} \end{cases} \tag{15}$$

Therefore, the computational complexity of a 5th-order Volterra inverse with wavelet filter length N is

$$\mathbf{C}^{\text{inv}}_5(N) = 3 + 3(2N-3)\binom{2N-2}{2} + 5\binom{2N-1}{3}\binom{2N-2}{2} \tag{16}$$

Table 1 shows the computational complexity of a 5th-order Volterra inverse with different wavelet filter lengths. It can be seen that the complexity of the inverse increases dramatically with higher wavelet filter length. For system operating with high sampling rates, real time compensation using a Volterra inverse is thus limited to lower order, short memory cases. Therefore, we have to truncate the memory length of the Volterra inverse in order to reduce the complexity for real time implementation. In the next section, computer simulations will be presented to demonstrate the performance of the truncated Volterra inverse.

wavelet filter length, N	complexity
4	2853
6	38343
8	210577
10	749091
12	2060061
14	4777503

Table 1. Computational complexity of 5th-order Volterra inverse versus wavelet filter length

4. COMPUTER SIMULATIONS

The performance of the 5th-order Volterra inverse had been evaluated through computer simulations. The memory length of the inverse had been truncated to reduce the computational complexity. For simplicity, *p*th-order Volterra pre-inverse with memory span K will be denoted by ***Pinv(p,K)***. In the simulations, a 4-channel WPDM system was considered and 16-QAM signal constellation was adopted in each channel. The transmission channel was assumed to be an AWGN channel with no multipath propagation and traveling wave tube amplifier (TWTA) model [7] was used for the nonlinearities. Scaling functions generated by both Daubechies and Symlets filters of length $N = 14$ were used and the memory length of the 5th-order Volterra inverse was truncated to $K = 1, 3$ and 5. The symbol duration T_0 is normalized to one.

Simulation 1: In this simulation, we attempt to demonstrate the performance of the truncated 5th-order Volterra inverse. Figure 3 plots the signal constellations of the receiving data sequences of the 4-channel WPDM system under noise-free condition, for IBO = 8dB (input backoff ratio). The resulting constellations demonstrate the ability of the truncated 5th-order Volterra inverse to compensate the attenuation and rotation introduced by the amplifier's amplitude nonlinearity (AM/AM conversion) and phase nonlinearity (AM/PM conversion), respectively. Moreover, the inverse is capable of reducing the spread of the signal constellations as the memory length increases. Figures 4 shows the resulting probabilities of symbol error under AWGN channel. The probabilities were averaged over the 4 channels and the theoretical probability of symbol error of 16-QAM was also plotted for comparison. The results demonstrate again the capability of the 5th-order Volterra inverse in compensating the distortion caused by the nonlinear amplifier. It is noted that the symbol error probabilities for Pinv(5,5) are close to the theoretical 16-QAM bound under AWGN channel. By comparing the results, we found that WPDM system using Symlets filter performs better than the counterpart using Daubechies filter, which can be explained as follows.

First, define the squared sum of the *p*th-order kernel coefficients of wavelet filter length N as

$$C_N^{(p)} = \sum_{m_1=-(2N-2)}^{2N-2} \cdots \sum_{m_p=-(2N-2)}^{2N-2} |h_p[m_1, \cdots, m_p]|^2 \tag{17}$$

and the squared sum of the truncated *p*th-order kernel coefficients of memory length K as

$$C_K^{(p)} = \sum_{m_1=-(K-1)/2}^{(K-1)/2} \cdots \sum_{m_p=-(K-1)/2}^{(K-1)/2} |h_p[m_1, \cdots, m_p]|^2 \tag{18}$$

The ratio of the two squared sums, gives a measure of the approximation accuracy of the truncated *p*th-order kernel to the original kernel, denoted β_p

$$\beta_p = \frac{C_K^{(p)}}{C_N^{(p)}} \tag{19}$$

where $0 \leq \beta_p \leq 1$. Table 2 shows the values of β_3 and β_5 for the Daubechies and Symlets filters of length $N = 14$. It can be seen from the tables that, for both truncated 3rd- and 5th-order Volterra kernels with memory length $K = 1, 3$ and 5, the Symlets filter always results in a higher β_3 and β_5 ratios. The higher approximation accuracy thus accounts for the better performance illustrated above.

From the Volterra kernel defined in (4), we found that the kernel coefficients are the higher order autocorrelations of the scaling function. This suggested that it is possible to design a wavelet filter such that the energy of the coefficients is packed within a specific region of the truncated Volterra kernels, resulting in a better predistortion. Finally, Table 3 shows the tremendous reduction of computational complexity of the truncated 5th-order Volterra inverses, compared to the original one with memory span $K = 25$ (last row).

	predistorter	Pinv(5,1)	Pinv(5,3)	Pinv(5,5)
β_3	Daubechies 14	0.603	0.900	0.984
	Symlets 14	0.860	0.964	0.999
β_5	Daubechies 14	0.417	0.831	0.971
	Symlets 14	0.793	0.949	0.998

Table 2. Values of β_3 and β_5 for truncated 3rd- and 5th-order Volterra kernels with Daubechies and Symlets filters of length 14

predistorter	Pinv(5,1)	Pinv(5,3)	Pinv(5,5)	Pinv(5,25)
complexity	11	357	2853	4777503

Table 3. Computational complexity of truncated 5th-order Volterra inverses with different memory lengths

Simulation 2: From the Volterra channel model derived in [4], we noticed that the complex Taylor coefficient γ in (4) depend on the saturation voltage A_{sat}, of the nonlinear amplifier. In the previous example, perfect knowledge of the parameter A_{sat} was assumed, which is reasonable as the predistorter was placed in the WPDM transmitter before the nonlinear HPA. In this simulation, we will investigate the effects of parameter discrepancy of A_{sat} on the performance of the truncated 5th-order Volterra inverse. Figure 5 gives the resulting probabilities of symbol error for a 4-channel WPDM system with truncated 5th-order Volterra inverses Pinv(5,1) and Pinv(5,3), respectively. In this test, Symlets filter of length $N = 14$ are used and IBO = 8dB. In Figure 5, $[\pm 10\%]$ denote the cases with inaccurate parameter $\tilde{A}_{sat} = A_{sat} \pm 0.1 A_{sat}$. From the simulation results, an apparent degradation in symbol error rate (SER) can be observed for the $[+10\%]$ case. On the contrary, the $[-10\%]$ case results in a SER improvement which can again be explained by the better approximation of the truncated Volterra series to the nonlinear system. In fact, it had been proved by simulation that further decrease of the inaccurate parameter $\tilde{A}_{sat}$ will result in performance degradation similar to the $[+10\%]$ case.

5. CONCLUSION

Based on the Volterra series, the truncated 5th-order Volterra inverse is proposed for data predistortion of 16-QAM signal constellation of multi-carrier WPDM system. It is shown that the computational complexity of the truncated inverse is greatly reduced when compared to the original one. Computer simulations have demonstrated the ability of the truncated 5th-order Volterra inverse in compensating the attenuation, rotation and spreading of the signal constellation caused by the nonlinear amplifier, for 4-channel WPDM. Experiments on the effects of parameter discrepancy have also shown the robustness of the truncated 5th-order Volterra inverse. For wavelet filters of length $N = 14$, it is found that the truncated inverse performs better in the case of Symlets filter, which is shown to be related to the higher approximation accuracy of the Volterra kernels. As the Volterra kernels are the higher order autocorrelation coefficients of the scaling function, therefore, this provides the possibility of having an optimum wavelet filter design in which the approximation accuracy is maximized, leading to a better data predistortion.

6. REFERENCES

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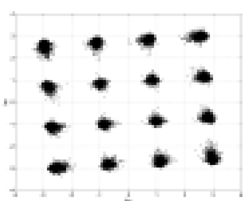
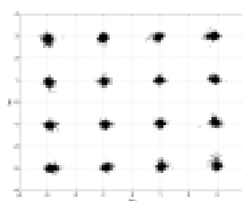
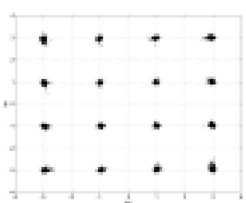


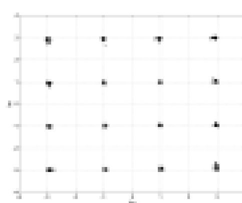
(a) No predistortion



(b) Pinv(5,1)

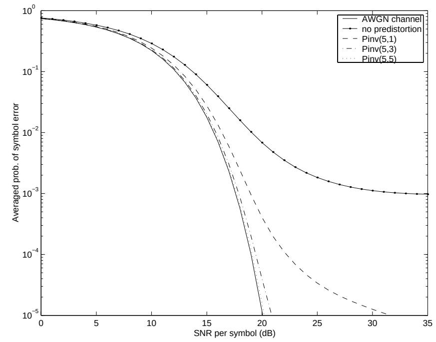


(c) Pinv(5,3)

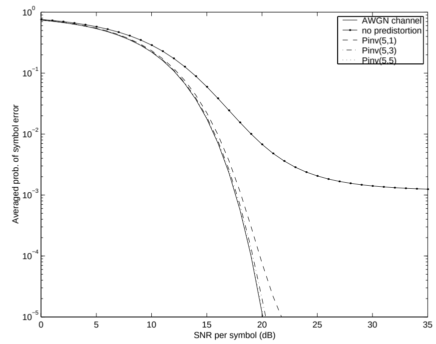


(d) Pinv(5,5)

Fig. 3. Simulated signal constellations of 4-channel WPDM with truncated 5th-order Volterra inverses of different memory lengths (IBO = 8dB, Symlets filter of length $N = 14$)

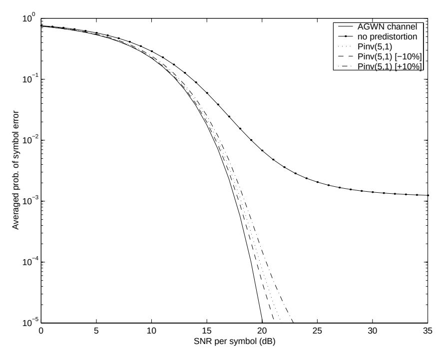


(a) Daubechies 14

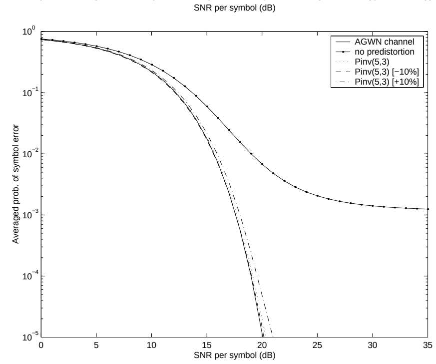


(b) Symlets 14

Fig. 4. Averaged probability of symbol error of 4-channel WPDM with truncated 5th-order Volterra inverses of different memory lengths (IBO = 8dB, wavelet filters of length $N = 14$)



(a) $K = 1$



(b) $K = 3$

Fig. 5. Averaged probability of symbol error of 4-channel WPDM with truncated 5th-order Volterra inverse with memory length K (IBO = 8dB, Symlets filter of length $N = 14$)