# DIRECTION-TIME TRANSMISSION FOR FAST FADING VECTOR CHANNEL

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## ABSTRACT

*Transmit diversity and space-time coding have received a lot of attention in recent years. In this paper, we propose a direction-time transmit scheme for a fast block fading channel which exploits the parametric structure of the time-varing vector channel. Specular propagation and local scattering are assumed at the base-station and mobile station respectively. The partial channel knowledge needed for the transmit design includes: the number of specular paths, path direction vectors and length of the coherent interval. Transmission matrix codewords are formed according to this partial channel information such that multiple channels are induced and symbols are spread over different transmission directions. Diversity gain and blind channel estimation are achieved simultaneously at the mobile station through equalization.*

## 1. INTRODUCTION

In recent years, multiple antennae have been employed at both the transmitter and receiver to achieve diversity and coding gain for quasi-static wireless channels [1]. The basic idea is to design space-time codeword which introduces redundancy across multiple antennae such that rank and determinant criteria are satisfied [1]. In such work, no channel state information is required at the transmitter and the construction of codewords is fixed. Diversity and coding gain are obtained at the receiver assuming perfect channel state information is available. Another significant study addressed a similar system but with mobile user [2] wherein the time-varying nature is characterized by a *coherent interval* (CI). The channel is assumed to be constant within each CI but varies independently from CI to CI. To combat time-varying fading, a coding scheme across both multiple antennae and multiple CIs is proposed. Though these insightful studies do reveal great potential of diversity gain in a wireless channel, they model the wireless channel by a random matrix. Each entry in the matrix is an independent complex Gaussian random variable. While this is approximately true for scenarios when antenna spacing is large and rich scattering exists, it generally cannot apply to a cellular network where different propagation patterns (line of sight, specular propagation and local scattering, etc.) coexist. In this paper, we consider a wireless channel in a cellular network equipped with smart antenna system (SAS). To provide proper service coverage, the base-station (BS) is normally mounted at a high place ( tower, building, etc ) to enable directional transceiving capable of more interference suppression, cell coverage and spectrum efficiency. The antenna separation for SAS must be no more than half a wavelength according to spatial Shannon sampling theorem to avoid spatial ambiguity, and the fading coefficients at different antennae are highly correlated.

In this paper, we consider a frequency flat fast fading vector channel model (Figure 1) with multiple antennae employed at both BS and mobile station (MS). This model was previously proposed in [3] and was employed to derive uplink channel parameter estimation in [4]. In this model, the MS are surrounded by a large number of local reflectors while the BS only sees a limited number of arriving/departing rays. The signal radiated from the BS enters the local scattering region around the MS through the specular propagation rays. Large buildings, terrain and other dominant reflectors produce the specular paths. Fading processes carried by different dominant paths are assumed to be independent and the number of paths are assumed to be less than the number of antennae at the BS. Due to the local scattering, signals received at different MS antennas are assumed to experience independent fading processes. The time-varying nature of the channel is characterized by the coherent interval (CI) of length $T$. The fading coefficients stay stationary within each CI and but vary from CI to CI. The length of CI depends on the Doppler spread $f_d$ of the channel by $T \simeq 1/f_d$.

We point out that methods proposed in [1, 2] don't exploit the channel state information. The codewords generated therein effectively radiate energy along all directions in the transmission space. This is a robust way of signal transmission. On the other hand, suppose we are given the directions of the dominant rays, we actually know the spatial distribution of power spectrum for a particular user. The more efficient way of delivering information would be to transmit along those directions. There are two ways of utilizing direction knowledge, one is sending the same symbol along all directions, another is sending different symbols along different directions. The independently faded rays provide transmit direction diversity which can be exploited at the receiver and the second method will be taken. Receive diversity is available by employing multiple antennae at the MS. With receiver diversity, blind channel estimation at the receiver is possible if transmitted signals are properly designed (will be clear later). Due to time-varying nature of the channel, blind channel estimation is highly desirable property, since the training sequence based estimation algorithm will fail. In the derivation of transmit design, we assume *direction vectors* and gains of dominant paths are available at the transmitter, and the length of CI for each path is known and remains stationary over multiple CIs. The assumptions are valid for TDD systems since data received during uplink can be used to estimate direction vectors [4] and Doppler frequency.

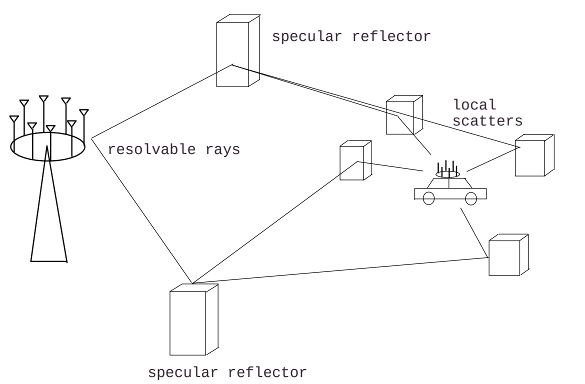


**Fig. 1**. Physical Basis for Fast Fading Vector Channel

The following notation is used: $(\cdot)^H$, $(\cdot)^T$, $(\cdot)^\dagger$, $\|\cdot\|_2$, $\|\cdot\|_F$ and **Tr** denote matrix Hermitian, transpose, pseudoinverse, 2-norm, F-norm and trace, respectively; $\odot$ denotes convolution, $\mathcal{CN}(0, \sigma^2\mathbf{I})$ denotes the distribution for independent complex Gaussian vector with variance $\sigma^2$.

## 2. VECTOR CHANNEL DOWNLINK AND PROBLEM STATEMENT

We consider a frequency non-selective vector channel for the forward link with $N$ transmitters and $M$ receivers. Suppose the number of dominant paths is $L(< N)$ and the BS has estimated the direction vectors (DOA) and the corresponding gains for all paths (e.g. using uplink data). We absorb the path gains into the direction vectors which are denoted as $\mathbf{A} = [\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_L}]^T$. The received discrete time signal at the $m^{th}$ antenna can be expressed as

$$y_m(k) = (\sum_{l=1}^{L} b_{ml}\mathbf{a_l})^T \sum_{i=1}^{P} \mathbf{w_i} s_i(k) + v_m(k), m = 1, \ldots, M \quad (1)$$

where $P$ is the number of symbols sent for each channel use, $\mathbf{w_i}$ is the spatial weight for the $i$th symbol, $b_{mj} \sim \mathcal{CN}(0, 1)$ is the fading coefficient corresponding to the $j^{th}$ specular path and seen by the $m^{th}$ mobile antenna and $v_m \sim \mathcal{CN}(0, \sigma^2)$ is additive noise for the $m^{th}$ mobile antenna. Defining $\mathbf{y}_k = [y_1(k) \ldots y_M(k)]^T$, $\mathbf{v}_k = [v_1(k) \ldots v_M(k)]^T$, $\mathbf{b}_m = [b_{m1} \ldots b_{mL}]^T$, $\mathbf{B} = [\mathbf{b}_1 \ldots \mathbf{b}_M]^T$, $\mathbf{W} = [\mathbf{w}_1 \ldots \mathbf{w}_P]$ and $\mathbf{s}_k = [s_1(k) \ldots s_P(k)]^T$, we can rewrite (1) in more compact form by stacking $M$ equations and obtain

$$\mathbf{y}_k = \mathbf{BAWs}_k + \mathbf{v}_k \quad (2)$$

Since the channel remains stationary during any CI, (2) is valid for $T$ continuous channel uses, which gives

$$\underbrace{[\mathbf{y}_1, \ldots, \mathbf{y}_T]}_{\mathbf{Y}} = \underbrace{\mathbf{BAW}}_{\mathbf{H}} \underbrace{[\mathbf{s}_1, \ldots, \mathbf{s}_T]}_{\mathbf{S}} + \underbrace{[\mathbf{v}_1, \ldots, \mathbf{v}_T]}_{\mathbf{V}} \quad (3)$$

We refer to modulating each row of $\mathbf{S}$ with the respective column in $\mathbf{W}$ as *direction coding*. For each CI, the matrix $\mathbf{WS}$ forms a *direction-time* block code and $\mathbf{BA}$ represents spatial channel. The problem is to design code block $\mathbf{S}$ and weight $\mathbf{W}$ such that diversity gain can be achieved at the MS without the knowledge of the channel matrix $\mathbf{BA}$ and weight $\mathbf{W}$. The total power constraint on all directions requires $\mathbf{Tr}(\mathbf{W}^H\mathbf{W}) \leq 1$. Note that $\mathbf{W}$ also serves as a channel shaping matrix in view of the receiver.

## 3. PARAMETER DEPENDENT CODEWORD CONSTRUCTION

The multiple antennae at the MS provide diversity to recover the symbol sequence or channel coefficients without CSI knowledge, provided the block codeword $\mathbf{S}$ is properly constructed. If only one antenna is available at the MS, the code block can be sent over $M$ CIs to introduce diversity. Given $T$ as the length of a CI and the set of direction vectors $\mathbf{A}$, we form the codeword using $N = T + L - 1$ sysmbols as follows $(P = L)$,

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & \cdots & s_T \\ s_2 & s_3 & \cdots & s_{T+1} \\ \vdots & \vdots & \ddots & \vdots \\ s_L & s_{L+1} & \cdots & s_N \end{bmatrix} \quad (4)$$

where each row of $\mathbf{S}$ is sent using a different weight vector. The designed code block serves the following multiple purposes: (i) except for the head and tail symbols, all symbols are transmitted along at least two directions ($P > 1$) with different effective fading coefficients; (ii) the flat fading channel is converted into a equivalent frequency selective channel. This can be seen by rewriting (3) as

$$\mathbf{Y} = \mathbf{HS} + \mathbf{V} \tag{5}$$

where $\mathbf{H} = \mathbf{BAW}$ is the equivalent channel matrix. Then equalization at the receiver can produce diversity gain [5]; (iii) both the channel coefficients and symbols can be blindly estimated if certain conditions on $\mathbf{S}$ and $\mathbf{H}$ are satisfied [6].

As for the design of $\mathbf{W}$, we rewrite (5) as follows

$$\mathbf{Y} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_L] \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \vdots \\ \mathbf{s}_L^T \end{bmatrix} + \mathbf{V} \tag{6}$$

where $[\mathbf{h}_1, \mathbf{h}_2, \cdots \mathbf{h}_L] = [\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_L]\mathbf{AW}$. Note that $\{\mathbf{b}_i \sim \mathcal{CN}(0, \mathbf{I}), i = 1, \ldots, L\}$ are independent random vectors. To obtain diversity, we want $\{\mathbf{h}_i\}$ to be uncorrelated. The condition is satisfied when

$$\mathbf{AW} = diag\{p_1, p_2, \ldots, p_L\} \tag{7}$$

where $p_1, \ldots, p_L$ are effective direction powers. At this point, we consider equal direction power, so $\mathbf{AW} = p\mathbf{I}$ and $p$ is determined by the power constraint. Another design consideration is the weight vectors $\{\mathbf{w}_i, i = 1, \ldots, L\}$ should all fall into the subspace spanned by $\{\mathbf{a}_i^*, i = 1, \ldots, L\}$, otherwise, partial energy cannot reach the desired user and power efficiency is reduced. Assume $L < N$ and incorporate total power constraint $\mathbf{Tr}(\mathbf{W}^H\mathbf{W}) \leq 1$, a solution is given by

$$\mathbf{W} = \mathbf{A}^\dagger / (\mathbf{Tr}(\mathbf{AA}^H)^{-1})^{1/2} \tag{8}$$

accordingly $p = 1/(\sum_{i=1}^{L} 1/\sigma_i^2)^{1/2}$ gives power delivered along each direction where $\sigma_i$ is non-zero singular value of $\mathbf{A}$. We call the above the *pseudoinverse weight*. The weight can also be chosen as $\mathbf{W} = \mathbf{A}^H / (\mathbf{Tr}(\mathbf{AA}^H)^{-1})^{1/2}$ which gives *conjugate weight*. In case $\mathbf{A}$ has orthogonal rows, the above two weights are equivalent.

## 4. PERFORMANCE MEASURES

The system can be implemented in two ways: (i) direct symbol estimation; (ii) channel estimation and equalization. For direct blind symbol estimation at the receiver, the algorithm proposed in [6] is employed. The algorithm is deterministic in nature and the diversity combining is embeded in the algorithm. The normalized root-mean square error(RMSE) is adopted as a performance measure

$$RMSE = \frac{1}{\|\mathbf{s}\|} \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \|\hat{\mathbf{s}}_i - \mathbf{s}\|} \tag{9}$$

An alternative is to first blindly estimate channel coefficients $\mathbf{H}$, then employ equalization techniques to recover the transmitted symbols. Maximum Likelihood Sequence Estimation (MLSE) is optimal in the sense of minimizing error probability. The detection performance is determined by the minimum distance over all possible error sequences. From (4) and (5), channel outputs can be written as

$$y_{mn} = \sum_{k=0}^{L-1} h_{mk} s_{n-k}, m = 1, \ldots, M, n = 1, \ldots, T \tag{10}$$

Then the distance between the two sequences $\mathbf{y}^{(1)}, \mathbf{y}^{(2)}$ is

$$d^2(\mathbf{y}^{(1)}, \mathbf{y}^{(2)}) = \sum_{m=1}^{M} \|\mathbf{h}_m \odot (\mathbf{s}^{(1)} - \mathbf{s}^{(1)})\|_2^2 \tag{11}$$

From (11), if only one symbol (excluding head and tail symbols) is in error, then $\mathbf{e} = \mathbf{s}^{(1)} - \mathbf{s}^{(1)} = [0, \cdots, 0, e, 0, \cdots, 0]$ and $d^2(\mathbf{y}^{(1)}, \mathbf{y}^{(2)}) = |e|^2 \|\mathbf{H}\|_F^2$, which means $ML$ order diversity gain has been achieved. If there are multiple symbol errors but their separation is larger than $L$, we still have $ML$ order diversity gain. When $T$ is large, the probability of high order diversity gain is large. In general, we need to search all error sequences to find the minimum distance

$$d_{min}^2 = \min_{\mathbf{e}} \sum_{m=1}^{M} \|\mathbf{h}_m \odot \mathbf{e}\|_2^2 \tag{12}$$

Since we use finite MLSE, the Viterbi algorithm is employed to find $d_{min}$.

## 5. COMPUTER SIMULATIONS

A computer simulation has been conducted to illustrate the performance of the transmission scheme. We consider the following fast fading channel: the BS has a circular array with 8 antennas, the MS has 5 antennas, there exist $L$ specular paths between BS and MS, the length of CI is $T = 10$, the channel coefficients are Rayleigh distributed and are independent for different CIs. The mean SNR at each receiving antenna is defined as $10 \log{(P_{tot}/\sigma^2)}$ where $P_{tot}$ is the average power delivered from all transmission antennas and $\sigma^2$ is noise power. We first consider $L = 2$ and SNR = $15$ dB and $500$ consecutive CIs. The received eye pattern is shown in Figure 2. In Figure 3, we show the RMSE of output symbols versus SNR for $L = 2$. So for adequate SNR, the system can work pretty well. To further illustrate the diversity gain, we use the Viterbi algorithm to calculate the minimum distance over all error events for a different number of paths and normalize the results with minimum distance $d_{MRC}$ corresponding to maximum ratio combining (MRC). The cumulative distribution (C.D.F.) of the normalized distances are shown in Figure 4. It shows that as number of paths $L$ increases, the diversity gain from transmission design cannot increase as fast as that from receive MRC. As pointed out in the last section, if $L$ increases while the length of CI remains constant, the probability of multiple symbol errors falling into one code segment of length $L$ will increase and thus the relative diversity gain is reduced. On the other hand, increasing the length of CI will reduce the error probability.

## 6. CONCLUSION

We have considered the channel parameter based transmission scheme for a fast fading vector channel. The spatial energy distribution of MS w.r.t BS is utilized to design downlink transmission. A direction-time coding scheme is proposed (i) to convert the flat fading channel into a frequency-selective channel; (ii) to orthogonalize all directions; (iii) and where each symbol is transmitted $L$ times using different direction vectors. Multiple antennae at the MS or multiple transmissions of the same code block at the BS introduce multiple channels at the receiver. Then blind channel estimation is possible which is highly desirable in the fast fading scenario. Diversity gain is achieved through equalization techniques at the receiver. The length of CI and the number of paths will determine the relative diversity gain w.r.t. MRC. Further work is needed to include the case of unequal direction power, to analyze the the relation between spatial vector design and diversity gain and to design an efficient detection scheme at the receiver.

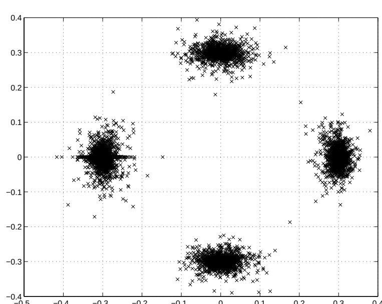

**Fig. 2**. Receiver Output Constellations for SNR = 15dB, L = 3, M = 5, N= 8, T = 10

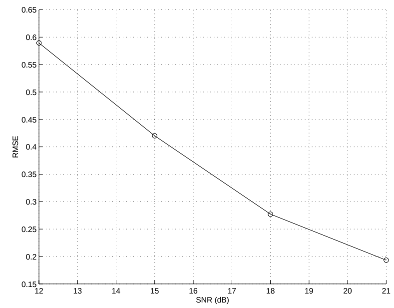


**Fig. 3**. RMSE vs. SNR L=2, M = 5, N= 8, T = 10

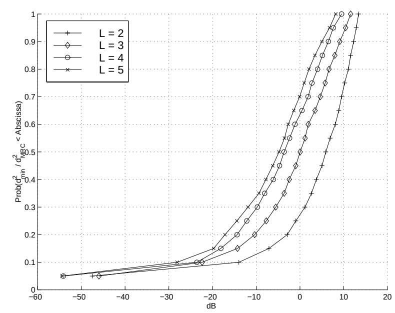


**Fig. 4**. C.D.F. of the normalized minimum distance for different number of paths, M = 5, N= 8, T = 10

## 7. REFERENCES


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