TIME AND SCALE EVOLUTIONARY EVD AND DETECTION

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Abstract:

Optimal detection of a known signal in nonstationary noise requires tracking the eigenvalue decomposition (EVD) of the noise data over time. To take advantage of information in the long-term, as well as short-term, correlation lags we turn to EVD over wavelet subspaces. In this paper, we develop a multirate EVD updating method over multiresolution subspaces and find maximum detectability nodes on wavelet binary full-tree structures. We use theoretical analysis to justify the effectiveness of a hyperspectrum for noise based on time and scale evolutionary EVDs. We also show results obtained with simulated 1/ f noise and noise collected by hydrophones of an underwater sonar communication system. Initial results are encouraging as they clearly indicate many subspaces, where detectability is significantly higher than in the original space prior to wavelet decomposition.

I. INTRODUCTION

Optimal detection of signals in noise requires the computation of noise eigenvalues and vectors, which form the Karhunen-Loeve (KL) basis. This is a computationally complex operation and is subject to numerical instabilities when the size of the covariance matrix is large. Fixed transforms such as the Discrete Cosine Transform (DCT) and Discrete Wavelet Transform (DWT) [1] are acceptable approximations to the KL transform and work for specific random processes. Many natural or human generated noise environments are non-stationary. Some examples are the different types of ambient acoustic noise through which a cellular telephone [5] user communicates and underwater ambient noise [1]. Under such circumstances, a fixed transform is not sufficient; and there are benefits to be gained from using the KL transform, which is based on noise statistics.

The underlying reason for obtaining eigenvalues and vectors of the noise is to whiten the noise, and follow it by matched filtering. If the signal can be designed, then the probability of error in the detection process is reduced by choosing the signal to reside in the subspace spanned by the eigenvector corresponding to the smallest eigenvalue. In such cases, matched filtering is equivalent to finding the projection of the incoming signal on the signal subspace. For a complete development of subspace matching techniques, the reader is referred to [9][8]. In a non-stationary environment, the KL transform matrix must be updated as well. A thorough and comprehensive treatment of the different techniques for updating EVD and Singular Value Decompositions (SVD) can be found in [7]. Update techniques surveyed share a common objective: What is the "best" way to find the new covariance matrix given the current covariance matrix and new data. Does the rank of the update go up, down or stay the same? Increasing the rank, computing the eigenvalues and then reducing the rank as determined by the eigenvalues is theoretically risk free, but computationally very expensive. In practice, increase in size of the covariance matrix subjects the EVD to numerical instabilities. On the other hand, confining the rank to be a small number, even when it is coupled by dominant subspace tracking, does not take into consideration the long-term correlations of the data.

In this paper, we develop an algorithm for detection on sets of nested subspaces: Multiresolution (MR) subspaces on the outside and KL subspaces on the inside. The process, which is described in the next section, allows for optimal detection, uses small KL matrixes, and takes into consideration long and short-term autocorrelation lags. It is numerically stable and allows for flexibility in signal design that may be combined with coding and encryption.

Detection of a known signal over multiresolution subspaces was analyzed in our previous works [3][4] and applied successfully to voice activity detection in [5]. In this paper, the proposed work is extended to include time and scale evolution of EVDs. It is applied to simulated 1/f noise and real underwater noise. We discuss detection and signal design strategies based on the resulting time and scale evolutionary EVDs.

This paper is organized as follows. We review the optimal detection problem in Section II. Section III, we highlight detection over MR subspaces. In Section IV, we discuss the evolution of detectability over scales and eigenvalues over time and show computation with simulated 1/f noise and real underwater ambient noise. Conclusion section follows.

II. REVIEW OF OPTIMAL DETECTION

A binary detection problem [10] of a known signal $\mathbf{s}=[s[1] \quad s[2] \quad \ldots \quad s[N]]^T$, in noise, $\mathbf{n}$, is described by two hypotheses: $H_1 : \mathbf{x=s+n}$ and $H_0 : \mathbf{x=n}$ where $\mathbf{n}$ is a zero-mean Gaussian random noise. The noise autocorrelation matrix $\mathbf{R}$ has the EVD decomposition $\mathbf{R=Q \Lambda Q^T}$, where $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues, λ_k. The likelihood ratio (LR) test is derived from probabilistic considerations to minimize the risk of making a wrong decision. Expressions for LR are useful and efficient when stated in terms of noise statistics. The difference between LR under the two hypotheses is used as a measure of detectability, given by

$$d^2 = \sum_{k=1}^{N} \frac{|s^{/}[k]|^2}{\lambda_k}, \qquad (1)$$

where s'_k are the elements of $\mathbf{Q^T s}$. The sufficient statistic required for making a decision is the first term of the exponent of LR as in equation,

$$g = \sum_{k=1}^{N} \frac{x^{/}[k]s^{/}[k]}{\lambda_k}. \qquad (2)$$

Decision is made by comparing g to a threshold determined by the probability of false detection and d^2.

III. DETECTION OVER MR SUBSPACES

For notational clarity, we refer to Figure 1, where all the node subscripts are defined as a number pair. Figure 2 shows a subtree starting at level j, branching into j+1 and its synthesis stage. Signal $\bar{\mathbf{x}}_{j,k}$ at node (j,k) is decomposed to its approximation projection, $\bar{\mathbf{x}}_{j+1,2k}$, and its detail, $\bar{\mathbf{x}}_{j+1,2k+1}$, so that $\bar{\mathbf{x}}_{j,k}= \bar{\mathbf{x}}_{j+1,2k}+\bar{\mathbf{x}}_{j+1,2(k+1)}$. The approximation coefficients are $\mathbf{x}_{j+1,2,k}$, and the detail coefficients are $\mathbf{x}_{j+1,2k+1}$. Subsequent decompositions of the approximation coefficient form what is known as a half-binary tree wavelet decomposition. If the detail coefficients are also decomposed, the structure is said to form a full binary tree as shown in Figure 1. The approximation (detail) coefficients are generated by filtering $\mathbf{x}$ by $\mathbf{h}$ ($\mathbf{g}$), followed by decimation by two. See, e.g. [1] for further clarification. The process can be represented as a matrix $\mathbf{H}$ ($\mathbf{G}$) operating on the input signal vector $\mathbf{x}$. Upsampling and filtering by $\mathbf{h'}$ (representing h[-n]) ($\mathbf{g'}$) can also be represented using a matrix $\mathbf{H'}$ ($\mathbf{G'}$). With this notation we have

$$\mathbf{x}_{1,0}=\mathbf{Hx} \qquad \mathbf{x}_{1,1}=\mathbf{Gx} \qquad (3a)$$

$$\bar{\mathbf{x}}_{1,0}=\mathbf{H'x}_{1,0}=\mathbf{H'Hx} \qquad \bar{\mathbf{x}}_{1,1}=\mathbf{G'x}_{1,1}=\mathbf{G'Gx}. \qquad (3b)$$

Repetition of this process on $\mathbf{x}_{1,0}$ and $\mathbf{x}_{1,1}$ produces the subsequent projections. Matrices $\mathbf{H}$, $\mathbf{H'}$, $\mathbf{G}$ and $\mathbf{G'}$ are sparse. Further, using the symmetric matrices $\mathbf{T}_h=\mathbf{HH'}$ and $\mathbf{T}_g=\mathbf{GG'}$, all subsequent projections can be written as repeated operations of these matrices.

There are other ways of representing wavelet transformations. Use of this format allows us to compute the covariance matrices of projections and coefficients, which are needed for further analysis. It also shows that it is sufficient to limit the analysis to a two-stage transform. Let $\mathbf{R}_x=E[\mathbf{xx}^T]$ be the NxN covariance matrix of $\mathbf{x}$. $\mathbf{R}_{1,0}$, $\mathbf{R}_{1,1}$, $\bar{\mathbf{R}}_{1,0}$ and $\bar{\mathbf{R}}_{1,1}$ are defined similarly. We have

$$\mathbf{R}_{1,0}=\mathbf{HR}_x\mathbf{H'} \qquad \mathbf{R}_{1,1}=\mathbf{GR}_x\mathbf{G'} \qquad (4a)$$

$$\bar{\mathbf{R}}_{1,0}=\mathbf{H'R}_{1,0}\mathbf{H}= \mathbf{H'HR}_x\mathbf{H'H} \qquad (4b)$$

$$\bar{\mathbf{R}}_{1,1}=\mathbf{G'R}_{1,1}\mathbf{G}= \mathbf{G'GR}_x\mathbf{G'G}. \qquad (4c)$$

$\mathbf{H}$ and $\mathbf{G}$ are size MxN matrices where M<N. In particular, M≅N/2 reflecting convolution, followed by decimation. $\mathbf{H'}$ and $\mathbf{G'}$ are of size NxM and reflect upsampling followed by convolution. For orthogonal wavelets, $\mathbf{H}$ ($\mathbf{G}$) is the left pseudo inverse of $\mathbf{H'}$ ($\mathbf{G'}$) so that $\mathbf{HH'=GG'=I}_M$, the identity matrix of size M.

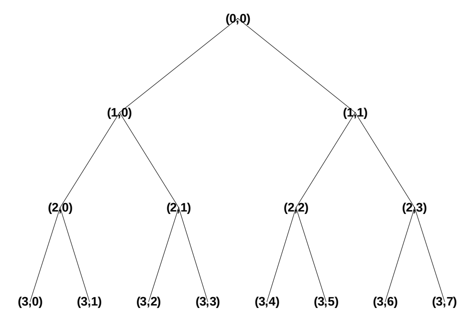

Figure 1. Full-tree wavelet decomposition to define notation.

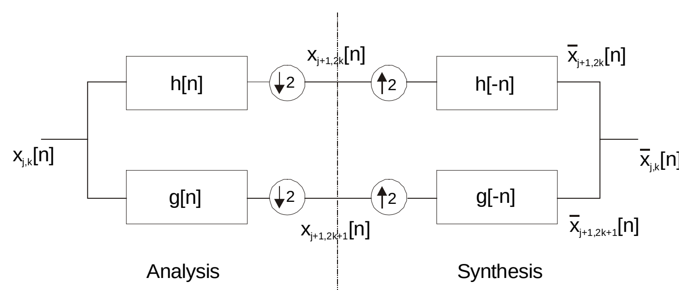

Figure 2. Analysis/synthesis of wavelet coefficients

The wavelet decomposition is a complete representation of a signal. The sets of coefficients on any cutset of the analysis subtree contain complete information about the input signal, as they can be followed up by the synthesis section to reconstruct the input signal. It is well known and clear that the branching can continue indefinitely or it can be terminated at any subbranch. With reference to the full binary tree structure shown in Figure 1, we seek answers to the following questions:

1. Does there exist a set of subspaces that maximize detectability?
2. If such a set exists, how is it determined and how does it vary with time?

Theoretical and empirical studies have shown that a set of subspaces exists where detectability increases significantly. To find the set requires a non-linear search algorithm as can be seen from the results in Section IV.2. When either noise statistics or signal varies with time, the set of optimum detectability varies. To track this set requires updating of the EVD of the subspaces, as is described in Section IV.3. In the following section, we state theoretical background on the EVD and detectability over multiresolution subspaces.

IV. EVD AND DETECTABILITY OVER MR SUBSPACES

The full-tree representation shown in Figure1 is the analysis section of the filter banks decomposition of a signal. It has a corresponding synthesis part, which is shown for one scale in Figure 2. The outputs of the synthesis filters are the called the projections of the signal. The entire analysis/synthesis structure gives us much flexibility in choosing nodes on which to consider noise and signal characterization and detectability. Given by Eq. 1, computation of detectability at any node requires the computation of the eigenvalues and eigen-vectors of the noise process at that node. It is important to know how they are related.

IV.1.Eigenvalues

In particular, we want to know how the eigenvalues of the coefficients and the projections are related. The eigenvalues of the projection covariance matrix $\overline{\mathbf{R}}_{1,0}$ are the roots of the polynomial

$$\left|\lambda \mathbf{I} - \overline{\mathbf{R}}_{1,0}\right| = \left|\lambda \mathbf{I} - \mathbf{H}' \mathbf{R}_{1,0} \mathbf{H}\right| \qquad (5a)$$

are the same as the roots of

$$\left|\mathbf{R}_{1,0}\right| \left|\lambda \mathbf{I} - \mathbf{H}' \mathbf{R}_{1,0} \mathbf{H}\right| . \qquad (5b)$$

Applying to this, the identity

$$\left|\mathbf{A}\right|\left|\mathbf{E} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B}\right| = \left|\mathbf{E}\right|\left|\mathbf{A} - \mathbf{B}\mathbf{E}^{-1}\mathbf{C}\right| \qquad (6)$$

with $\mathbf{A} = \mathbf{R}^{-1}{}_{1,0}, \mathbf{E} = \lambda\mathbf{I}, \mathbf{C} = \mathbf{H}', \mathbf{B} = \mathbf{H}$, yields

$$\left|\lambda \mathbf{R}_{1,0}{}^{-1} - \mathbf{H}\mathbf{H}'\right| = \left|\mathbf{R}_{1,0}{}^{-1}\right|\left|\lambda \mathbf{I} - \mathbf{R}_{1,0}\right| \qquad (7)$$

where use has been made of **HH'=I**.

This proves that the eigenvalues of $\overline{\mathbf{R}}_{1,0}$ are identical to the eigenvalues of $\mathbf{R}_{1,0}$, and the remaining N-M eigenvalues are zero. Thus we deduce that not only can we, but also we must confine our analysis to the coefficients.

IV.2. Detectability

Given by Eq.1, detectability is a metric of distance between the vector **s'** and $\gamma = [1/\lambda_1, 1/\lambda_2, ..., 1/\lambda_N]$ where **s'** is the KL transform (with respect to the noise EVD) of the signal **s.** If noise is white, then d^2 is the signal-to-noise ratio, otherwise it is a quantity between 0 and infinity and has to do with the relative distribution of the signal and eigenvalues. The relationship between detectability at scales j and j+1 is based on the principle of energy conservation. For simplicity, we will consider scale j=0. Let the KL-transform of the signal coefficients at nodes (0,0), (1,0) and (1,1) be given **s**, $\mathbf{s}_0$ and $\mathbf{s}_1$ respectively. Also, let the vectors of noise eigenvalues be given as γ , γ^0, γ^1 respectively. We have

$$\lambda_k = \lambda_k^0 + \lambda_k^1 , \qquad (8)$$

$$s^2[k] = [s_0^2[k] + s_1^2[k]] \text{ and} \qquad (9)$$

$$d^2 = \frac{s^2[k]}{\lambda_k}, \quad d_i^2 = \frac{s_i^2[k]}{\lambda^i{}_k} \text{ for i=0,1.} \qquad (10)$$

If noise is white, the detectability in parent-children subspaces are related by

$$d^2 = \frac{\sigma_0^2}{\sigma^2} d_0{}^2 + \frac{\sigma_1^2}{\sigma^2} d_1{}^2 \qquad (12)$$

where $\sigma^2 = \sigma_0^2 + \sigma_1^2$ are the respective noise variances. Clearly, detectability in any subspace is highly affected by the noise eigenvalues. A redistribution causing one of them to be close to zero may result in near perfect detection. This is a very encouraging reason to do detection over multiresolution subspaces. Tables 1 and 2 show detectability computed at each node of the full binary tree for simulated pink noise and ambient ocean noise. The signal used was a windowed sinusoid.

Nd	d^2	Nd	d^2	Nd	d^2	Nd	d^2
(0,0)	12.5	(1,0)	54.7	(2,0)	1.42	(3,0)	0.0143
		(1,1)	1.99	(2,1)	20.3	(3,1)	5.3
				(2,2)	1.32	(3,2)	0.576
				(2,3)	4.12	(3,3)	45.5
						(3,4)	0.0384
						(3,5)	2.83
						(3,6)	0.0354
						(3,7)	12.3

Table 1. Detectability of sinusoidal signal in pink noise.

Detectability (d^2) is shown at each node (Nd) numbered according to the convention defined by Figure 1. We note that for pink noise, detectability of 12.5 at node (0,0) nearly quadruples at node (1,0) and reaches a local maximum, suggesting that no further decomposition past the first stage is necessary. Data in Table 2 were obtained from ambient ocean noise recorded by a hydrohone. Local maximum for detectability is reached at node (2,0). The two sets of data have the same signal-to-noise ratio.

Nd	d^2	Nd	d^2	Nd	d^2	Nd	d^2
(0,0)	45	(1,0)	17.6	(2,0)	72.3	(3,0)	2.14
		(1,1)	0.07	(2,1)	33.5	(3,1)	65.8
				(2,2)	0.08	(3,2)	0.05
				(2,3)	0.08	(3,3)	68.4
						(3,4)	0.002
						(3,5)	0.037
						(3,6)	0.001
						(3,7)	0.217

Table 2. Detectability of sinusoidal signal in sonar ocean noise.

IV.3. Time evolution of the noise eigenvalues

Since detectability is highly affected by noise distribution, it follows naturally that we observe the variation of the noise eigenvalues with time. The motivation for this computation is to see if the change was significant and updating the EVD at the nodes of the wavelet tree was necessary. A sample computation at node (1,0) for ocean noise is given in Figure 3. We note that a dominant eigenvalue fluctuates significantly in time at that node.

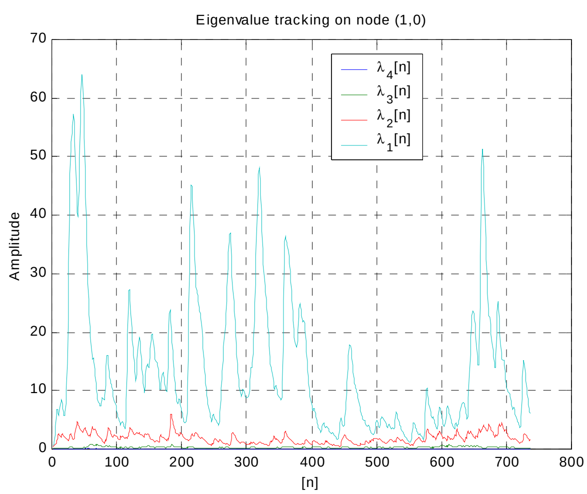

Figure 3. Time evolution of eigenvalues in node (1,0).

V. CONCLUSION

A method of detection of a known signal in nonstationary Gaussian noise has been proposed. It has been argued that applying optimal detection techniques over multiresolution subspaces has advantages over traditional subspace matching or whitening plus matched filtering methods. They are increased detectability and numerical stability. We have shown theoretically and empirically that there exist multiresolution subspaces that show significant increase in detectability, that they can be found by a search through the full-tree wavelet decomposition structure. The significance of updating the EVD of the noise at the nodes of the wavelet decomposition tree has been evidenced, indicating the need for a scale and time evolutionary spectral representation for reliable detection.

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