HIGH THROUGHPUT WIDEBAND SPACE-TIME SIGNALING USING CHANNEL STATE INFORMATION

Eko N. Onggosanusi, Barry D. Van Veen, and Akbar M. Sayeed

Department of Electrical and Computer Engineering
University of Wisconsin–Madison, Madison, WI 53706

ABSTRACT

An orthogonal decomposition of a general wideband space-time multipath channel is derived assuming antenna arrays at both the transmitter and receiver. This decomposition provides a framework for efficiently managing the available space-time channel dimensions using channel state information at the transmitter and receiver. Signaling and receiver designs for high throughput applications are derived using this decomposition. For a fixed throughput system, we investigate a power allocation scheme that minimizes the effective bit-error rate. In addition, a strategy to maximize the average throughput is discussed.

1. INTRODUCTION

Use of antenna arrays at both the base station and mobile is envisioned in future wireless communication systems. Channel information can be used at the transmitter to design efficient signaling schemes (see e.g. [1, 3]). Channel information may be obtained at the transmitter via several means. In time-division duplexing (TDD), the uplink and downlink channels are reciprocal so the transmitter can estimate the channel using pilot and/or data symbols transmitted by the receiver. In frequency-division duplexing (FDD), a feedback channel may be used to relay channel information estimated by the receiver back to the transmitter. In [1] we derive an optimal wideband multi-antenna signaling scheme that minimizes the bit-error rate (***BER***) given either channel *state* information (CSI) or channel *statistics*.

The results in [1] are extended in this paper for high throughput systems. An orthogonal decomposition of the general L-path wideband space-time channel is derived assuming P-transmit and Q-receive antennas. This decomposition serves as a framework for efficiently managing the degrees of freedom in the space-time channel to optimize any combination of BER and throughput criteria. We study the trade-off between BER and throughput by deriving a power allocation scheme that minimizes the BER for a fixed throughput assuming BPSK modulation in each sub-channel. In addition, a strategy for choosing the instantaneous throughput to maximize the average throughput is proposed.

The rest of the paper is organized as follows. The space-time channel model and orthogonal decomposition are given in Sections 2 and 3, respectively. Signaling and receiver designs that trade throughput for BER are outlined in Section 4 and 5, respectively.

Superscript T, H, and $*$ indicate matrix transpose, matrix conjugate transpose, and complex conjugation, respectively. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. The symbol $\otimes$ denotes Kronecker product and vec$(\mathbf{A})$ is formed by stacking the columns of matrix $\mathbf{A}$ into a vector [2]. The i-th largest eigenvalue and the corresponding eigenvector of matrix $\mathbf{A}$ is denoted by $\lambda_i[\mathbf{A}]$ and $\mathbf{ev}_i[\mathbf{A}]$, respectively. We assume that perfect estimates of CSI are available at the transmitter and receiver. We also assume that the maximum transit time across the array is assumed small compared to the inverse bandwidth of the signal.

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2. GENERAL SINGLE-USER MULTI-ANTENNA FRAMEWORK

Consider a system with P transmit and Q receive antennas. We assume that each component of the transmitted signal $\mathbf{x}(t) = [\, x_1(t)\,,\ x_2(t)\,, \cdots,\ x_P(t)\,]^T$ has duration T and bandwidth B, which propagates over an L-path P-input, Q-output fading channel with delays $\{\tau_1, \cdots, \tau_L\}$. We consider a typical wideband multipath channel where $\tau_l \ll T$ for all l. Hence, intersymbol interference is negligible and the baseband signal at the q-th receive antenna over one symbol interval can be written as

$$\begin{aligned} r_q(t) &= \left[\, \mathbf{x}^T(t-\tau_1) \cdots \mathbf{x}^T(t-\tau_L) \,\right] \mathbf{h}_q + n_q(t) \quad (1)\\ \mathbf{h}_q^T &\stackrel{def}{=} \left[\, \mathbf{h}_{1q}^T \ \cdots \ \mathbf{h}_{Lq}^T \,\right], \text{ and}\\ \mathbf{h}_{lq}^T &\stackrel{def}{=} \left[\, h_{lq1} \ \cdots \ h_{lqP} \,\right]. \end{aligned}$$

The channel coefficient h_{lqp} corresponds to the l-th path between the q-th receive and p-th transmit antenna. The additive noise vector $n_q(t)$ is assumed to be zero mean circularly symmetric complex Gaussian with $E[n_q(t)n_q^*(t')] = \sigma^2\delta(t-t')$. We assume for a single transmitted bit stream b that the p-th antenna waveform $x_p(t)$ has the following form:

$$x_p(t) = \sqrt{\rho}\, b \sum_{i=0}^{N-1} s_p[i]\omega(t - i/B)\,, \quad 0 \le t < T \quad , \tag{2}$$

where $\omega(t)$ is the (unit-energy) chip waveform of duration $1/B$, ρ is the transmit power, and $N = TB$. Here $s_p[i]$, $i = 0, 1, \cdots, N-1$, represents the signature sequence corresponding to the p-th antenna. Since $x_p(t)$ is bandlimited to B, it suffices to assume that $\tau_l = d_l/B$, $d_l \in \{0, 1, \cdots, N-1\}$. We sample $r_q(t)$ in (1) at the rate $\frac{1}{B}$:

$$\begin{aligned} \mathbf{r}_q &\stackrel{def}{=} [r_q(0), r_q(1/B), \cdots, r_q((N-1)/B)]^T \quad ,\\ \mathbf{s}_p &\stackrel{def}{=} [s_p[0], s_p[1], \cdots, s_p[N-1]]^T \quad ,\\ \mathbf{S} &\stackrel{def}{=} [\, \mathbf{s}_1 \cdots \mathbf{s}_P \,] \quad . \end{aligned} \tag{3}$$

Now, define $\mathbf{\Delta}_{d_l} \in \mathbb{C}^{N \times N}$ as the time-shift matrix corresponding to the path delay d_l. We assume the delay is cyclic, so that $\mathbf{\Delta}_{d_l}$

is circulant. The circulant assumption is reasonable for $d_l \ll N$, and is exact if a cyclic prefix is transmitted. Furthermore, define $\boldsymbol{\Delta} \stackrel{def}{=} [\boldsymbol{\Delta}_{d_1} \cdots \boldsymbol{\Delta}_{d_L}] \in \mathbb{C}^{N\times NL}$, $\mathbf{s} = \text{vec}(\mathbf{S}) \in \mathbb{C}^{NP}$, and $\mathbf{H}_q \stackrel{def}{=} [\mathbf{h}_{1q} \cdots \mathbf{h}_{Lq}] \in \mathbb{C}^{P\times L}$ such that $\mathbf{h}_q = \text{vec}(\mathbf{H}_q)$. Then, from (1) – (2) and applying the identity $\text{vec}(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})\text{vec}(\mathbf{X})$ [2] twice, we have

$$\mathbf{r}_q = \sqrt{\rho}\, b\boldsymbol{\Delta}\left(\mathbf{I}_L \otimes \mathbf{S}\right)\mathbf{h}_q + \mathbf{n}_q = \sqrt{\rho}\, b\ \mathcal{H}_q\mathbf{s} + \mathbf{n}_q\ , \quad (4)$$

where $\mathcal{H}_q = \boldsymbol{\Delta}\left(\mathbf{H}_q^T \otimes \mathbf{I}_N\right)$ and $\mathbf{n}_q \sim \mathcal{N}_C\left[\mathbf{0}, \sigma^2\mathbf{I}_N\right]$.

3. SPACE-TIME CHANNEL DECOMPOSITION

The overall MIMO space-time channel may be represented as $\mathcal{H} \stackrel{def}{=} \left[\begin{array}{ccc}\mathcal{H}_1^T & \cdots & \mathcal{H}_Q^T\end{array}\right]^T \in \mathbb{C}^{NQ\times NP}$. The number of available dimensions for transmission N_{dim} is the rank of $\mathcal{H}$. Assuming that the channel coefficients $\{h_{lqp}\}$ are not perfectly correlated, $N_{dim} = N \times \min(P, Q)$ w.p.1. Our goal is to design transceivers that access all N_{dim} dimensions. This can be accomplished when a singular value decomposition (SVD) of $\mathcal{H}$ is available analogous to [3]. However, as in [3], there is no closed-form expression for the overall $\mathcal{H}$. In this paper, instead of using an SVD for $\mathcal{H}$, we present a closed-form SVD for $\mathcal{H}_q$ and show that via appropriate system designs, all N_{dim} dimensions can be accessed. The proof is given in [4].

Theorem 1 *Define*

$$\mathbf{c}_n \stackrel{def}{=} \frac{1}{\sqrt{N}}\left[\begin{array}{cccc}1 & e^{-j2\pi n/N} & \cdots & e^{-j2\pi(N-1)n/N}\end{array}\right]^T \in \mathbb{C}^N$$

$$\mathbf{g}_{n,q} \stackrel{def}{=} \frac{1}{\sqrt{N}}\left[\begin{array}{c}\sum_{l=1}^L h_{lq1}e^{-j2\pi n d_l/N} \\ \vdots \\ \sum_{l=1}^L h_{lqP}e^{-j2\pi n d_l/N}\end{array}\right]^* \in \mathbb{C}^P . \quad (5)$$

Then, $\mathcal{H}_q \in \mathbb{C}^{N\times NP}$ admits the following SVD*:*

$$\mathcal{H}_q = \sum_{n=0}^{N-1} \sigma_{n,q}\mathbf{c}_n\mathbf{v}_{n,q}^H\ , \quad (6)$$

$$\sigma_{n,q} \stackrel{def}{=} \|\mathbf{g}_{n,q}\|\ ,\quad \mathbf{v}_{n,q} \stackrel{def}{=} \frac{\mathbf{g}_{n,q}}{\|\mathbf{g}_{n,q}\|} \otimes \mathbf{c}_n \quad (7)$$

□

Notice that the p-th element of $\mathbf{g}_{n,q}$ is the complex conjugate of the frequency response of the channel between the q-th receive and p-th transmit antenna at frequency $\frac{2\pi}{N}n$.

Consider implementing the maximum likelihood (ML) or maximum ratio combining (MRC) receiver for the bit b. We may decompose the MRC receiver into two stages: matched filtering with the channel coefficients and combining across all received antennas ($\tilde{\mathbf{r}} \stackrel{def}{=} \sum_{q=1}^Q \mathcal{H}_q^H\mathbf{r}_q$), followed by matched filtering with the signature code $\mathbf{s}$. Substitute for $\mathbf{r}_q$ to obtain

$$\tilde{\mathbf{r}} = \sqrt{\rho}\, b\left(\sum_{q=1}^Q \mathcal{H}_q^H\mathcal{H}_q\right)\mathbf{s} + \sum_{q=1}^Q \mathcal{H}_q^H\mathbf{n}_q \quad (8)$$

Now apply Theorem 1 and the identity $(\mathbf{X}_1 \otimes \mathbf{X}_2)(\mathbf{Y}_1 \otimes \mathbf{Y}_2) = \mathbf{X}_1\mathbf{Y}_1 \otimes \mathbf{X}_2\mathbf{Y}_2$ to get

$$\sum_{q=1}^Q \mathcal{H}_q^H\mathcal{H}_q = \sum_{n=0}^{N-1} \boldsymbol{\Gamma}_n \otimes \mathbf{c}_n\mathbf{c}_n^H \in \mathbb{C}^{PN\times PN} \quad (9)$$

$$\boldsymbol{\Gamma}_n \stackrel{def}{=} \sum_{q=1}^Q \mathbf{g}_{n,q}\mathbf{g}_{n,q}^H \in \mathbb{C}^{P\times P} \quad (10)$$

where $\boldsymbol{\Gamma}_n$ is the overall spatial matrix at frequency $\frac{2\pi}{N}n$. Since the rank of $\boldsymbol{\Gamma}_n$ in (10) is $\min(P,Q)$, it is clear from (9) that $\sum_{q=1}^Q \mathcal{H}_q^H\mathcal{H}_q$ is of rank $N_{dim} = N \times \min(P,Q)$ w.p.1. It is easy to verify that the N_{dim} eigenvectors of $\sum_{q=1}^Q \mathcal{H}_q^H\mathcal{H}_q$ are

$$\mathbf{w}^{((i-1)N+n)} \otimes \mathbf{c}_n\ ,\quad n \in \{0,1,\cdots,N-1\}\ ,$$
$$i \in \{1,2,\cdots,\min(P,Q)\} \quad (11)$$
$$\text{where}\quad \mathbf{w}^{((i-1)N+n)} = \mathbf{ev}_i\left[\boldsymbol{\Gamma}_n\right]\ .$$

The corresponding eigenvalues are $\gamma_{(i-1)N+n+1} = \lambda_i\left[\boldsymbol{\Gamma}_n\right] > 0$. These N_{dim} eigenmodes represent all the available *orthogonal space-time sub-channels* for any MRC based receiver.

Notice that to compute all the eigenvectors in (11), the most costly operation is finding the eigenvectors of N different $P\times P$ matrices. This is much less complex than computing the singular modes of the $NQ \times NP$ matrix $\mathcal{H}$ since N is large and P is small in practice.

4. SINGLE-USER WITH MINIMUM BER

Maximum (selection) diversity gain is obtained by transmitting only via the most dominant sub-channel [1]. In this case,

$$\mathbf{s} = \mathbf{w} \otimes \mathbf{c}_{\overline{n}}\ , \quad (12)$$
$$\mathbf{w} = \mathbf{ev}_1\left[\boldsymbol{\Gamma}_{\overline{n}}\right]\ ,\quad \overline{n} = \arg\max_{n=0,\cdots,N-1} \lambda_1\left[\boldsymbol{\Gamma}_n\right]\ . \quad (13)$$

Notice that only *one* dimension is used in any one symbol duration to achieve minimum BER. This signaling scheme can be implemented as shown in Figure 1(a) with $n = \overline{n}$.

For receiver design, we assume BPSK modulation ($b \in \{\pm 1\}$). To simplify receiver complexity, we exploit Theorem 1 as follows. It is easy to show using the identity $(\mathbf{X}_1 \otimes \mathbf{X}_2)(\mathbf{Y}_1 \otimes \mathbf{Y}_2) = \mathbf{X}_1\mathbf{Y}_1 \otimes \mathbf{X}_2\mathbf{Y}_2$ and the orthogonality of $\{\mathbf{c}_n\}_{n=0}^{N-1}$ that

$$\mathcal{H}_q\left(\mathbf{w} \otimes \mathbf{c}_{\overline{n}}\right) = \left(\mathbf{g}_{\overline{n},q}^H\mathbf{w}\right)\mathbf{c}_{\overline{n}} \quad (14)$$

Hence, the test statistic Z can be written as:

$$Z = \left(\mathbf{w} \otimes \mathbf{c}_{\overline{n}}\right)^H \sum_{q=1}^Q \mathcal{H}_q^H\mathbf{r}_q = \sum_{q=1}^Q \left(\mathbf{w}^H\mathbf{g}_{\overline{n},q}\right)\mathbf{c}_{\overline{n}}^H\mathbf{r}_q\ . \quad (15)$$

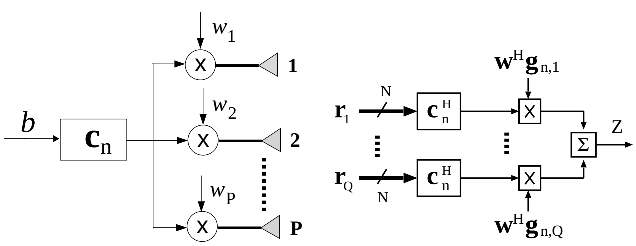
This can be implemented as shown in Figure 1(b) with $n = \overline{n}$.

5. HIGH THROUGHPUT SINGLE-USER SYSTEMS

We assume a fixed modulation scheme, so the system throughput is determined by the number of streams transmitted simultaneously. The throughput is bounded by N_{dim}. To transmit M data streams via the channel, we choose a transmitted signal of the form

$$\sum_{m=1}^M \sqrt{\rho_m}\, b_m\ \mathbf{s}^{(m)}\ , \quad (16)$$

where $\{\mathbf{s}^{(m)}\}$ are chosen to be a subset of the eigenvectors of $\sum_{q=1}^Q \mathcal{H}_q^H\mathcal{H}_q$ given in (11). At the receiver, different streams can be separated due to the orthogonality of $\{s^{(m)}\}$. The transmitter and receiver for this maximum throughput scheme may be implemented as shown in Figure 1 (a) and (b), respectively, for each data stream with n and $\mathbf{w}$ chosen accordingly. Without loss of generality, we assume that $\sigma^2 = 1$. Hence, ρ_m is the transmit power for the m-th stream normalized by the noise variance σ^2.



(a) Transmitter (b) Receiver

Fig. 1. Single-user minimum BER system for BPSK.

Throughput is maximized by using all the N_{dim} dimensions for data transmission. However, this comes at the expense of BER since the available power has to be distributed between N_{dim} streams. Instead of maximizing throughput, one may trade throughput for lower BER by choosing to transmit with $M < N_{dim}$. Since CSI is available at the transmitter, the sub-channel gains $\{\gamma_m\}_{m=1}^{N_{dim}}$ can be determined. Without loss of generality, assume that

$$\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_M \geq \gamma_{M+1} \geq \cdots \geq \gamma_{N_{dim}} > 0 \,. \quad (17)$$

Clearly, the most power efficient way to achieve a throughput of M is to use the M sub-channels with the highest gains. We define the *effective* BER of an M-stream system as follows:

$$BER_{eff}^{(M)} \stackrel{def}{=} \frac{1}{M}\sum_{m=1}^{M} BER\left(\rho_m\gamma_m\right) \,. \quad (18)$$

where $\rho_m\gamma_m$ is the received SNR corresponding to the m-th stream and $BER(\rho_m\gamma_m)$ is the BER. The effective BER reflects the average system performance across M sub-channels. The transmit power allocated for all streams $\{\rho_m\}_{m=1}^{M}$ satisfies the constraint $\sum_{m=1}^{M}\rho_m = \rho_{TOT}$. To achieve throughput of M, we require $\rho_m > 0$ for $m \in \{1, \cdots, M\}$ since $\rho_m = 0$ indicates that no transmission occurs on sub-channel m. We assume BPSK modulation, so that $BER(\rho_m\gamma_m) = \mathcal{Q}\left(\sqrt{2\rho_m\gamma_m}\right)$, where $\mathcal{Q}(x) \stackrel{def}{=} \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-u^2/2}du$.

5.1. Fixed Throughput Criterion

For a given throughput of M, we choose $\{\rho_m\}_{m=1}^{M}$ to minimize $BER_{eff}^{(M)}$ with respect to $\{\rho_m\}_{m=1}^{M}$. Although minimizing the effective BER ensures the best total performance over all sub-channels, it may result in some sub-channels with extremely low received SNR $\rho_m\gamma_m$. This may happen when the total transmit power ρ_{TOT} is low. To prevent this, a worst-case SNR constraints is employed. Thus, we pose the following optimization problem:

$$\{\overline{\rho}_m\}_{m=1}^{M} = \arg\min_{\rho_1,\cdots,\rho_M} \sum_{m=1}^{M} \mathcal{Q}\left(\sqrt{2\rho_m\gamma_m}\right) \quad (19)$$

$$\text{s.t.} \quad \sum_{m=1}^{M}\rho_m = \rho_{TOT} \;, \quad (20)$$

$$\rho_m\gamma_m \geq c_m \;, \quad m = 1, \cdots, M \quad (21)$$

The constant c_m is chosen such that $\mathcal{Q}\left(\sqrt{2c_m}\right)$ is the worst-case BER for sub-channel m.

The above optimization problem can be solved via the Kuhn-Tucker conditions. Constraints (20) and (21) imply that a solution exists if and only if

$$\rho_{TOT} \geq \rho_{co,M} \quad , \quad \rho_{co,M} \stackrel{def}{=} \sum_{m=1}^{M}\frac{c_m}{\gamma_m} \,. \quad (22)$$

where $\rho_{co,M}$ denotes the *cut-off* transmit power for a throughput of M. Hence, to maintain a relative throughput of M for different channel realizations, ρ_{TOT} may need to be adjusted accordingly. The *exact* solution of (19) is given in [4] using an iterative algorithm. An approximate solution can be obtained by replacing the exact BER for each sub-channel in (19) with its Chernoff bound. In this case, we minimize the upper bound $BER_{eff}^{(M)} \leq \frac{1}{2M}\sum_{m=1}^{M}\exp\left(-\rho_m\gamma_m\right)$. Using Kuhn-Tucker conditions, we obtain the following closed-form solution for ρ_m, $m = 1, \cdots, M$ assuming (22) holds:

$$\tilde{\rho}_m = \frac{\max\left(c_m, \log\gamma_m - \tilde{\mu}\right)}{\gamma_m} \quad (23)$$

where $\tilde{\mu}$ is chosen to satisfy the power constraint $\sum_{m=1}^{M}\tilde{\rho}_m = \rho_{TOT}$. We term this solution the *Chernoff-based* power allocation.

Another simple sub-optimal power allocation scheme that satisfies the constraints in (20) and (21) assuming (22) holds can be obtained as follows:

$$\hat{\rho}_m = \frac{c_m}{\gamma_m} + \frac{1}{M}\times\left(\rho_{TOT} - \rho_{co,M}\right) \,. \quad (24)$$

That is, after satisfying the minimum SNR constraint in each sub-channel, the remaining power is distributed equally across all sub-channels. We term this scheme *uniform* power allocation.

5.2. Maximum Throughput Criterion

In this section, we consider an adaptive throughput scheme where the instantaneous throughput is maximized subject to constraints (20) and (21). Let the set of 'allowable' throughput values be in the set $\mathcal{M}$. We assume $0 \in \mathcal{M}$ to allow 'no-transmission' when the channel undergoes deep fades such that the BER requirement can not be achieved for a given ρ_{TOT}. For each realization of sub-channel gains $\{\gamma_m\}$, we choose the largest M such that

$$\rho_{co,M} \leq \rho_{TOT} \,. \quad (25)$$

Note that the maximum throughput criterion is not coupled to any power allocation scheme, but only requires $\rho_m \geq \frac{c_m}{\gamma_m}$. Thus, one may use the minimum effective BER or uniform allocation scheme described above to choose the ρ_m.

5.3. Examples

For all the examples, we assume a $P = Q = 2$, $L = 3$ system with $d_l \in \{0, 1, 2\}$, $N = 16$, and hence $N_{dim} = 32$. The channel coefficients $\{h_{lqp}\}$ are IID and $\mathcal{N}_C\left[0, 1/QL\right]$ (Rayleigh fading).

The system is required to achieve the worst-case BER of $\varepsilon = 10^{-2}$ on each sub-channel. Hence, $c_m = \left(\mathcal{Q}^{-1}(\varepsilon)\right)^2/2 \ , \quad m = 1, \cdots, M$, where $\mathcal{Q}^{-1}(x)$ is the inverse of $\mathcal{Q}(x)$.

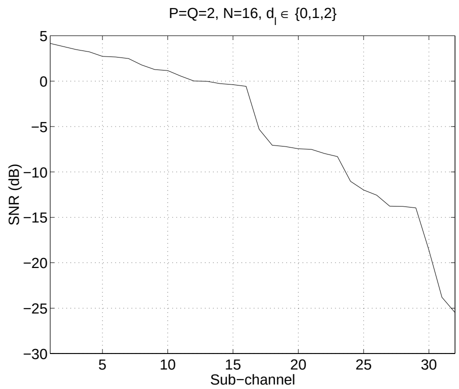


Fig. 2. A channel realization.

We compare the minimum exact and Chernoff-bounded effective BER to uniform power allocation for one channel realization. The resulting sub-channel SNR values are depicted in Figure 2. A comparison to the effective BER obtained using Chernoff bound and uniform power allocations is shown in Figure 3. Observe that the loss of performance due to uniform power allocation compared to the minimum effective BER solution is more pronounced as M increases. Also, the Chernoff approximation introduces negligible performance loss.

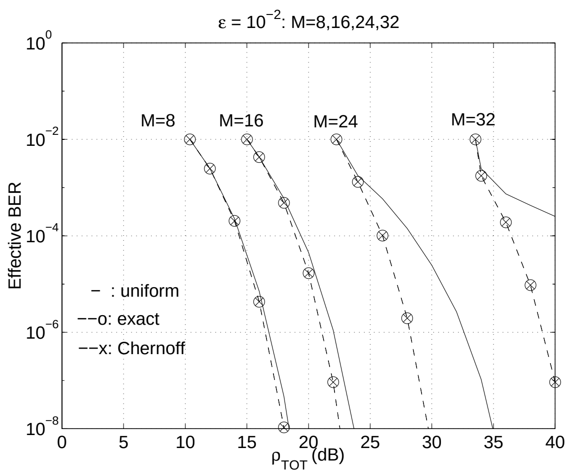
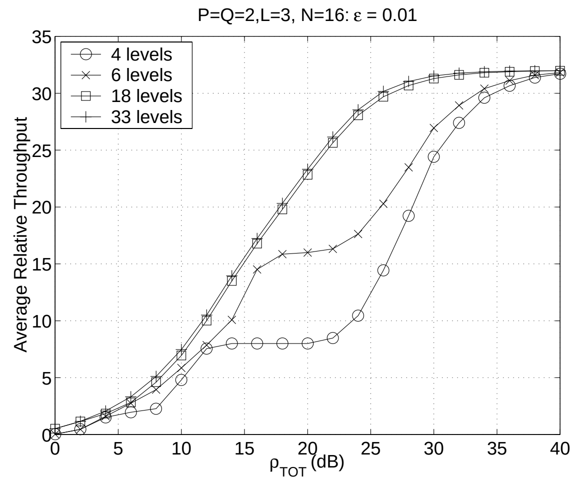


Fig. 3. Comparing $BER_{eff}^{(M)}$ of different schemes.

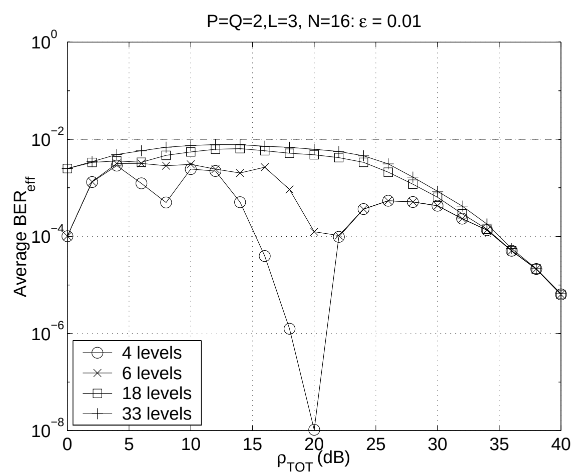
To demonstrate the notion of adaptive throughput, we use the Chernoff-bound power allocation scheme. Four different sets of allowable relative throughputs are used:

$$\begin{aligned} \mathcal{M}_1 &= \{0, 2, 8, 32\} && \text{(4 levels)} \\ \mathcal{M}_2 &= \{0, 2, 4, 8, 16, 32\} && \text{(6 levels)} \\ \mathcal{M}_3 &= \{0, 1, 2, 4, 6, 8, \cdots, 30, 32\} && \text{(18 levels)} \\ \mathcal{M}_4 &= \{0, 1, 2, 3, \cdots, 31, 32\} && \text{(33 levels)}\ . \end{aligned}$$

The average relative throughput and BER_{eff} are depicted in Figure 4 (a)–(b). Observe that larger sets result in better average throughput for any ρ_{TOT} and the resulting BER are closer to the worst-case requirement. With small sets excess power tends to reduce the effective BER rather than increase the number of channels, while with the larger sets increases in ρ_{TOT} tend to increase the number of channels, rather than reduce average BER_{eff}.



(a) Average throughput M



(b) Average BER_{eff}

Fig. 4. Adaptive throughput scheme.

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