SEQUENTIAL SIGNAL ENCODING AND ESTIMATION FOR DISTRIBUTED SENSOR NETWORKS

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ABSTRACT

We develop algorithms for sequential signal encoding from sensor measurements, and for signal estimation via fusion of channel-corrupted versions of these encodings. For signals described by state space models, we present optimized sequential binary-valued encodings constructed via threshold-controlled scalar quantization of a running Kalman filter signal estimate from the sensor measurements. We also develop methods for robust fusion from observations of these encodings corrupted by binary symmetric channels.

1. INTRODUCTION

In this paper we focus on signal estimation from binary-valued sequential encodings constructed from noisy measurements, where the encodings are observed through binary symmetric channels (BSC). Problems of this type arise in various distributed sensor networks, whereby each sensor must communicate its noisy signal measurements to a host with minimal delay. Inherent limitations in the available bandwidth the sensor apparatus and the modulation circuitry often dictate communication via a finite set of signal levels. Hence there is a need for methods for encoding the sensor measurements into a digital stream prior to communication over the channel with minimal delay.

We focus on the case that each sensor observes a noisy version of a signal described by a state-space model and produces a sequence of binary-valued encodings. The sequential encoders we construct operate on a running signal estimate obtained at the sensor via a Kalman Filter; this is consistent with [1], where it is shown that there is no loss of optimality in (batch-mode) encoding, if an encoder operates on the MMSE estimate from the data. In addition, it is consistent with the static signal case analysis for sequential encoding in [2], according to which, sequential encoding of an efficient running signal estimate from the measurements followed by a properly designed fusion rule asymptotically achieves the encoder-free MSE performance.

The encoding is obtained by adding a properly designed control input to the running sensor signal estimate followed by scalar quantization. Such encodings were considered in [3] for encoding noisy static signals. As shown in [3], in the static signal case for control inputs that are IID processes, there exists an optimal power level in terms of minimizing the signal estimate MSE from the encodings. Also, upon availability of a feedback channel from the host to the sensors, a feedback-based control input can be exploited to achieve the minimum possible MSE via these systems. In this paper we consider control inputs that are combinations of a random signal and a term due to feedback from the host. We present a framework for optimizing the control input and for fusing the channel-corrupted versions of the encodings into a signal estimate at the host.

The outline of the paper is as follows. In Sec. 2 we present the system model of interest. In Sec. 3 we state the performance metrics based on which the encoders and fusion rules are constructed. In Sec. 4 we focus on optimizing the encoder design. In Sec. 5 we propose fusion methods based on the channel-corrupted encodings. In Sec. 6 we present a simulation example whereby the information bearing signal is a first-order autoregressive process. Finally, some concluding remarks are included in Sec. 7.

2. SYSTEM MODEL

The model for the signal observations, as well as the channel model and the proposed encoding and fusion methods are shown in Fig. 1. We consider an L sensor setting, whereby the nth observation at the lth sensor is given by

$$s_l[n] = A[n] + v_l[n], \tag{1}$$

where the $v_l[n]$'s are independent zero-mean IID Gaussian random processes (GRPs) each with variance σ_v^2, and $A[n]$ denotes the information-bearing signal satisfying

$$A[n] = \mathbf{q}^T \mathbf{x}[n] \tag{2a}$$

where $\mathbf{q}$ is a known $M \times 1$ vector, and the vector process

$$\mathbf{x}[n] = \begin{bmatrix} x[n] & x[n-1] & \dots & x[n-M+1] \end{bmatrix}^T \tag{2b}$$

is a state space vector that satisfies the following dynamics

$$\mathbf{x}[n] = G\mathbf{x}[n-1] + \mathbf{h}u[n], \tag{2c}$$

and where the $M \times M$ matrix G and the $M \times 1$ vector $\mathbf{h}$ are known, and $u[n]$ is a zero-mean IID GRP with variance σ_u^2. We assume that the eigenvalues of G are less than 1 in magnitude yielding $\lim_{n\to\infty} \sigma_A^2[n] = \sigma_A^2 < \infty$.

The encoding at the lth sensor consists of adding a control input to a sensor estimate of $A[n]$ (formed from the available measurements at lth sensor) followed by scalar quantization. The encoder is depicted as part of the system in Fig. 1 where $\hat{A}_l[n|n]$ is the lth sensor estimate of $A[n]$ from all $s_l[k]$ for $k \leq n$, $w_l[n]$ denotes the lth sensor control input, and $y_l[n]$ is a binary-valued encoding, *i.e.*,

$$y_l[n] = \mathrm{sgn}(\hat{A}_l[n|n] + w_l[n]). \tag{3}$$

The encodings of the lth sensor are communicated to the host over a binary symmetric channel (BSC) with error probability P_e, so that the encodings received by the host satisfy

$$z_l[n] = \begin{cases} y_l[n], & \text{with probability } 1 - P_e \\ -y_l[n], & \text{with probability } P_e \end{cases} . \tag{4}$$

We assume that there is a broadcast channel so that the host can relay back to each sensor a control sequence $w_{\mathrm{host}}[n]$ obtained from all past received encodings; specifically, we assume that $w_l[n]$ in (3) is given by

$$w_l[n] = w_l^{\mathrm{fb}}[n] + w_l^{\mathrm{rn}}[n]\,, \tag{5}$$

where the $w_l^{\mathrm{rn}}[n]$'s are independent zero-mean IID GRPs with power level σ_w^2, and

$$w_l^{\mathrm{fb}}[n] = w_{\mathrm{host}}[n] + \sigma_{\mathrm{fb}}\tilde{r}_l[n], \tag{6}$$

and where the $\tilde{r}_l[n]$'s are independent zero-mean unit-variance IID GRPs and represent the feedback channel distortion.

Throughout the paper we use $\check{A}[n|m]$ to denote the host estimate of $A[n]$ from $\mathbf{z}^m$, *i.e.*, from all BSC corrupted encodings collected up to time m, and $\hat{A}_l[n|m]$ to denote the lth sensor estimate of $A[n]$ from $\mathbf{s}_l^m$, *i.e.*, from all lth sensor observations collected up to time m. The overall objective is to design the sensor encoders and the host fusion rule so as obtain an accurate estimate of $A[n]$ from the corrupted digital encodings $z_l[n]$. Specifically, this amounts to selecting rules for $\hat{A}_l[n|n]$ and $w_{\mathrm{host}}[n]$ as well as the random control input power level σ_w^2 at each sensor, and the host fusion rule $\check{A}[n|n]$ so as to minimize the MSE in the host estimate.

3. PERFORMANCE METRICS

In designing the sensor encoders we employ as our figure of merit the average information loss in estimating A[n] via $\mathbf{z}^n$ (all received encodings up to time n) instead of $\mathbf{s}^n$ (all sensor observations collected up to time n) [2], *i.e.*,

$$\bar{\mathcal{L}}(A[n]; n) \triangleq \frac{\bar{\mathcal{B}}(A[n], \mathbf{z}^n)}{\bar{\mathcal{B}}(A[n], \mathbf{s}^n)}, \tag{7}$$

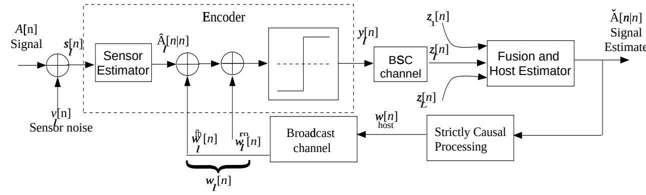

Fig. 1. Encoding via threshold-controlled scalar quantization of a running signal estimate, and signal estimation via channel-corrupted versions of these encodings.

where $\bar{\mathcal{B}}(A[n];\mathbf{x}) = E_{A[n]}\left[\mathcal{B}(A[n],\mathbf{x})\right]$, with $\mathcal{B}(A[n];\mathbf{x})$ denoting the Cramér-Rao bound for unbiased estimates of $A[n]$ from the vector of observations $\mathbf{x}$ [4], and where the expectation is with respect to the prior of $A[n]$.

To assess the quality of the encoding and fusion strategy, we employ the ratio of the host estimate MSE to the MSE of an efficient estimate of $A[n]$ given $\mathbf{s}^n$, *i.e.*,

$$\mathbb{L}(n) = \frac{\frac{1}{n}\sum_{i=1}^{n} E\left[\left(A[i] - \check{A}[i|i]\right)^2\right]}{\frac{1}{n}\sum_{i=1}^{n} E\left[\left(A[i] - \hat{A}[i|i]\right)^2\right]}\,, \tag{8}$$

where $\hat{A}[i|i]$ denotes the estimate obtained by optimal combining of the individual sensor estimates $\hat{A}_l[i|i]$ (given by $\hat{A}[i|i] = \sum_{l=1}^{L} \hat{A}_l[i|i]/L$ in this case).

4. ENCODING STRATEGY

As shown in Fig. 1 the encoder at the lth sensor consists of adding a control input to the lth sensor estimate $\hat{A}_l[n|n]$ prior to scalar quantization. Our objective in this section is to select the rule for the sensor estimator $\hat{A}_l[n|n]$ and the host control input $w_{\mathrm{host}}[n]$, as well as the power level σ_w^2 so as to minimize the average information loss.

4.1. Sensor Estimator

Consistent with [1, 2], we consider encoding at the lth sensor the MMSE estimate of $A[n]$ given $\mathbf{s}_l^n$. This estimate can be obtained via a Kalman filter, given by the following set of equations [5]

$$\hat{\mathbf{x}}_l[n] = G\hat{\mathbf{x}}_l[n{-}1] + \Gamma_l[n]\left(s_l[n] - \mathbf{q}^T G\hat{\mathbf{x}}_l[n{-}1]\right) \tag{9a}$$

$$\Gamma_l[n] = \hat{\Sigma}_l[n|n{-}1]\mathbf{q}\left(\mathbf{q}^T\hat{\mathbb{P}}_l[n|n{-}1]\mathbf{q} + \sigma_v^2 I\right)^{-1} \tag{9b}$$

$$\hat{\mathbb{P}}_l[n|n] = \left(I - \Gamma_l[n]\mathbf{q}^T\right)\hat{\mathbb{P}}_l[n|n-1] \tag{9c}$$

$$\hat{\mathbb{P}}_l[n|n-1] = G\hat{\mathbb{P}}_l[n-1|n-1]G^T + \sigma_u^2\mathbf{h}\mathbf{h}^T, \tag{9d}$$

initialized with $\hat{\mathbf{x}}_l[-1] = 0$ and $\hat{\mathbb{P}}_l[-1|-1] = \sigma_u^2 I$, where $\hat{\mathbb{P}}_l[n|k]$ denotes the covariance matrix of $\hat{\mathbf{x}}_l[n|k]$ and where we have used the shorthand notation $\hat{\mathbf{x}}_l[n] \stackrel{\triangle}{=} \hat{\mathbf{x}}_l[n|n]$ for convenience. The lth sensor estimate and its MSE are given by $\hat{A}_l[n|n] = \mathbf{q}^T \hat{\mathbf{x}}_l[n|n]\mathbf{q}$, and $\hat{\sigma}_l^2[n|n] = \mathbf{q}^T \hat{\mathbb{P}}_l[n|n]\mathbf{q}$, respectively. At steady state, the sensor estimate becomes [5], $\hat{A}_l[n|n] = A[n] + \hat{\sigma}_{\text{sens}}\tilde{e}_l[n]$, where $\tilde{e}_l[n]$ is a zero-mean unit-variance GRP and $\hat{\sigma}_{\text{sens}} = \lim_{n\to\infty} \hat{\sigma}_l[n|n]$.

4.2. Control Input Selection

We next select $w_{\text{host}}[n]$ and σ_w^2 so as to minimize the average information loss by exploiting the static-case analysis in [2]. Given that the encoding information loss is minimized if the binary quantizer threshold equals exactly the information-bearing signal [3], we wish to select $w_{\text{host}}[n]$ (constructed via $\mathbf{z}^{n-1}$) so as to make $A[n] + w_{\text{host}}$ as close to zero–the quantizer threshold in (3)–as possible. Consequently, the encoding information loss is optimized via

$$w_{\text{host}}[n] = -\check{A}[n|n-1]\,,$$

i.e., the negated one-step predictor of $A[n]$ given all past received encodings $\mathbf{z}^{n-1}$. Rewriting (2) as

$$A[n] = \mathbf{q}^T G\mathbf{x}[n-1] + \mathbf{q}^T \mathbf{h}u[n], \tag{10}$$

reveals that the signal $A[n]$ at time n can be viewed as a sum of a term $\mathbf{q}^T G\mathbf{x}[n-1]$ that can be predicted based on $\mathbf{s}_l^{n-1}$ (and thus $\mathbf{z}^{n-1}$), and a term $\mathbf{q}^T\mathbf{h}u[n]$ which is independent of all past measurements (and thus received encodings). Consequently, assuming sufficiently large L, so that

$$E\left[(\hat{A}[n|n] - \check{A}[n|n-1])^2\right] \ll \acute{\sigma}_u^2\,, \tag{11}$$

with $\acute{\sigma}_u^2 = \sigma_u^2 \mathbf{q}^T\mathbf{h}\mathbf{h}^T\mathbf{q}$, $w_{\text{host}}[n]$ can effectively cancel out the "predictable" term $\mathbf{q}^T G\mathbf{x}[n-1]$, but not the term $\mathbf{q}^T\mathbf{h}u[n]$ which is independent of all past encodings.

The average information loss (7) can often be further reduced by employing (in addition to feedback) random control inputs $w_l^{\text{rn}}[n]$ with properly selected power levels σ_w^2. In particular, the average information loss (7) for a given signal term $\mathbf{q}^T\mathbf{h}u[n]$ with power $\acute{\sigma}_u^2$, a given aggregate noise power $\hat{\sigma}_{\text{enc}}^2 = \hat{\sigma}_{\text{sens}}^2 + \sigma_{\text{fb}}^2$, a given P_e level for the BSC, and a given random control power level σ_w^2 satisfies

$$\bar{\mathcal{L}}(\acute{\sigma}_u, \sigma_\alpha, P_e, \sigma_v) \approx \frac{\pi\sigma_\alpha^2}{2\sigma_v^2}\left[\frac{1}{\sqrt{1-\frac{\acute{\sigma}_u^2}{\sigma_\alpha^2}}} + \frac{4P_e(1-P_e)}{(1-2P_e)^2\sqrt{1-\frac{2\acute{\sigma}_u^2}{\sigma_\alpha^2}}}\right]. \tag{12}$$

where $\sigma_\alpha = \sqrt{\sigma_w^2 + \hat{\sigma}_{\text{enc}}^2}$. The value of σ_α that minimizes the average information loss is given by

$$\sigma_\alpha^{\text{opt}} = \begin{cases} 1.1395\acute{\sigma}_u & , \quad P_e = 0 \text{ or } 1 \\ (1.1395)\sqrt{2}\acute{\sigma}_u & , \quad \text{otherwise} \end{cases}. \tag{13a}$$

yielding the optimal random control power level σ_w^{opt} as

$$\sigma_w^{\text{opt}} = \begin{cases} \sqrt{(\sigma_\alpha^{\text{opt}})^2 - (\hat{\sigma}_{\text{enc}})^2} & , \quad \sigma_\alpha^{\text{opt}} > \acute{\sigma}_u \\ 0 & , \quad \sigma_\alpha^{\text{opt}} < \acute{\sigma}_u \end{cases}. \tag{13b}$$

5. SIGNAL ESTIMATION

We consider a two-stage algorithm for fusing the received encodings given by the nonlinear set of measurement equations (3)–(4). First, fusion is applied in space by obtaining the MAP estimate of $A[n]$ based on the $L \times 1$ measurement vector $\mathbf{z}[n]$–consisting of all received encodings at time n, *i.e.*, $z_l[n]$ for $1 \leq l \leq L$–and given the prior $A[n] \sim \mathcal{N}(0, \sigma_A^2)$. The resulting MAP estimate $\hat{A}_{\text{MAP}}[n]$ sequence can be viewed as an alternative single measurement equation. Finally, the signal estimate is obtained via an extended Kalman Filter (EKF) given the state-space signal model and the MAP-based measurement equation.

The advantages of performing spatial fusion prior to an EKF implementation are readily evident when one considers a direct EKF implementation from (3)–(4); in that case the EKF degenerates, as implementation of the EKF equations requires a linearization of (3), which involves the derivative of the $\text{sgn}(\cdot)$ function. We first develop the MAP estimator and consequently construct the host EKF.

5.1. MAP Estimator

The MAP estimate of the nth sample of $A[n]$ from observation of the $L \times 1$ vector $\mathbf{z}[n]$ can be obtained via the following EM algorithm [6] [2]

$$\hat{A}_{\text{EM}}^{(k+1)}[n] = \frac{\hat{A}_{\text{EM}}^{(k)}[n] + \frac{\sigma_\alpha[K_1 - L\mathcal{Q}(z^{(k)}[n])]e^{-(z^{(k)}[n])^2/2}}{\sqrt{2\pi}L\mathcal{Q}(z^{(k)}[n])[1-\mathcal{Q}(z^{(k)}[n])]}}{1 + \frac{\sigma_\alpha^2}{L\acute{\sigma}_u^2}}, \tag{14}$$

where $z^{(k)}[n] = A_{\text{EM}}^{(k)}[n]/\sigma_\alpha$, $\mathcal{Q}(x) = (1/2\pi)\int_x^\infty e^{\frac{-x^2}{2}}dx$ and K_1 denotes the number of elements in the vector $\mathbf{z}[n]$ that are equal to 1. The MAP estimate is then given by $\hat{A}_{\text{MAP}}[n] = \lim_{k\to\infty} \hat{A}_{\text{EM}}^{(k)}[n]$.

For sufficiently large L, $\hat{A}_{\text{MAP}}[n]$ achieves the associated average encoding information loss. Specifically, $\hat{A}_{\text{MAP}}[n]$ becomes asymptotically (in L) Gaussian with mean $A[n]$ and variance $\sigma_{\hat{v}_z}^2[n] = \mathcal{B}(A[n]; z[n])/L$, i.e.,

$$\hat{A}_{\text{MAP}}[n] = A[n] + \hat{v}_z[n], \tag{15}$$

where $\hat{v}_z[n]$ is zero-mean GRP with variance $\sigma_{\hat{v}_z}^2[n]$ that depends on $A[n]$. As shown in [2], however, when the encoder is operating at the optimal aggregate noise level $\sigma_\alpha^{\text{opt}}$, $\mathcal{B}(A[n]; z[n])$ is effectively constant as a function of $A[n]$ over a wide range of $A[n]$ values. Consequently, $\sigma_{\hat{v}_z}^2[n]$ the variance of the MAP estimate at time n, is well approximated as $\sigma_{\hat{v}_z}^2[n] \approx \sigma_{\hat{v}_z}^2 = \bar{\mathcal{L}}(\acute{\sigma}_u, \sigma_\alpha^{\text{opt}}, P_e, \sigma_v)\,\sigma_v^2/L$.

5.2. Host Kalman Filter

By viewing (15) as a single measurement equation we can obtain a host signal estimate via an EKF; it is given by the

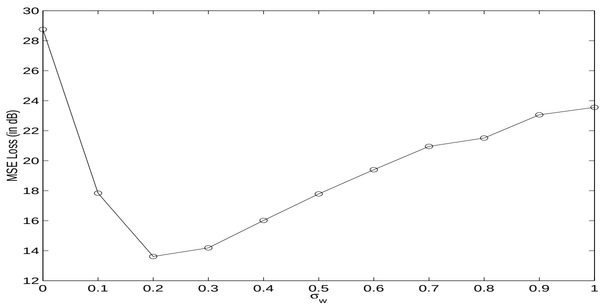

Fig. 2. MSE loss in estimating the AR(1) process (17) where $\sqrt{1-\rho^2} = 0.2$, $\sigma_A = 1$ and $P_e = 0$.

same equations (9) with $\hat{\mathbf{x}}_l[n|n]$, $\hat{\mathbb{P}}_l[n|n]$, $\Gamma_l[n]$, $s_l[n]$ and σ_v^2, replaced by $\check{\mathbf{x}}[n|n]$, $\check{\mathbb{P}}[n|n]$, $\mathbb{R}[n]$, $\hat{A}_{\text{MAP}}[n]$ and $\sigma^2_{\tilde{v}_z}[n]$, respectively. We can also represent the host Kalman filter estimate $\check{A}[n|n]$ obtained from $\mathbf{z}^n$ as follows

$$\check{A}[n|n] = A[n] + \check{\sigma}_{\text{hst}}\tilde{\gamma}[n] \quad (16)$$

where $\tilde{\gamma}[n]$ is a zero mean unit variance GRP and $\check{\sigma}^2_{\text{hst}}$ is the asymptotic average MSE of estimating $\check{A}[n|n]$ which is given by $\check{\sigma}^2_{\text{hst}} = \lim_{n\to\infty} \mathbf{q}^T \check{\mathbb{P}}[n|n]\mathbf{q}$.

6. SIMULATION AND RESULTS

As an illustration, we next present Monte-Carlo simulations for sequential encoding and estimation of a first order autoregressive (AR) process,

$$A[n] = \rho A[n-1] + \sigma_A\sqrt{1-\rho^2}\,\tilde{u}[n], \quad (17)$$

where $\tilde{u}[n]$ is a zero-mean unit-variance IID GRP, and where $|\rho| < 1$. In particular, we consider a network of $L = 10^3$ sensors, whereby the signal and sensor noise power levels are $\sigma_A = 1$ and $\sigma_v = 0.1$, respectively, and the power level of feedback channel distortion is $\sigma_{\text{fb}} = 0.01$.

Fig. 2 depicts the MSE loss (8) of the encoding and fusion algorithms we developed as a function of the power of the random control inputs. As the figure reveals, there is an optimal power level in terms of minimizing the MSE loss which is very accurately predicted by (13).

Fig. 3 depicts the MSE loss (8) as a function of ρ, for various BSC P_e levels. For any given P_e and ρ, the random control power level is selected according to (13b). As the figure reveals, for any given P_e, the MSE loss is minimized for $\rho = 1$; in this static case limit, $\acute{\sigma}_u = 0$, so $\sigma_w^{\text{opt}} = 0$, and hence the optimized control input strategy reduces to pure feedback control. At the other extreme ($\rho = 0$), we have the maximum MSE loss as feedback-based control does not provide any encoding benefits and the optimized control at each sensor consists of a purely random control input.

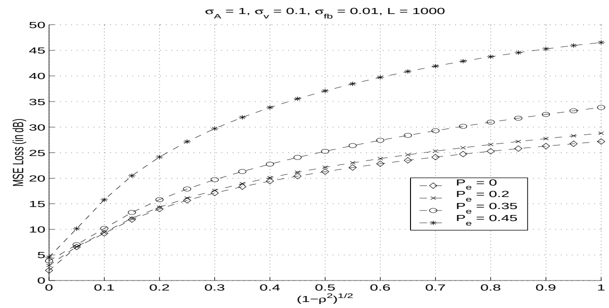

Fig. 3. MSE loss in estimating the AR(1) process (17) as a function of bandwidth, at various BSC P_e values.

7. CONCLUSION

We propose methods for sequential binary-valued encoding of noisy sensor measurements and fusion from channel corrupted versions of these encodings. The encodings we considered consist of adding to running MMSE signal estimate at each sensor a properly designed control input followed by scalar quantization. We showed how to optimize the sensor control inputs and the associated fusion rule so as to minimize the host estimate MSE. In particular, we presented an efficient fusion method, whereby the received encodings are first fused spatially to produce a single equivalent measurement sequence based on which an extended Kalman filter is constructed.

8. REFERENCES

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