WIDE BAND CHANNEL CHARACTERISATION IN COLOURED NOISE USING THE REVERSIBLE JUMP MCMC

Jean-René Larocque and James P. Reilly

Department of Electrical and Computer Engineering, McMaster University, Canada
larocque@reverb.crl.mcmaster.ca, reillyj@mcmaster.ca

ABSTRACT

This paper presents a novel approach for characterizing wide-band (CDMA) multiple dimensional channels for the wireless environment in arbitrarily coloured additive Gaussian noise. This characterization is sufficient for the specification of optimal multichannel space-time receivers. The proposed solution is defined in the Bayesian framework and uses the Reversible Jump Markov Chain Monte Carlo (MCMC) method to obtain estimates of the number of scatterers, their directions of arrival and their times of arrival. The developed method is applied to simulated and real measured data to verify the performance of the approach.

1. INTRODUCTION

In this paper, we address the problem of characterizing wireless multipath channels for CDMA receivers which use an array of antennas. We assume the propagation channel consists of a discrete number of independent Rayleigh-faded components, each with a distinct direction of arrival and relative delay time. We consider the special case where the noise covariance matrix of the antenna elements is unknown and arbitrarily coloured. Our objective is to estimate the number of scatterers, the corresponding delay times (TOAs), and the directions of arrival (DOAs) of the multipath components using only the data received from an antenna array in a wideband scenario. This characterization is sufficient for the construction of optimal multiple channel space-time receiver structures, e.g. BLAST receivers. Such characterizations are also sufficient for estimating channel capacity, etc.[1]

A Bayesian approach is proposed, where the undesired nuisance parameters representing the instantaneous amplitudes and the known noise covariance matrix are integrated out, after assigning suitable noninformative priors. The resulting posterior distribution is highly nonlinear in the parameters, making it difficult to achieve a global optimum by ordinary numerical techniques. Moreover, the number of parameters of the distribution depends on the unknown model order. Well known methods, such as AIC or MDL, cannot be used to determine the model order, as the noise is coloured.

MCMC methods e.g. [2][3] have been quickly gaining the attention of the signal processing community, especially for problems which involve nonlinear and/or high dimensional models. Here, we propose the reversible jump MCMC method [4] for the estimation of our desired parameters. This technique is capable of exploring a parameter space of varying model order, thus allowing an estimate of the number of scatterers to be made. Under benign conditions, it can be proved the global optimum is always achieved with this method. Simulation results, and results obtained from real measurements, verify the usefulness of the proposed method.

2. DEVELOPING THE POSTERIOR DISTRIBUTION FOR COLOURED NOISE

For the purposes of this presentation, we assume all relative delays of the individual scattering components are upper bounded by the symbol duration, and the delay spread from each scattering point is small compared to the chip duration ΔT. (The treatment can be readily extended so that these restrictions can be relaxed). Let the number of chips in one symbol in the received CDMA data sequence be P. Then, the data received over the nth symbol from an array of M antenna elements consists of a complex data matrix $\boldsymbol{Y}(n) \in \mathcal{C}^{M\times P}$ given by

$$\boldsymbol{Y}(n) = \boldsymbol{S}(\boldsymbol{\phi})\boldsymbol{A}(n)\boldsymbol{T}(\boldsymbol{\tau}) + \boldsymbol{\Omega}(n) \;\; n = 1, \ldots, N, \qquad (1)$$

where the matrix $\boldsymbol{S}(\boldsymbol{\phi}) \in \mathcal{C}^{M\times k_o}$ is a function of the directions of arrival $\boldsymbol{\phi}$ and is the standard steering matrix; k_o is the (unknown) number of scattering components; the diagonal matrix $\boldsymbol{A}(n) \in \mathcal{C}^{k_o\times k_o}$ contains the quickly–varying signal amplitudes at the nth symbol; the structure of the matrix $\boldsymbol{T}(\boldsymbol{\tau}) \in \mathcal{C}^{k_o\times P}$ is given later; the elements of the matrix $\boldsymbol{\Omega}(n) \in \mathcal{C}^{M\times P}$ are distributed as $\mathrm{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_w)$, where $\boldsymbol{\Sigma}_w$ is unknown, and N is the number of symbols over which the data is collected.

Each row of the matrix $\boldsymbol{T}(\boldsymbol{\tau})$ consists of all zero elements except for a single one in the pth position. This element indicates that the relative delay of the corresponding scattering component is $p\Delta T$.

Our objective is to jointly estimate the number of scatterers, k_o, their directions of arrival, $\boldsymbol{\phi} \in [0, 2\pi]^{k_o}$ and their times of arrival represented by the vector of integers, $\boldsymbol{\tau} \in [0, P]^{k_o}$. It is assumed that the scatterers have unique DOAs and TOAs, to avoid degeneracy of the model. Since the number of scatterers k_o is unknown, the dimension of the parameters $\boldsymbol{\phi}$ and $\boldsymbol{\tau}$ are also unknown. We therefore denote them as $\boldsymbol{\phi}_k$ and $\boldsymbol{\tau}_k$ respectively, where k is the hypothesized number of signals. It is assumed that $\boldsymbol{\phi}_k$ and $\boldsymbol{\tau}_k$ remain stationary over the N snapshots, and the amplitudes $\{\boldsymbol{a}(n)\}$ are *iid* between snapshots.

The model described by (1) can be rearranged, to a more familiar form, using Kronecker algebra, as follows:

$$\boldsymbol{y}(n) = \boldsymbol{Z}(\boldsymbol{\tau}, \boldsymbol{\phi})\boldsymbol{b}(n) + \boldsymbol{\omega}(n) \quad n = 1, \ldots, N, \qquad (2)$$

with:

$$\boldsymbol{y}(n) = \mathrm{vec}(\boldsymbol{Y}(n)) \qquad (3)$$

$$\boldsymbol{b}(n) = \mathrm{vec}(\boldsymbol{A}(n)) \qquad (4)$$

$$\boldsymbol{Z}(\boldsymbol{\tau}, \boldsymbol{\phi}) = \boldsymbol{T}^t(\boldsymbol{\tau}) \otimes \boldsymbol{S}(\boldsymbol{\phi}). \qquad (5)$$

where vec$(\cdot)$ is the *vec* operator. Furthermore, noting that the matrix $\boldsymbol{A}(n)$ is diagonal, the vector $\boldsymbol{b}(n)$ defined previously only operates on a few columns of $\boldsymbol{Z}$. Regrouping these usefull columns in a new matrix $\boldsymbol{H}(\boldsymbol{\tau},\boldsymbol{\phi})$:

$$\boldsymbol{y}(n)=\boldsymbol{H}(\boldsymbol{\tau},\boldsymbol{\phi})\boldsymbol{a}(n)+\boldsymbol{\omega}(n) \quad n=1,\ldots,N, \tag{6}$$

where $\boldsymbol{a}(n)$ holds the diagonal elements of $\boldsymbol{A}(n)$, and the matrix $\boldsymbol{H}(\boldsymbol{\tau},\boldsymbol{\phi})$ defines the space-time structure of the multipath. This form is more familiar and can now easily be analyzed in the Bayesian framework, as in [3].

Since the N snapshots are *iid*, the total likelihood function of all the data can be expressed in the following form:

$$p(\boldsymbol{Y}|\boldsymbol{\phi},\boldsymbol{\tau},\boldsymbol{A},\boldsymbol{\sigma}_w,k)=\frac{1}{\pi^{NMP}|\boldsymbol{\Sigma}_w|^N}\times$$
$$e^{-\sum_{n=1}^{N}(\boldsymbol{y}(n)-\boldsymbol{H}(\boldsymbol{\phi},\boldsymbol{\tau})\boldsymbol{a}(n))^H\boldsymbol{\Sigma_w^{-1}}(\boldsymbol{y}(n)-\boldsymbol{H}(\boldsymbol{\phi},\boldsymbol{\tau})\boldsymbol{a}(n))}. \tag{7}$$

To proceed with the integration of the nuisance parameters $\boldsymbol{\Sigma}$ and $\boldsymbol{A}$, we first define an orthonormal matrix $\boldsymbol{U}(\boldsymbol{\phi},\boldsymbol{\tau},k)\in\mathcal{C}^{MP\times MP}$ as in [5][6]

$$\boldsymbol{U}(\boldsymbol{\phi},\boldsymbol{\tau},k)=\;[\underset{MP\times k}{\boldsymbol{U}_s(\boldsymbol{\phi},\boldsymbol{\tau},k)} \quad \underset{MP\times(MP-k)}{\boldsymbol{U}_\nu(\boldsymbol{\phi},\boldsymbol{\tau},k)}], \tag{8}$$

where $\boldsymbol{U}_s(\boldsymbol{\phi},\boldsymbol{\tau},k)\in\mathcal{H}$, the signal subspace, and $\boldsymbol{U}_\nu(\boldsymbol{\phi},\boldsymbol{\tau},k)\in\mathcal{N}$, the noise subspace. We now transform the received data $\boldsymbol{y}(n)$ into $\mathcal{H}$ and $\mathcal{N}$ to form a signal component $\boldsymbol{z}_s(n)$ and a noise component $\boldsymbol{z}_\nu(n)$ respectively as

$$\boldsymbol{z}_s(n)=\boldsymbol{U}_s^H(\boldsymbol{\phi},\boldsymbol{\tau},k)\boldsymbol{y}(n) \tag{9}$$

and

$$\boldsymbol{z}_\nu(n)=\boldsymbol{U}_\nu^H(\boldsymbol{\phi},\boldsymbol{\tau},k)\boldsymbol{y}(n). \tag{10}$$

The new parameters $\boldsymbol{z}_s(n)$ and $\boldsymbol{z}_\nu(n)$ are both Gaussian. For arbitrary $\boldsymbol{\Sigma}_w$, the noise components of $\boldsymbol{z}_s(n)$ and $\boldsymbol{z}_\nu(n)$ may not be uncorrelated; however, for the sake of tractable analysis, we assume them to be so. In the neighborhood of the true values of the parameters, it can then be shown [3] that $\boldsymbol{z}_s(n)$ is distributed as $\mathrm{N}(\tilde{\boldsymbol{a}},\boldsymbol{C})$, where the covariance matrix $\boldsymbol{C}$ and mean $\tilde{\boldsymbol{a}}$ are to be defined [3], and $\boldsymbol{z}_\nu(n)$ is distributed as $\mathrm{N}(\mathbf{0},\boldsymbol{W})$, with covariance matrix $\boldsymbol{W}\triangleq\mathrm{E}\{\boldsymbol{U}_\nu^H\boldsymbol{y}_n\boldsymbol{y}_n^H\boldsymbol{U}_\nu\}$. The joint likelihood function of $\boldsymbol{z}_s$ and $\boldsymbol{z}_\nu$ is then given as:

$$\begin{aligned}&p(\boldsymbol{Z}_s,\boldsymbol{Z}_\nu|\tilde{\boldsymbol{A}},\boldsymbol{\phi},\boldsymbol{\tau},k,\boldsymbol{W}^{-1})\approx\pi^{-Nk}|\boldsymbol{C}^{-1}|^N\\&\times\exp\left\{-\sum_{n=1}^{N}(\boldsymbol{z}_s(n)-\tilde{\boldsymbol{a}}(n))^H\boldsymbol{C}^{-1}(\boldsymbol{z}_s(n)-\tilde{\boldsymbol{a}}(n))\right\}\\&\times\pi^{-N(M-k)}|\boldsymbol{W}^{-1}|^N\exp\left\{-\sum_{n=1}^{N}\boldsymbol{z}_\nu^H(n)\boldsymbol{W}^{-1}\boldsymbol{z}_\nu(n)\right\}.\end{aligned} \tag{11}$$

To complete the model, prior distributions are chosen to be non-informative where possible. When convenient, we also choose the structural form of these distributions for their desirable conjugate properties. The priors distributions are described as follows:

- $\tilde{\boldsymbol{A}}$ is assigned a conjugate non-informative prior distribution described as a Gaussian function with a large covariance matrix $\boldsymbol{D}$ (compared to $\boldsymbol{C}$), and zero mean. Thus,

$$p(\tilde{\boldsymbol{A}}|\boldsymbol{\phi}_k,k,\boldsymbol{W}^{-1})=\prod_{n=1}^{N}\mathrm{N}(\mathbf{0},\boldsymbol{D}) \tag{12}$$

  where $\boldsymbol{D}\triangleq d^2\boldsymbol{I}_k$, which assumes the projected signals are independent with the same large variance. The choice of the hyper-parameter d is discussed at length in [3].

- The prior distributions for both $\boldsymbol{\phi}$ and $\boldsymbol{\tau}$ are chosen to be uniform:

$$p(\boldsymbol{\phi}|k)=U[0,2\pi]^k \qquad p(\boldsymbol{\tau}|k)=\frac{1}{P^k}. \tag{13}$$

- The prior on k is chosen to be Poisson with expectation Λ (a flat prior over a range $[0,k_{max}]$ is another option, but tends to slow down the convergence):

$$p(k)=\Lambda^k e^{-\Lambda}/k! \tag{14}$$

- $\boldsymbol{W}^{-1}$: We use a non-informative multi-dimensional Jeffreys' prior for the unknown transformed noise covariance matrix [5][3]

$$p(\boldsymbol{W}^{-1}\mid\boldsymbol{\phi},k)\propto|\,\boldsymbol{W}^{-1}\,|^{-(M-k)} \tag{15}$$

The posterior distribution, after carrying out the integration of the nuisance parameters and ignoring the constant terms, is then (see [3] for more details):

$$\begin{aligned}p(k,\boldsymbol{\phi},\boldsymbol{\tau}|\boldsymbol{Z}_\nu)\propto&\frac{\pi^{\frac{1}{2}(MP-k)(MP-k-1)}\prod_{i=1}^{MP-k}\Gamma(N-i+1)}{(2\pi P/\Lambda)^k k!(d^2)^{kN}}\\&\times\left|N\hat{\boldsymbol{W}}\right|^{-N},\end{aligned} \tag{16}$$

with $N\hat{\boldsymbol{W}}(\boldsymbol{\phi},\boldsymbol{\tau},k)\triangleq\sum_{n=1}^{N}\boldsymbol{z}_\nu(n)\boldsymbol{z}_\nu(n)^H$. Take note that this function depends only on the slowly varying parameters of interest. The objective is to estimate the parameters of this highly non-linear function, as the Maximum A Posteriori (MAP) estimates:

$$\{\hat{k},\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\tau}}\}=\arg\max_{k,\boldsymbol{\phi},\boldsymbol{\tau}\in\Theta}p(k,\boldsymbol{\phi},\boldsymbol{\tau}|\mathbf{Z}_v). \tag{17}$$

3. THE REVERSIBLE JUMP MCMC ALGORITHM

We now propose an MCMC method to perform the Bayesian computation in extracting the parameters of interest from the posterior distribution (16).

A powerful MCMC technique is the "reversible jump MCMC" from Green [4], which is a variation of the Metropolis Hastings method, for the cases where the model order is a part of the unknown parameters. With this method, assume at the ith iteration, we are in state $\boldsymbol{\gamma}_k^{(i)}$. A candidate $\boldsymbol{\gamma}_k^\star$ for the next state of the chain is drawn at random from a so-called proposal distribution $q(\cdot|\cdot)$, which may be conditional on $(\boldsymbol{\gamma}_k^{(i)})$. The distribution $q(\cdot|\cdot)$ is chosen to be easy to draw samples from, and also ideally, to closely approximate the desired posterior distribution. The candidate samples are randomly accepted according to an acceptance ratio that ensures reversibility, and therefore the invariance of the Markov chain with respect to the desired posterior distribution. In the case where the model order is unknown, we allow the dimension of $\boldsymbol{\gamma}_k^\star$ to vary at each iteration. As such, we choose our set of proposal distributions to correspond to the following set of moves, according to [2]:

1. the *birth* move, valid for $k<M-1$. Here, a new scatterer point is proposed at random on $(0,2\pi]\times(0,P)$.

2. the *death* move, valid for $k > 0$. Here, a randomly chosen scatterer point is removed.

3. the *update* move, valid for $k > 0$. Here, the parameters describing the multipath characteristics are updated for a fixed value of k.

The probabilities for choosing each move are denoted u_k, b_k and d_k, respectively, such that $u_k + b_k + d_k = 1$ for all k, chosen in accordance with [4]. The distinguishing features of each move are now summarized in the following subsections.

3.1. Update move

Here, we assume that the current state of the algorithm is $(\boldsymbol{\phi}_k, \boldsymbol{\tau}_k, \{k\})$. When the update move is selected, the algorithm samples only on the space of $(\boldsymbol{\phi}_k, \boldsymbol{\tau}_k)$, sequentially for all the parameters, for k fixed. This is a Metropolis-one-at-the-time approach. The proposal distribution of the parameters is assumed separable (even though in practice these two parameters are dependant, this assumption is justified for the purpose of proposing candidate states):

$$q(\boldsymbol{\phi}, \boldsymbol{\tau}) = q_1(\boldsymbol{\phi}) q_2(\boldsymbol{\tau}) \quad (18)$$

In this case, the proposal distribution $q_1(\cdot|\cdot)$ for the candidate DOA parameter $\boldsymbol{\phi}^\star$ is two-fold: with a certain probability, it is assigned to be the multidimensional uniform distribution over $[0, 2\pi]$, for a fully random exploration; or it is a perturbation of the actual state, for a local exploration. This hybrid approach gives the best performance in terms of rate of convergence and variance of the estimate [2]. For the TOA parameter, the proposal distribution $q_2(\cdot|\cdot)$ is simply a uniform distribution. The acceptance ratio $r = r_{update}$ from (16) for the update move is therefore:

$$r_{update}(\boldsymbol{\phi}_k^\star, \boldsymbol{\tau}_k^\star, k, \boldsymbol{\phi}_k, \boldsymbol{\tau}_k, k) = \frac{|N\hat{\boldsymbol{W}}(\boldsymbol{\tau}^\star, \boldsymbol{\phi}^\star, k)|^{-N}}{|N\hat{\boldsymbol{W}}(\boldsymbol{\tau}, \boldsymbol{\phi}, k)|^{-N}} \quad (19)$$

The candidate $\{\boldsymbol{\phi}^\star, \boldsymbol{\tau}^\star\}$ is then accepted as the current state with probability $\alpha_{update} = \min[r_{update}, 1]$, according to the procedure described in more detail in [2].

3.2. Birth and Death moves

In the death move case, we assume the current state is in $(\boldsymbol{\phi}_{(k+1)}, \boldsymbol{\tau}_{(k+1)}, \{k+1\})$, and we wish to determine whether the next state is in $(\boldsymbol{\phi}_k, \boldsymbol{\tau}_k, \{k\})$ at the next iteration. This involves the removal of a scatterer point, which is chosen randomly amongst the $(k+1)$ existing points. The candidate state $(\boldsymbol{\phi}_k^\star, \boldsymbol{\tau}_k^\star, k)$ is proposed and accepted with probability $\alpha = \min[r_{death} 1]$, given as:

$$\begin{aligned} r_{death}(\boldsymbol{\phi}_k^\star, \boldsymbol{\tau}_k^\star, k, \boldsymbol{\phi}_{k+1}, \boldsymbol{\tau}_{k+1}, k+1) = \\ \frac{|N\hat{\boldsymbol{W}}(\boldsymbol{\tau}_k^\star, \boldsymbol{\phi}_k^\star, k)|^{-N}}{|N\hat{\boldsymbol{W}}(\boldsymbol{\tau}_{k+1}, \boldsymbol{\phi}_{k+1}, k+1)|^{-N}} \\ \times \pi^{(MP-k-1)} \Gamma(N - MP + k + 1)(k+1)(d^{2N}) \end{aligned} \quad (20)$$

Similarly, in the birth move case, we assume the current state is $(\boldsymbol{\phi}_k, \boldsymbol{\tau}_k, \{k\})$ and we wish to determine whether the next state is in $(\boldsymbol{\phi}_{(k+1)}, \boldsymbol{\tau}_{(k+1)}, \{k+1\})$. This involves the addition of a new scatterer point, which is proposed uniformly over $(0, 2\pi] \times (0, P)$.

The acceptance ratio $r = r_{birth}$ for the birth move can be verified to be:

$$\alpha_{birth} = \min[1, \frac{1}{r_{death}}]. \quad (21)$$

The MCMC sampling process is repeated for many iterations, to provide a histogram which approximates the posterior pdf (16). The MAP estimate of the parameters are readily obtained.

4. SIMULATION RESULTS

The proposed algorithm is now applied to simulation data, generated for $k_o = 2$ scatterers with the parameters described in table 4.1. The receiver array is composed of 5 elements. The amplitudes are *iid* Rayleigh distributed over $N = 125$ received symbols (or snapshots) for $P = 25$, with an SNR of 5dB. The noise is coloured with an AR filter, the poles for which are $0.95e^{-j1.07\pi}$ and $0.95e^{-j0.88\pi}$.

Scatterers	DOA (deg)	TOA (bins)	Amplitude (dB)
S1	65°	8	10
S2	20°	2	10

Table 4.1: Parameters of the multipath for simulated data

The Reversible Jump MCMC scheme goes through 1500 iterations after a burn-in period of 300 iterations. The results, as found by the algorithm, are summarized in figure 1. It is clear from the histograms that the DOA and TOA parameters concentrate around their true values. The posterior probability of the number of scatterers being $k = 2$ was evaluated at 75% (due to a high noise level), as summarized in table 4.2. Clearly, the algorithm correctly identified the parameters of the simulated multipath scenario. Further simulation results for the DOA case only are presented in [7].

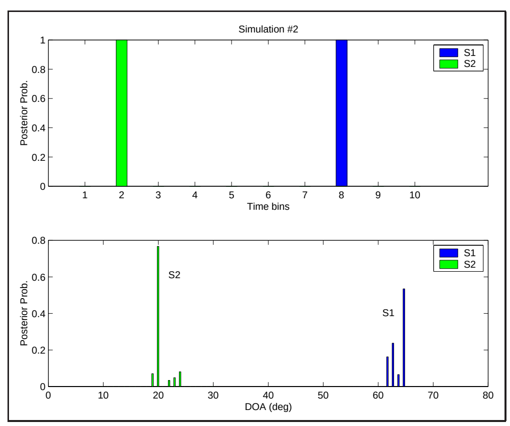

Figure 1: Simulations: Histogram of the TOA (top); Histogram of DOA (bottom).

5. APPLICATION TO REAL-LIFE PROBLEM

In this section, we apply our proposed scheme to real-life outdoor propagation measurements with a typical data set collected on Mc-

$p(\hat{k}=i)$ %	1	2	3
Simulations	0	75	25
Measurements	1	98	1

Table 4.2: Posterior estimate of the number of paths using MCMC

Master University campus.

The channel impulse response, in time and space, is measured directly in the time domain from an $M = 8$ elements experimental receiver by transmitting a wideband CDMA signal and correlating the received signal with the known transmitted sequence, at each channel of the receiving array. The transmitted signal is a 255 chip pseudonoise (PN) sequence at 5 MHz IF bandwidth. It was observed that the maximum excess delay of the channel can be considered by using only $P = 25$ chips. An initial calibration was first realized, based on measurements with the antenna array inside an anechoic chamber. The measurements were conducted with the receiving base station at different locations and at different heights, in a pico-cell scenario that offers rich multipath characteristics with severe fading.

For the purpose of demonstrating the proposed algorithm, $N = 20$ received data symbols (or snapshots) are used, measured from a position where the multipath characteristics have been geographically observed to be 2 rays incident approximatively from angles of arrival of 30° and 135°, as shown on figure 2. Figure 3 shows

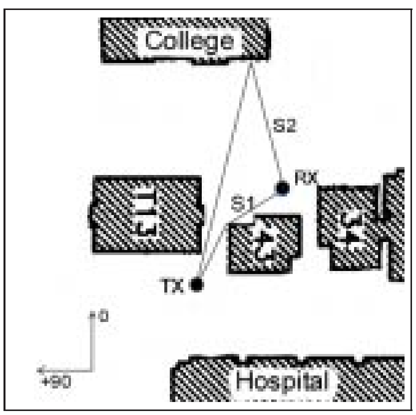

Figure 2: Map describing the geometry of the setup.

typical results for 1000 iterations of the Reversible Jump MCMC Sampler. It is clear that the algorithm identifies the two major multipath components, in time and in direction.

6. CONCLUSION

In this paper, a new and innovative approach to channel characterisation is presented. In the Bayesian framework, using a Markov Chain Monte Carlo method to perform the optimisation, the slow varying parameters of the multipath ($\boldsymbol{\phi}$ and $\boldsymbol{\tau}$), as well as the number of scatterers, are jointly estimated. The nuisance parameters (amplitudes and unknown noise variance) are integrated out analytically, for a more efficient sampling sheme. The proposed method has been verified using simulations and real-life propagation measurements.

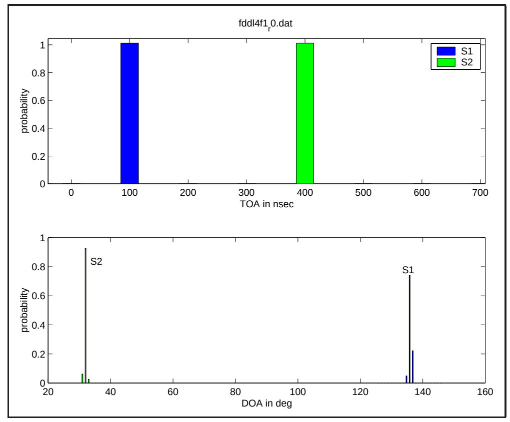

Figure 3: Measurements: Histogram of the TOA (top); Histogram of the DOA (bottom).

7. ACKNOWLEDGEMENTS

The authors are most greatful to Dr. Christophe Andrieu for the helpful discussions. The authors wish to acknowledge and thank the following institutions for their support of this project: Mitel Corporation, Kanata, Ontario, Canada; The Centre for Information Technology Ontario (CITO); and the Natural Sciences and Engineering Research Council of Canada (NSERC).

REFERENCES

[1] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, 1998.

[2] C. Andrieu and A. Doucet, "Joint Bayesian model selection and estimation of noisy sinusoids via reversible jump MCMC," *IEEE Transactions on Signal Processing*, vol. 47, p. 2667, Oct. 1999.

[3] J.-R. Larocque and J. P. Reilly, "Reversible jump MCMC for joint detection and estimation of directions of arrival in coloured noise," *submitted for publications to IEEE Transactions on Signal Processing*, 2000.

[4] P. Green, "Reversible jump Markov Chain Monte Carlo computation and Bayesian model determination," *Biometrika*, vol. 82, no. 4, pp. 711–732, 1995.

[5] K.-M. Wong, J. P. Reilly, Q. Wu, and S. Qiao, "Estimation of the directions of arrival of signals in unknow collelated noise, part 1: The MAP approach and its implementation," *IEEE Transactions of Signal Processing*, vol. 40, pp. 2007–2017, August 1992.

[6] C.-M. Cho and P. M. Djuric, "Bayesian detection and estimation of cisoids in colored noise," *IEEE Transactions on Signal Processing*, vol. 43, pp. 2943–2951, Dec. 1995.

[7] J.-R. Larocque and J. P. Reilly, "Reversible jump MCMC for joint detection and estimation of directions of arrival in coloured noise," *Proceedings of the IMA International Conference on Mathematics in Signal Processing*, 2000.