Blind Identification and Equalization of FIR MIMO Channels by BIDS

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Abstract

This paper presents an algorithm of blind identification and equalization of finite-impulse-response and multiple-input and multiple-output (FIR MIMO) channels driven by colored signals. This algorithm is an improved realization of a concept referred to as blind identification via decorrelating subchannels (BIDS). This BIDS algorithm first constructs a set of decorrelators which decorrelate the output signals of subchannels, and then estimates the channel matrix using the transfer functions of the decorrelators, and finally recovers the input signals using the estimated channel matrix. This BIDS algorithm in general assumes that the channel matrix is irreducible and the input signals are mutually uncorrelated and of sufficiently diverse power spectra. However, for channel matrix identification, this BIDS algorithm only requires the channel matrix to be nonsingular (i.e., full rank almost everywhere as opposed to everywhere) and column-wise coprime. Such a channel matrix may have zeros and be of non-minimum phase.

1. Introduction

Blind identification and/or equalization of FIR MIMO channels driven by colored signals are a fundamental problem one encounters in many applications such as radar targets identification, wireless communications, and microphone arrays for speech enhancement. In this paper, we are interested in the development of the second-order statistics based algorithms for blind identification and equalization of FIR MIMO channels. The subspace algorithm [1] and the matrix pencil (MP) algorithm [2], require the channel matrix to be irreducible and column-reduced in addition to other conditions. The algorithms shown in [3-4] only handle square channel matrices with unit diagonal elements. The idea of blind identification via decorrelating subchannels (BIDS) was first presented in [5-6] where the first BIDS algorithm, to be referred to as BIDS-1, was developed. The BIDS-1 algorithm requires the channel matrix to be irreducible, which is a much weaker condition than those mentioned above. The BIDS-1 algorithm first constructs subchannel decorrelators, then forms a set of single-input-multiple-output (SIMO) subchannels, and finally uses a SIMO channel algorithm to recover the input signals. In this paper, we show a second realization of BIDS, referred to as BIDS-2. The BIDS-2 algorithm differs from the BIDS-1 algorithm in that the former computes the channel matrix directly from the decorrelators before the input signals are estimated. The estimated channel matrix by BIDS-2 is asymptotically exact if the channel matrix is nonsingular and column-wise coprime (which can be a non-minimum phase matrix). The BIDS-2 algorithm avoids the errors that the BIDS-1 algorithm accumulates in constructing the SIMO subchannels, and provides a significant improvement of robustness against noise.

Section 2 lays down the basic problem and provides some fundamental identifiability conditions. Section 3 describes the BIDS-2 algorithm. Section 4 illustrates the performance of the BIDS-2 algorithm. Due to space limitation, a large amount of details can not be given in this paper but are all available in [8].

2. The Problem

An FIR MIMO channel is described by:

$$\mathbf{y}(n) = \mathbf{H}_z(z)\mathbf{x}(n) + \mathbf{w}(n) \qquad (1)$$

where $\mathbf{x}(n)$ is the $I\times 1$ input vector; $\mathbf{y}(n)$ the $J\times 1$ output vector, $\mathbf{w}(n)$ the noise vector; and $\mathbf{H}_z(z)$ is a polynomial matrix, i.e., $\mathbf{H}_z(z) = \sum_{l=0}^{L_H} \mathbf{H}(l) z^{-1}$, which is called the channel matrix. The problem here is to estimate $\mathbf{x}(n)$ and/or $\mathbf{H}_z(z)$ using $\mathbf{y}(n)$. Without loss of generality, all time-domain parameters in (1) are real valued. The following theorem is proved in [7].

Theorem 1: An FIR MIMO channel is identifiable up to a scaling and permutation using the second-order statistics of the channel output if

- The channel matrix $\mathbf{H}_z(z)$ is irreducible (i.e., of full column rank everywhere in the complex z-plane except at $z=0$); and
- The input power spectral matrix $\mathbf{S}_{\mathbf{xx}}(z)$ is diagonal and of distinct diagonal (polynomial or rational) functions.

Proof: See [7].

Theorem 1 is a strong result. But the conditions of the theorem are too weak for all existing algorithms. In fact, developing an algorithm that yields the exact channel identification under those conditions has not been successful. Some additional condition appears necessary for developing a practical algorithm. In the next section, we show the BIDS-2 algorithm, which requires the input signals to be mutually uncorrelated and of *sufficiently diverse* power spectra. With sufficiently diverse power spectra, the BIDS-2 algorithm can yield the exact channel matrix even if the channel matrix has zeros and is of non-minimum phase.

3. Blind Identification via Decorrelating Subchannels

3.1 Decorrelating Subchannels

All BIDS algorithms assume that the input signals are mutually uncorrelated and there are more output signals than input signals, i.e., $J > I$. Note that to reduce (or remove asymptotically) the noise effect, the condition $J > I$ is necessary in general. The noise effect on the second-order statistics of the channel output can be eliminated (asymptotically) if the noise is spatially white. For convenience, we now write

$$\mathbf{y}(n) = \mathbf{H}(z)\mathbf{x}(n) \qquad (2)$$

where the noise term is removed. We have also dropped the subscript "z" from $\mathbf{H}_z(z)$. Let $\mathbf{S}_i$ be a $I \times J$ selection matrix. All BIDS algorithms first form subchannel output vectors as

$$\mathbf{y}_i(n) \hat{=} \mathbf{S}_i \mathbf{y}(n) \qquad (3)$$

where i = 1, 2, …, M, and $M \hat{=} J!/[(J-I)!I!]$ is the total number of such subchannels. For each i, the BIDS algorithms then search for a decorrelator $\mathbf{G}_i(z)$ such that the power spectral matrix $\mathbf{S}_{\mathbf{u}_i\mathbf{u}_i}(z)$ of

$$\mathbf{u}_i(n) \hat{=} \mathbf{G}_i(z)\mathbf{y}_i(n) \qquad (4)$$

is diagonal. Let $\mathbf{G}_i(z) = \sum_{l=1}^{L_G} \mathbf{G}_i(l) z^{-l}$. Then, we can write

$$\mathbf{u}_i(n) = \begin{bmatrix} \mathbf{G}_i(0) & \mathbf{G}_i(1) & \cdots & \mathbf{G}_i(L_G) \end{bmatrix} \begin{bmatrix} \mathbf{y}_i(n) \\ \mathbf{y}_i(n-1) \\ \vdots \\ \mathbf{y}_i(n-L_G) \end{bmatrix}$$

or simply, $\mathbf{u}_i(n) = \overline{\mathbf{G}}_i \overline{\mathbf{y}}_i(n)$. The autocorrelation matrix of $\mathbf{u}_i(n)$ can be computed as

$$\hat{\mathbf{R}}_{\mathbf{u}_i\mathbf{u}_i}(\tau) = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{u}_i(n)\mathbf{u}_i^T(n-\tau)$$

It follows that $\hat{\mathbf{R}}_{\mathbf{u}_i\mathbf{u}_i}(\tau) = \overline{\mathbf{G}}_i \hat{\mathbf{R}}_{\overline{\mathbf{y}}_i\overline{\mathbf{y}}_i}(\tau) \overline{\mathbf{G}}_i^T$ where $\hat{\mathbf{R}}_{\overline{\mathbf{y}}_i\overline{\mathbf{y}}_i}(\tau)$ is computable from the available data $\mathbf{y}(n)$. In fact, if the noise is spatially and temporally white, the noise effect can be removed asymptotically from $\hat{\mathbf{R}}_{\overline{\mathbf{y}}_i\overline{\mathbf{y}}_i}(\tau)$ via eigenvalue decomposition. The cost function for constructing the decorrelator can be defined as the mean squared values of the off-diagonal elements of $\hat{\mathbf{R}}_{\mathbf{u}_i\mathbf{u}_i}(\tau)$ over a sufficient range of τ, i.e.,

$$E_i = \frac{1}{L_u I(I-1)} \sum_{\tau=0}^{L_u} \sum_{j \neq k} \left(\mathbf{g}_{i,k}^T \hat{\mathbf{R}}_{\overline{\mathbf{y}}_i\overline{\mathbf{y}}_i}(\tau) \mathbf{g}_{i,j} \right)^2$$

where $\mathbf{g}_{i,j}$ is defined by

$$\overline{\mathbf{G}}_i^T = \begin{bmatrix} \mathbf{g}_{i,1} & \mathbf{g}_{i,2} & \cdots & \mathbf{g}_{i,I} \end{bmatrix}.$$

The cost function E_i is a non-quadratic function of $\overline{\mathbf{G}}_i$. But it is quadratic with respect to each (individual) row of $\overline{\mathbf{G}}_i$. So, a simple algorithm consists of a sequence of the following sweeps until convergence. During each sweep, the rows of $\overline{\mathbf{G}}_i$ are updated sequentially. When updated, each row of $\overline{\mathbf{G}}_i$ is constrained to have a constant norm but minimizes E_i with all other rows fixed. The above procedure may be referred to as alternating projection. One can also apply gradient based algorithms here as well. But in general, E_i has local minima. Hence, a good initialization is required. Ideally, the minimum of E_i is zero (for large N). In our simulation shown later, $\overline{\mathbf{G}}_i$ is initialized in a neighborhood of an ideal $\overline{\mathbf{G}}_i$, and the alternating projection procedure is applied to search for the optimal $\overline{\mathbf{G}}_i$.

We now go back to (4) and discuss the conditions under which the decorrelation will lead to the desired result. We can rewrite (4) as

$$\mathbf{u}_i(n) = \mathbf{G}_i(z)\mathbf{y}_i(n) = \mathbf{G}_i(z)\mathbf{H}_i(z)\mathbf{x}(n) \qquad (5)$$

where $\mathbf{H}_i(z) = \mathbf{S}_i\mathbf{H}(z)$. We will show next that with a proper choice of $\deg(\mathbf{G}_i(z))$, $\mathbf{G}_i(z)\mathbf{H}_i(z)$ is diagonalizable by $\mathbf{G}_i(z)$. If $\mathbf{G}_i(z)\mathbf{H}_i(z)$ is diagonal, we see that each element of $\mathbf{u}_i(n)$ corresponds to a distinct input signal, and hence $\mathbf{G}_i(z)$ is also a signal separator. It is such separators $\mathbf{G}_i(z)$ that will be further exploited by the BIDS algorithms. However, a decorrelator is not necessarily a separator unless a diversity condition on $\mathbf{S}_{\mathbf{xx}}(z)$ is satisfied. This condition will also be shown next.

Lemma 1: Provided $\deg(\mathbf{G}_i(z)) \geq (I-1)\deg(\mathbf{H}_i(z))$, there always exists a $\mathbf{G}_i(z)$ such that $\mathbf{G}_i(z)\mathbf{H}_i(z)$ is diagonal.

Proof: This is a fact easy to prove [8].

Definition of diversity: Given two polynomials, we say that the distinction of one polynomial from the other is the number of *distinct* zeros of the first polynomial that are not shared by the second polynomial. The diversity of two polynomials is defined to be the larger distinction of the two polynomials. The diversity of two diagonal functions of a power spectral matrix is defined to be half the diversity of the two functions. The diversity of a power spectral matrix is defined to be the minimum diversity between any two diagonal functions of the matrix. The diversity of $\mathbf{S}_{\mathbf{xx}}(z)$ will be denoted by $div(\mathbf{S}_{\mathbf{xx}}(z))$.

Theorem 2: Let $\mathbf{C}(z)$ be a nonsingular (i.e., full rank almost everywhere) $I \times I$ polynomial matrix. The diagonalization of $\mathbf{C}(z)\mathbf{S}_{\mathbf{xx}}(z)\mathbf{C}(z^{-1})^T$ implies the diagonalization of $\mathbf{C}(z)$ up to a row permutation if

$$div(\mathbf{S}_{\mathbf{xx}}(z)) > (I-1)\deg(\mathbf{C}(z)). \qquad (6)$$

Proof: See [8].

Corollary 1: Provided that $\mathbf{G}_i(z)\mathbf{H}_i(z)$ is nonsingular, the diagonalization of $\mathbf{S}_{\mathbf{u}_i\mathbf{u}_i}(z)$ implies the diagonalization of $\mathbf{G}_i(z)\mathbf{H}_i(z)$ up to a row permutation if

$$\deg(\mathbf{G}_i(z)) < \frac{div(\mathbf{S}_{\mathbf{xx}}(z))}{I-1} - \deg(\mathbf{H}_i(z)). \qquad (7)$$

Proof: It follows from Theorem 2.

Note that $\mathbf{G}_i(z)\mathbf{H}_i(z)$ is nonsingular if and only if $\mathbf{S}_{\mathbf{u}_i\mathbf{u}_i}(z)$ is nonsingular (assuming that $\mathbf{S}_{\mathbf{xx}}(z)$ is nonsingular). Hence, the nonsingularity condition can be verified even with unknown $\mathbf{H}(z)$.

Assuming that the conditions of Lemma 1 and Corollary 1 are met, we can now find a $\mathbf{G}_i(z)$ by diagonalizing $\mathbf{S}_{\mathbf{u}_i\mathbf{u}_i}(z)$ such that $\mathbf{G}_i(z)\mathbf{H}_i(z)$ is diagonal up to an unknown row permutation. The row permutation on each $\mathbf{G}_i(z)$ can be determined using the correlations among $\mathbf{u}_i(n)$ [8].

Without loss of generality, we can now assume that $\mathbf{G}_i(z)$ is available such that

$$\mathbf{G}_i(z)\mathbf{H}_i(z) \hat{=} \mathbf{C}_i(z) = diagonal.$$

By removing the greatest common divisor (GCD) from each row of $\mathbf{G}_i(z)$ [9], $\mathbf{G}_i(z)$ becomes row-wise coprime. There are several possible ways to exploit $\mathbf{G}_i(z)$ to estimate the channel matrix and/or the input signals. The BIDS-1 algorithm [6] is one of them. The BIDS-2 algorithm is another as shown below.

3.2 BIDS-2 Algorithm

The BIDS-2 algorithm estimates the channel matrix $\mathbf{H}(z)$ from the decorrelators $\mathbf{G}_i(z)$, and then estimates the input signals using the estimated $\mathbf{H}(z)$. Recall that we can obtain the row-wise coprime nonsingular matrices $\mathbf{G}_i(z)$, for i = 1, 2, …, M', such that $\mathbf{G}_i(z)\mathbf{H}_i(z)$, for i = 1, 2, …, M', are diagonal. Furthermore, given the assumption that $\mathbf{H}(z)$ is nonsingular (full column rank for almost all z), we have the following lemmas (all proved in [8]).

Lemma 2: Each (nonzero) row of $\mathbf{H}(z)$ must be a row of an $I \times I$ square submatrix $\mathbf{H}_i(z)$ that is nonsingular (i.e., with full rank for almost all z).

Lemma 3: All the submatrices $\mathbf{H}_i(z)$ that are nonsingular are "chained" together in the sense that every two nonsingular submatrices share a common row either within the two submatrices or with another.

Lemma 4: If $\mathbf{H}_i(z)$ is nonsingular, so is the corresponding $\mathbf{G}_i(z)$ (provided that all rows of $\mathbf{G}_i(z)$ are constrained to be nonzero for almost all z, and $\mathbf{S}_{\mathbf{xx}}(z)$ is nonsingular diagonal).

We now define

$$\mathbf{H}_i(z) = [\mathbf{h}_{i,1}(z) \quad \mathbf{h}_{i,2}(z) \quad \cdots \quad \mathbf{h}_{i,I}(z)]$$

$\mathbf{G}_{i,p}(z)$ = $\mathbf{G}_i(z)$ without its pth row.

Then, we know that for i = 1, 2, …, M',

$$\mathbf{G}_{i,p}(z)\mathbf{h}_{i,p}(z) = 0 \qquad (8)$$

Since $\mathbf{G}_{i,p}(z)$ has the rank I-1 for almost all z, the solution to $\mathbf{G}_{i,p}(z)\hat{\mathbf{h}}_{i,p}(z) = 0$ for each i is $\hat{\mathbf{h}}_{i,p}(z) = \mathbf{h}_{i,p}(z) f_{i,p}(z)$ where $f_{i,p}(z)$ is a scalar polynomial. From Lemma 3, we know that $\mathbf{h}_{i,p}(z)$ for i = 1, 2, …, M' are "chained" together through shared elements. Hence, the solution to

$$\mathbf{G}_{i,p}(z)\hat{\mathbf{h}}_{i,p}(z) = 0 \text{ for all } i = 1, 2, \ldots, M' \qquad (9)$$

where $\hat{\mathbf{h}}_{i,p}(z)$ has the same overlapping (or chained) pattern as $\mathbf{h}_{i,p}(z)$, is then $\hat{\mathbf{h}}_{i,p}(z) = \mathbf{h}_{i,p}(z) f_p(z)$ where $f_p(z)$ is independent of i. In other words, (9) yields the pth column of the channel matrix $\mathbf{H}(z)$ up to a common polynomial. A detailed implementation of (9) is available in [8]. This leads to the following lemma.

Lemma 5: If $\mathbf{H}(z)$ is nonsingular (full column rank for almost all z) and column-wise coprime (each column is a coprime vector), then each column of $\mathbf{H}(z)$ can be found uniquely (up to scaling) from (9).

Like the BIDS-1 algorithm, the BIDS-2 algorithm yields the exact input signals in the absence of noise if $\mathbf{H}(z)$ is irreducible and the subchannel decorrelation is ideal (which requires the diversity condition). However, unlike the BIDS-1 algorithm, the BIDS-2 algorithm yields the exact $\mathbf{H}(z)$ if $\mathbf{H}(z)$ is nonsingular and column-wise coprime and the diversity condition is satisfied.

Such a matrix $\mathbf{H}(z)$ can have zeros and even be non-minimum-phase. But for estimating the input signals using $\mathbf{H}(z)$, we need it to be irreducible [8]. Once $\mathbf{H}(z)$ is found, we can check if $\mathbf{H}(z)$ is irreducible or not.

4. Simulation

We considered the following data model:

$$\mathbf{y}(n) = \mathbf{H}(z)\mathbf{x}(n) + \mathbf{w}(n)$$

where $\mathbf{w}(n)$ is white Gaussian, $\mathbf{x}(n)$ is a moving average random process with auto-correlation length 12, and the channel matrix is

$$\mathbf{H}(z) = \begin{bmatrix} -0.1615 & 0.0311 & 0.9587 \\ 0.2661 & 1.9975 & 0.0417 \\ -1.2879 & 0.5255 & 1.9072 \\ 2.0602 & 0.5089 & 1.0875 \end{bmatrix} + \begin{bmatrix} 1.1339 & 1.1339 & 0.9994 \\ 0.8219 & 0.8219 & -1.9034 \\ -0.2925 & -0.2925 & 0.2828 \\ 0.1754 & 0.1754 & 0.5940 \end{bmatrix} z^{-1}$$

Note that this channel matrix is irreducible but not column-reduced.

The BIDS-2 algorithm is tested against the BIDS-1 algorithm [6] and the matrix pencil (MP) algorithm [2]. The data length was chosen to be 10000. The mean squared errors (MSE) of the estimated channel matrix and the estimated input signals were computed over 50 independent runs. Figures 1 and 2 show, respectively, the channel estimation error and the signal estimation error in terms of the signal-to-noise ratio (SNR). The channel estimation error of the BIDS-2 algorithm is very small over a wide range of SNR because of the large data length. The MP algorithm suffers badly because the channel matrix is not column-reduced.

5. References

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Figure 1: Channel estimation errors

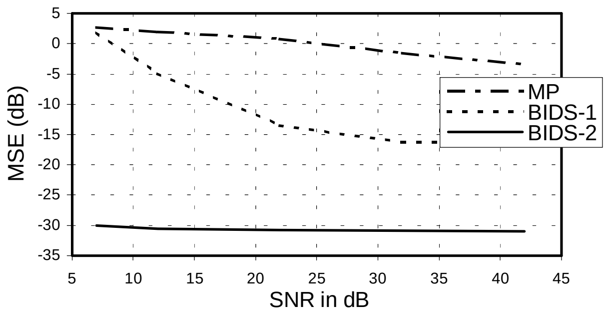

Figure 2: Signal estimation errors