MUTUAL COUPLING EFFECTS ON THE CAPACITY OF MULTIELEMENT ANTENNA SYSTEMS*

Thomas Svantesson, Anders Ranheim

Department of Signals & Systems
Chalmers University of Technology, SWEDEN.

ABSTRACT

A study of the capacity of multiple element antenna systems is presented, with particular emphasis on the effect that mutual coupling between the antenna elements has on the capacity. The results presented here shows, contrary to some earlier claims, that correlation between different channel coefficients as a function of antenna spacing, can in fact decrease when the mutual coupling effect is accounted for. As a consequence, capacity also improves. A realistic channel model is used to perform simulations to support these claims.

1. INTRODUCTION

The topic of multiple-input multiple-output (MIMO) communications systems have received considerable attention in recent years, and in particular the study of the capacity of such systems. Several authors have shown that the capacity gains resulting from the use of multi-element antenna (MEA) systems are potentially very significant [3, 8]. A common assumption in the study of such systems is that the fading coefficients between different pairs of transmit-receive antennas are independent and identically distributed (i.i.d.). However, in practice the signals received by different antennas will be correlated which will reduce the capacity.

Measurement campaigns have been reported [5] that show a substantial capacity increase when using MEA systems, as long as elements are placed sufficiently far apart. Since the capacity of MEA systems is strongly dependent upon the number of transmit and receive elements available, it is highly desirable to use as many antennas as possible. On the other hand, one typically has a limited amount of space/volume to distribute the antenna elements over. Unfortunately, closely spaced elements increase the correlation and thus decrease the capacity.

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The effects on the correlation and thus also the capacity gain when using a MIMO system with a small inter-element antenna spacing will be investigated in this paper. Previously published studies in this area [1, 8] have ignored the fact that small inter-element antenna spacing will cause mutual coupling between elements. Mutual coupling is well known in the antenna community, but rather unknown in signal processing circles. In principle, the received voltage on each element will depend not only on the incident field, but also on the voltages on the other elements. This effect becomes significant at inter-element spacings of less than half a wavelength, and thus needs to be included in a correlation/capacity study when closely spaced elements are employed.

The general belief is that mutual coupling will deteriorate the channel, increase the correlation and reduce the achievable capacity. For instance, it was been stated in [3] that since mutual coupling increases with reduced antenna spacing, it will also cause problems for achieving high capacity. In this paper, it is found that mutual coupling, on the contrary, actually can increase the channel capacity for scenarios with closely spaced antennas. It will be shown that for a typical realistic scattering model, mutual coupling can in fact have a decorrelating effect on the channel coefficients, and thereby also improve the capacity.

2. SYSTEM MODEL

Consider a communication link with n_T transmit antennas and n_R receive antennas. Some important assumptions used throughout are

- There is only a single user transmitting at any given time, so the received signal is corrupted by additive white Gaussian noise only.
- The communication is carried out in frames/packets of finite time-span, and the coherence-time of the channel is longer than the packet duration.

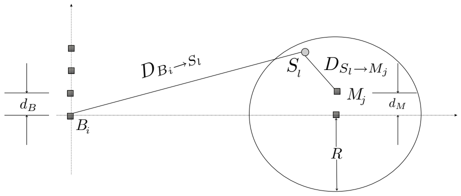

Figure 1: Geometry of channel

- The bandwidth of the transmitted signal is less than the coherence-bandwidth of the channel, i.e. the fading is frequency-flat.

The following discrete-time vector/matrix model for the relation between input signal $\mathbf{s}_t$ and received signal $\mathbf{r}_t$ can then be formulated as

$$\mathbf{r}_t = \mathbf{H}\mathbf{s}_t + \mathbf{v}_t, \tag{1}$$

where $\mathbf{r}_t = [r_t^{(1)}, \ldots, r_t^{(n_R)}]^T$, $\mathbf{s}_t = [s_t^{(1)}, \ldots, s_t^{(n_T)}]^T$, $\mathbf{v}_t$ is spatially and temporally additive white Gaussian noise (AWGN) with unit variance, and t is a discrete-time index. The $n_R \times n_T$ channel matrix $\mathbf{H}$ is made up of elements $h_{i,j}$ as follows

$$\mathbf{H} = \begin{pmatrix} h_{1,1} & \ldots & h_{1,n_T} \\ \vdots & \ddots & \vdots \\ h_{n_R,1} & \ldots & h_{n_R,n_T} \end{pmatrix}, \tag{2}$$

where $h_{i,j}$ denotes the channel coefficient between the j:th transmit antenna and the i:th receiver element. In principle, any channel model that accurately includes the spatial dimension could be used to investigate the correlation properties of two spatially separated antennas. For an excellent overview, see [2].

A simple, yet detailed, channel model that includes the spatial dimension is presented in [7], where a circular disc of uniformly distributed scatterers is placed around the mobile. In Figure 1, a simple illustration of the scatter disc and the orientation of the mobile and base station is shown. Based on this model, the elements of the channel matrix in (2) is generated as follows. Assume there are L scatterers S_l, $l = 1, \ldots, L$, uniformly distributed on a disc of radius R, centered around the mobile The channel parameter $h_{i,j}$ connecting transmit element j and receive element i is thus

$$h_{i,j} = \sum_{l=1}^{L} \alpha_l \exp\left(-j\frac{2\pi}{\lambda} \cdot (D_{B_i \to S_l} + D_{S_l \to M_j})\right) \tag{3}$$

where $D_{B_i \to S_l}$ and $D_{S_l \to M_j}$ are the distances from base station antenna i to scatterer l, and scatterer l to mobile antenna j, respectively, as shown in Figure 1. Also, α_l is the scattering coefficient from scatterer l, and is modeled as a normal complex random variable, with zero mean and unit variance. The channel matrix is finally normalized such that $\|\mathbf{H}\|_F^2 = n_T$. Thus, the increased antenna gain due to the use of multiple antennas are not included [1].

3. MUTUAL COUPLING

The principal function of an antenna is to convert an electromagnetic field into an induced voltage or current to be measured. However, the measured voltage at each antenna element will depend not only on the incident field, but also on the voltages on the other elements. Essentially, the received voltage on each element will induce a current on the element which in turn radiates a field which affects the surrounding element, i.e. the elements are said to be mutually coupled.

Mutual coupling is well known in the antenna community, since coupling between antenna elements is one of the most important properties to consider in antenna design. However, this phenomenon is is rarely accounted for or studied in the signal processing or communications literature. It is a simple matter to include the coupling effect in the model for the received voltage, by inserting a mutual coupling matrix, modyfying (1) to

$$\mathbf{r}_t = \mathbf{C}\mathbf{H}\mathbf{s}_t + \mathbf{v}_t. \tag{4}$$

It is then natural to include the coupling effects into the channel by combining the two terms into a new channel matrix $\mathbf{H}'$. Note that (4) only includes coupling at the receiving antenna elements. In the scenario depicted in Figure 1, several closely spaced elements at the transmitter (mobile) will also experience mutual coupling. Thus, including this effect at both the transmitter and receiver, the expression for the channel becomes

$$\mathbf{H}' = \mathbf{C}_b \mathbf{H} \mathbf{C}_m, \tag{5}$$

where the coupling matrix at the base $\mathbf{C}_b$ is $n_R \times n_R$ and the corresponding matrix at the mobile $\mathbf{C}_m$ is $n_T \times n_T$. Using fundamental electromagnetics and circuit theory, the coupling matrix of an array antenna can be written as [4]

$$\mathbf{C} = (Z_A + Z_T)(\mathbf{Z} + Z_T\mathbf{I})^{-1}, \tag{6}$$

where Z_A is the antenna impedance, Z_T is the impedance of the measurement equipment at each element, and $\mathbf{Z}$ is the mutual impedance matrix. This expression can be used for any array antenna, but for many types of elements, an analytical expression for the mutual impedance matrix and the antenna impedance is difficult to obtain. However, one noticable exception is the case of dipoles which will be used as antenna elements in the communications scenarios investigated

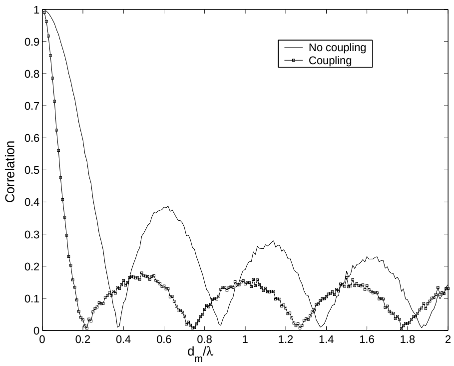

Figure 2: The effect of the coupling on the correlation between paths $E[h_{1,1}, h_{1,2}]$ versus different element spacings d_m at the mobile. The base station separation is $d_b = 0.5\lambda$ and the scatter disc radius $R = 200\lambda$.

here. A more detailed derivation of the received voltages of an array of thin finite dipoles where coupling is included can be found in [9]. In the following, the impedance of the measurement equipment Z_T is chosen as the complex conjugate of the dipole impedance in order to reduce the powerloss. The effect of the coupling on the correlation between paths $E[h_{1,1}, h_{1,2}]$ is shown in Figure 2 for different element spacings d_m at the mobile. Interestingly, the correlation between these two channel coefficients decreases faster when coupling is included in the calculations. In fact, it is possible to cut the element separation in half due to the coupling at the 0.1 correlation level (0.4λ to 0.2λ). Thus, the coupling phenomenon actually decorrelates the signals by acting as an additional "channel". Interestingly, it was recently reported in [6] that coupling can in fact decrease the Bit-Error-Rate performance on a Nakagami fading channel. In summary, the mutual coupling may, contrary to common belief, actually decrease the correlation level between channel coefficients and thereby also increase the channel capacity.

4. CHANNEL CAPACITY

Considering a (n_T, n_R) MEA system, with channel matrix $\mathbf{H}$, the channel capacity is given by the expression

$$\begin{aligned} C &= \log_2(|\mathbf{I}_{n_R} + \rho\frac{\mathbf{HH}^*}{n}|) \\ &= \sum_{k=1}^{n} \log_2(1 + \rho\frac{\lambda_k}{n}) \end{aligned} \tag{7}$$

where $\mathbf{H}$ is the $n_R \times n_T$ channel matrix, $n = \min(n_T, n_R)$, and λ_k are the eigenvalues of $\mathbf{HH}^*$. Note that this expression assumes that the available transmit power ρ is uniformly allocated to the n_T transmit elements, which is the practical approach when the transmitter has no knowledge of the channel.

It is easily realized from this expression that a large capacity hinges on the presence of a rich scattering environment, being directly related to the rank of the channel matrix. Conversely, little or no scattering will result in a channel matrix of unit rank and thus low capacity.

In the event that there is coupling between the antenna elements of the mobile, the channel matrix should be modified to $\mathbf{H}_{cm} = \mathbf{HC}$, where $\mathbf{C}$ is given in (6). The capacity in this case thus becomes

$$C = \log_2(|\mathbf{I}_{n_R} + \rho\frac{\mathbf{H}_{cm}\mathbf{H}_{cm}^*}{n}|). \tag{8}$$

Likewise, modifications for the case of coupling at the base or both at base-and-mobile, are similarly straightforward, as discussed in Section 3. In what follows, the capacity is computed for a large number of channel realizations, each with a random location of scattering elements within the disc. The capacity thus becomes a random variable, and we can compute its complentary cumululative distribution function (ccdf) as well as the outage cpacity, where we vary the parameters of interest.

We consider a scenario where the mobile has $n_T = 2$ antenna elements, and is placed at broadside relative to the $n_R = 4$ element base-array, and at a distance of $D = 300\lambda$ away from it. For each channel realization, a total of 100 scatterers are placed randomly and uniformly ditributed on a disc of radius $R = 200\lambda$, centered on the mobile. The mobile has a total power $\rho = 10$ and 1000 channel realizations were generated for each value of d_m to compute the required statistics.

Figure 3 shows the ccdf:s both with and without mutual coupling between the mobile antenna elements, for two different values of mobile antenna separation, d_m. The separation between base-elements was held constant at $d_b = 0.5\lambda$. It also shows the capacity for the case of idealized i.i.d. channel coefficients [3]. If one ignores the effect of mutual coupling, it can be seen that one suffers a significant capacity loss by placing the mobile antennas closer together, i.e. from $d_m = 0.5\lambda$ to $d_m = 0.1\lambda$, as would be expected. The more interesting result is that when the coupling effect is accounted for, the difference in capacity is essentially identical for these two values of mobile antenna separation. In short, the advantage we lose from the decreased antenna separation is more than compensated for when we also take the coupling between the same elements into account. It should be mentioned that these observations hold true in the event that we also include the coupling between the base antenna alements. Figure

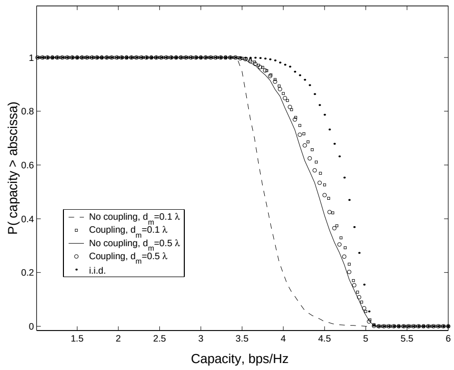

Figure 3: Capacity with/without coupling at mobile, for different values of mobile antenna separation, d_m.

4 shows the 10 % outage capacity $C_{0.1}$, i.e. there is a probability of 0.10 that the capacity is less than what is seen in the plot. First, notice how the capacity when coupling is included is higher for all but a couple of isolated values of d_m, as compared to the no coupling case. These results should be seen in conjunction with the correlation plot in Figure 2; The largest disparity between the two outage capacity curves occur at values of d_m for which the corresponding correlation curves also have a large disparity, in favor of the system which includes the coupling effect.

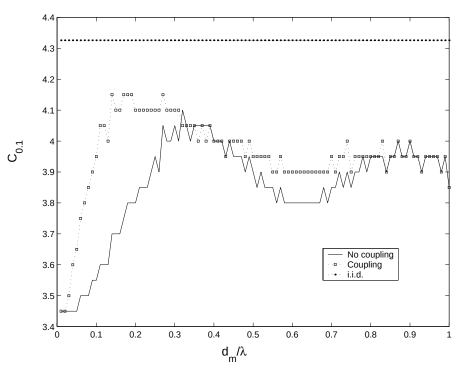

Figure 4: 10 % Outage capacity as a function of antenna spacing

Other scenarios were also simulated to support the generality of the above conclusions to different propagation conditions. In summary, in the case of very localized scattering, the elements of the channel matrix will have a high degree of correlation regardless of whether coupling is included or not. Hence, there is little or nothing to be gained from an MEA system in terms of capacity improvements in such scenarios. For the other extreme, when the scattering environment is sufficiently rich to approach the i.i.d. assumption, coupling between antenna elements will clearly only degrade the performance. The results presented here have focused on more realistic scenarios that fall between these two extremes.

5. CONCLUSIONS

The capacity of multiple element antenna systems was studied, focusing on the effect of antenna spacing at the mobile terminal. In particular, the effect of mutual coupling between antenna elements was considered, and its effect on the correlation between channel coefficients and thereby on the capacity of such systems. Contrary to earlier claims regarding the effect of mutual coupling on capacity of MEA systems, we showed results to support the conclusion that coupling can in fact have a decorrelating effect on the channel coefficients and thereby also increasing the capacity.

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