A SINGLE-PARAMETER ADAPTIVE COMB FILTER

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ABSTRACT

The study is concerned with a single-parameter adaptive comb filter (ACF), a multi-notch filter with periodically located nulls. The filter is suggested to retrieve a waveform modeled by superposition of harmonics, in particular, periodic non-sinusoidal signal. Using trigonometric constraints between the signal fundamental frequency and over tones results in a non-linear estimation problem. In the present study, the parameter adjustment relies on the extended Kalman filter scheme. Particularly, the 2, 3 and 4 notch ACF are derived and tested under different conditions. Given a multi-tone scenario, the ACF significantly outperforms common adaptive multi-notch filter.

1. INTRODUCTION

To eliminate (or retrieve) the signal components periodically located across the frequency band is a problem frequently encountered in different applications. Given the signal component frequencies, one usually applies a multi-notch (or, specifically, comb) filter [1].

If the component frequencies are unspecified beforehand, one may use a common adaptive notch filter (ANF) [2-4] with a desired number of zeros. A reasonable choice is the Nehorai's minimal parameter ANF with constrained poles and zeros where the number of parameters equals the number of notches [3]. Another choice is the Regalia-type ANF [4].

The objective of the current study is to modify the ANF into an adaptive comb filter (ACF) that agrees with the signal model rich in harmonics. Indeed, applying the multi-notch ANF to a superposition of over tones, one may utilize explicit trigonometric relationships between the fundamental frequency and its multiples. By this means only one parameter (basic frequency) is necessary to build the desired comb filter.

Combining the IIR notch filter with proper trigonometric constraints results in essentially nonlinear models that should be treated by a nonlinear estimation technique. An extended Kalman filter (EKF) is applied to adjust the ACF parameter.

Next we determine the ACF model by convolving a cascade of single-parameter sections related to a fundamental frequency, and build a corresponding adaptation procedure relying on an EKF-like technique. Major questions behind this study are whether an ACF provides benefits over a conventional ANF and whether an ACF is applicable to a signal with a large number of over tones.

2. THE ANF MODEL

The Nehorai minimal parameter ANF has the IIR form

$$W\left(z^{-1}\right)=L\left(z^{-1}\right)/L\left(\rho z^{-1}\right) \qquad (1)$$

where the nominator and denominator are specified by the polynomial

$$L(u)=1+a_1u+\ldots+a_nu^n+\ldots+a_1u^{2n-1}+u^{2n} \qquad (2)$$

where $(a_1,\ldots,a_n)$ is the n-vector of the filter coefficients, and ρ is a parameter that defines the notch sharpness. The Nehorai ANF needs n coefficients to extract n sinusoids.

For a single null, (1) reduces to the single-parameter section

$$W\left(z^{-1}\right)=\frac{1+bz^{-1}+z^{-2}}{1+\rho bz^{-1}+\rho^2z^{-2}} \qquad (3)$$

where b=-2cos2πfT, the filter coefficient, f is the notch frequency, and T the sampling period.

Respectively, a multi-notch filter (1) can be defined as a product of single-parameter sections

$$W\left(z^{-1}\right)=\prod_k\frac{1+b_kz^{-1}+z^{-2}}{1+\rho b_kz^{-1}+\rho^2z^{-2}} \qquad (4)$$

where b_k corresponds to a certain notch, k=1,2,…, n.

In the presence of super harmonics, their frequencies are multiples of the fundamental frequency and therefore a single parameter can specify the filter. The cases of 2, 3 and 4 over tones are of particular interest.

3. THE COMB FILTER

We start with a signal consisting of the basic frequency f and its 2^{nd} harmonic, the case when a 2-notch ANF (ANF2) is common. The ANF2 with model (1) needs two parameters, a_1 and a_2. However, both terms can be expressed via a single parameter, say b. Let us consider ANF2 in the form (4) with k=1,2. If $b=b_1=-2\cos\Omega$ (where $\Omega=2\pi f$T), then $b_2=-2\cos 2\Omega$. Using the familiar trigonometric identity

$$\cos 2\Omega=2\cos^2\Omega-1 \qquad (5)$$

results in the constraint

$$b_2=2-b^2 \qquad (6)$$

Substituting (6) into the 2^{nd} section of (4) and convolving two sections gives the model (1) with

$$a_1=2+b-b^2,\ a_2=2+2b-b^3 \quad (7)$$

So, the nulls of 2-notch ACF (ACF2) depend on the single parameter b specified by the fundamental frequency f.

Next, the ANF for *three* notches (ANF3) has the structure (4) with 3 coefficients: b_1, b_2 and b_3. Similarly, using a proper trigonometric identity

$$\cos 3\Omega=4\cos^3\Omega-3\cos\Omega \quad (8)$$

results in the relationship

$$b_3=b^3-3b \quad (9)$$

Substituting (6) and (9) into the model (4) and convolving *three* sections gives the ACF3 parameters

$$\begin{aligned} a_1&=2+2b-b^2+b^3\\ a_2&=3-4b-3b^2+4b^3+b^4-b^5,\\ a_3&=4-4b-8b^2-2b^3+5b^4-b^6 \end{aligned} \quad (10)$$

Next, the *four* notch filter ANF4 can be expressed in the form (4) with coefficients $b_1,\ldots, b_4$. Using the identity

$$\cos 4\Omega=8\cos^4\Omega-8\cos^2\Omega+1 \quad (11)$$

one obtains

$$b_4=-b^4+4b^2-2 \quad (12)$$

Substituting (6), (9) and (12) into (4), and convolving four sections gives the ACF4 parameters

$$\begin{aligned} a_1&=-b^4+b^3+3b^2-2b,\\ a_2&=-b^7+b^6+5b^5-5b^4-6b^3+7b^2,\\ a_3&=b^9-b^8-8b^7+6b^6+22b^5-12b^4-21b^3+9b^2+2b,\\ a_4&=b^{10}-9b^8-2b^7+30b^6+10b^5-44b^4-12b^3+26b^2-2 \end{aligned} \quad (13)$$

In a similar manner, one can specify a higher order ACF.

4. THE PARAMETER ESTIMATION TECHNIQUE

Since the ACF models are strongly nonlinear, a proper filtering technique is necessary to adjust the filter parameter. In particular, one may try an EKF [5] rather than the conventional Gauss-Newton method [1-4]. So, there is a question of interest whether the model's nonlinearity affects the ACF behavior.

Due to (1), the ANF inputs and outputs are related as

$$y_iL(z^{-1})=e_iL(\rho z^{-1}) \quad (14)$$

where y_i and e_i are the filter input and output, respectively, at i^{th} instant of time. The latter expression can be rearranged so that to isolate the current observation y_i in the left-hand side:

$$y_i=-AY^T+A\Lambda E^T+e_i \quad (15)$$

where $A=[a_1,\ldots,a_n,\ldots,a_1,\ 1]$, $Y=[y_{i-1},\ldots,\ y_{i-n},\ldots,\ y_{i-2n}]$, $E=[e_{i-1},\ldots,\ e_{i-n},\ldots,\ e_{i-2n}]$, and $\Lambda=\mathrm{diag}(\rho,\ldots,\rho^{2n})$.

The relationship (15) can be viewed as the filter observation function (first and second terms) contaminated by the noise term e_i. The ANF parameters, a_1 through a_n, are unknowns and so (15) is a nonlinear function of the form

$$y_i = h_i(a_1,\ \ldots,\ a_n\ ) + e_i \quad (16)$$

The observation function h_i relies on the past inputs $y_{i-1},\ldots,\ y_{i-2n}$ and outputs $e_{i-1},\ldots,\ e_{i-2n}$. Both sets play role of the observation function parameters. In practice, the unknown output noise sequence may be replaced by the corresponding ANF residuals.

After translating an ANF to single-parameter ACF, the term b can be viewed as the EKF state. The simplest way is to model the time varying parameter as the 1st order autoregression [5].

With another approach, the slowly changing parameter b and its several derivatives (b', b'',…) may be viewed as states of the Kalman-like tracking filter [6]. Then the system state vector equation becomes

$$x_i=Fx_{i-1}+ q_i \quad (17)$$

where $x_i=[b,\ b',\ldots]^T$ denotes the system state vector at i^{th} instant, F - state transition matrix, and q_i - system noise term [6].

The filter observation model then can be presented as

$$y_i=h_i(x_i)+ e_i \quad (18)$$

Given a *linear* system (17) and *nonlinear* observation function (15), one may apply the EKF equations

$$\bar{x}_i=F\hat{x}_{i-1} \quad (19)$$

$$\bar{y}_i=h_i(\bar{x}_i) \quad (20)$$

$$\hat{x}_i=\bar{x}_i+K_i(y_i-\bar{y}_i) \quad (21)$$

$$K_i=\tilde{P}_iH^T\left(H\tilde{P}_iH^T+R\right)^{-1} \quad (22)$$

$$\tilde{P}_i=FP_{i-1}F+Q \quad (23)$$

$$P_i=(I-K_iH)^T\tilde{P}_i(I-K_iH)+K_iRK_i^T \quad (24)$$

where Q and R are the covariance matrices of the process and measurement noise, respectively, and H is the gradient of the observation function h_i with respect to (w.r.t.) the system state variables. Thus, differentiating both sides of (14) w.r.t. b gives

$$y'L(z^{-1})+ yL(z^{-1})'=eL(\rho z^{-1})'+e'L(\rho z^{-1}) \quad (25)$$

Note that in the stable mode, the output error represents a white noise and its sensitivity to b may be ignored. So, the 2nd term in the right-hand side of (25) drops resulting in

$$y'= L(z^{-1})^{-1}[\ -yL(z^{-1})' + eL(\rho z^{-1})'\ ] \quad (26)$$

Thus the gradient H that can be presented in the form

$$H=G(\Lambda V^T -X^T) \quad (27)$$

(or, approximately [2], $H= -GX^T$)

where

$$\begin{aligned} &G=\mathrm{grad}\{A\}\ \text{w.r.t.}\ b\\ &X=[\xi_{i-1},\ldots,\xi_{i-n}]\\ &V=[\varepsilon_{i-1},\ldots,\varepsilon_{i-n}] \end{aligned} \quad (28)$$

with

$$\xi_i=y_i/L(z^{-1}),\quad \varepsilon_i=e_i/L(z^{-1}) \quad (29)$$

However, it was found useful to replace the latter terms with

$$\xi_i=y_i/L(rz^{-1}),\quad \varepsilon_i=e_i/L(rz^{-1}) \quad (29a)$$

where r, $\rho\le r\le 1$ distinguishes a more general narrow-band ARMA by contrast to a sine-based model [3].

Eqs. (27)-(29) together with (19)-(24) define an EKF-based ACF. Note that in accordance with (20)-(21), the filter residual is found due to the *predicted b*. However, it is reasonable to recompute the current residual using an *updated b*. For this purpose, steps (20) and (21) can be iterated (at least once).

The term ρ is defined as recommended in [3]. Next, the noise covariance starts from a sufficiently large magnitude and then gradually reduces to a smaller value that suites to a stable mode.

As seen, for any type of ACF the parameter estimator holds a general structure based on the EKF equations (19)-(24) and expressions (27)-(29). A peculiarity of each particular case (1-4 nulls) is reflected only in the model of h_i and its gradient H.

5. SIMULATIONS

Next we check the ACF performance and compare it with an ordinary ANF under different conditions.

A generated signal comprises the unit-amplitude sinusoid of 0.112 normalized frequency and its super harmonics. Harmonics vary their amplitudes due to a particular scenario. The signal is contaminated by an additive, zero-mean, 0.1 rms white noise. By this means, $SNR=10^2/2=50$ (or, equivalently, 16 dB).

With each scenario, we apply a particular ACF or ANF to a certain type of signal. The EKF runs with the single-state dynamic model. Computation of ρ starts with 0.8 and approaches 0.995 with forgetting factor 0.99 [3]. The observation noise covariance $R=\sigma^2$, where $\sigma=4$. Initial covariance P=0.01. The system noise covariance $Q=10^{-5}$. Each scenario is averaged over 100 independent runs (with the same initial conditions). At the first stage, we intentionally determine a slower and smoother convergence in order to enhance differences between scenarios.

Figs. 1 and 2 show performance of the ANF1 and ANF2, respectively, under a 2-tone scenario. As the 2nd harmonic magnifies (curves 1, 2 and 3 correspond to amplitudes 0, 0.5 and 1, respectively), both filters dramatically deteriorate.

It is not the case with a comb filter. Unlike the ANF, ACF2 (Fig. 3) not only remains smooth, but also, as the 2nd harmonic magnifies, converges significantly faster.

Figs. 4 and 5 show the performance of ACF3 and ACF4 applied to the 3-tone and 4-tone signals, respectively, while the highest harmonic amplitude is 0 (curve 1), 0.5 (curve 2) and 1 (curve 2). These figures display that as the highest harmonic magnifies, the ACF improves convergence rate. However, for higher order filters the benefit becomes less significant. So, the ACF3 or ACF4 may be sufficient for practical purposes. All ACF demonstrate a fast and smooth unbiased convergence.

In the next scenario, the observation noise is set smaller ($\sigma=0.4$) in order to increase the convergence rate and reduce bias. The initial frequency is given a true value in order to exclude the initial error contribution on the filter output.

Fig. 6 displays the standard deviation (std) experienced by the ACF2 in the 2-tone scenario. As the 2nd tone is zero (curve 4), the output std, as expected [3], is slightly higher than the corresponding CRB (curve 1). As far as the 2nd tone amplifies (curves 2 and 3 correspond to amplitudes 1 and 0.5, respectively), the ACF2 drastically changes its behavior for short-length data, in particular, the filter considerably decreases output variance for shorter records. At initial stage, up to nearly 200 steps, the filter experiences a std even smaller than predicted by the CRB for given SNR. It may be explained so that utilizing higher harmonics yields proper increase in the SNR. Note that the shorter is the data length, the higher is the benefit provided by the increasing 2nd amplitude. For longer periods, the curves 2, 3 and 4 in Fig. 6 gradually converge.

Other simulations (not presented due to space limitation) exhibit the filter advantages in more detail.

6. FIELDS OF APPLICATION

The ACF is an efficient tool to retrieve a non-sinusoidal signal, a problem encountered in a wide range of applications.

Thus, the Sagnac effect utilized in the fiber-optic gyroscope (FOG) [7] yields a phase-modulated signal with significant super harmonics.

Another example is the dithering technique when a vibrating high-power and high-frequency signal is applied to the sensing block in order to avoid a non-linear (lock-in) zone. The block output usually comprises higher harmonics that should be accurately deleted in order to recover the original input.

One more application field is the photoplethysmographic signal (PPS) encountered in the pulse oxymetry [8]. The PPS is definitely a non-sinusoidal waveform and has 4-5 significant harmonics. The accurate spectrum shape is of importance to distinguish between useful components and artifacts.

The list of ACF applications may be continued.

7. CONCLUSIONS

In this study, the conventional ANF was adjusted to a signal model comprising a basic sinusoid and several super harmonics. Trigonometric relationships between the fundamental tone and its multiples are used as the filter constraints. This modification results in a particular type of adaptive multi-notch filter, ACF. Given the multi-harmonic signal scenario, an ACF clearly outperforms ANF. Thus, as the higher harmonics amplify, an ACF with appropriate number of notches improves its convergence rate, whereas an ANF, a notch filter of the same order but with independent nulls deteriorates. Evidently, in the former case, with an ACF, a super harmonic contributes in the signal power, whereas in the latter case, with an ANF, it magnifies an interference.

It is noteworthy that as the number of nulls is less than the number of harmonics the standard ANF considerably degrades in the convergence rate. It means that with a parallel or cascade ANF implementation each section that retrieves a particular harmonic is affected by other tones. It is not the case with an ACF when all notches are in agreement with the basic frequency.

An ACF may be efficiently used to recover non-sinusoidal waveforms encountered in a multitude of applications.

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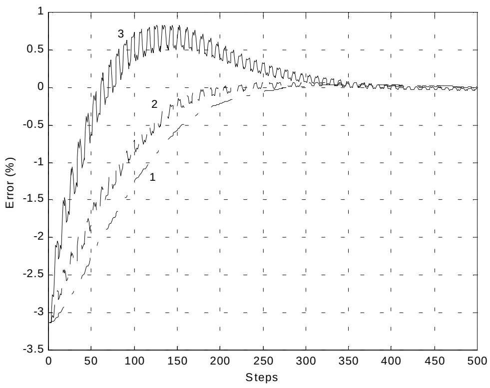

Fig. 1. ANF1. 2nd tone 0 (dashed curve 1), 0.5 (dashdot 2) and 1 (solid 3).

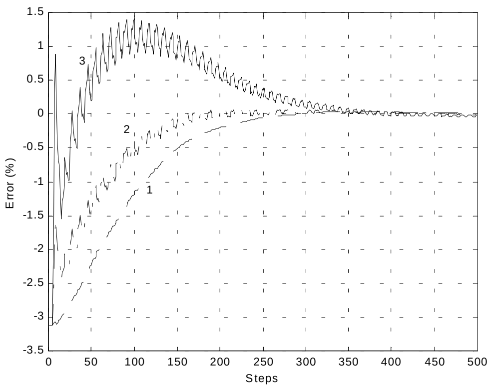

Fig. 2. ANF2. 2nd tone 0 (dashed curve 1), 0.5 (dashdot 2) and 1 (solid 3).

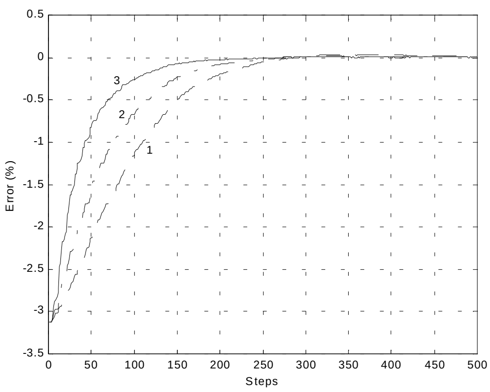

Fig. 3. ACF2. 2nd tone 0 (dashed curve 1), 0.5 (dashdot 2) and 1 (solid 3).

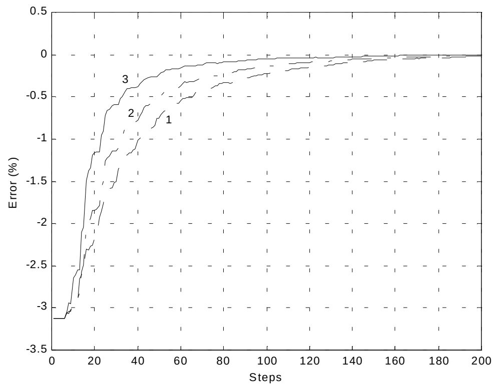

Fig. 4. ACF3. 3rd tone 0 (dashed curve 1), 0.5 (dashdot 2) and 1 (solid 3).

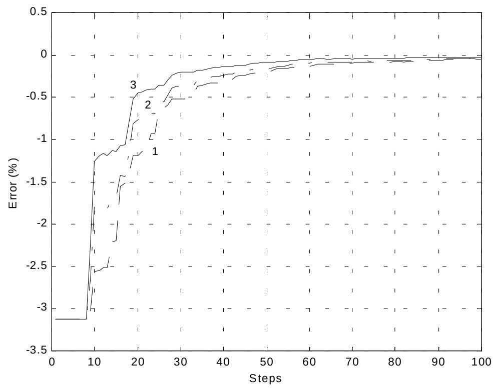

Fig. 5. ACF4. 4th tone 0 (dashed curve 1), 0.5 (dashdot 2) and 1 (solid 3).

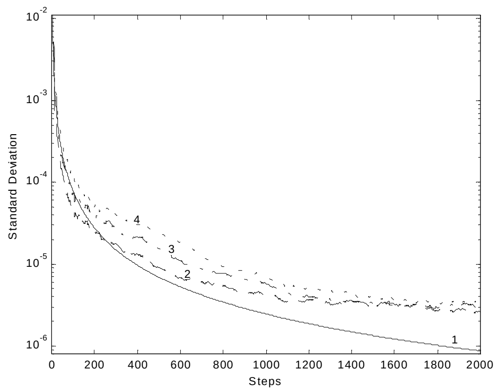

Fig. 6. CRB (solid curve 1) & ACF2 standard deviation. 2nd tone 0 (dotted 4), 0.5 (dashdot 3) and 1 (dashed 2).