MARKOV MODELING OF TRANSIENT SCATTERING AND ITS APPLICATION IN MULTI-ASPECT TARGET CLASSIFICATION

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ABSTRACT

Transient scattered fields from a general target are composed of wavefronts, resonances and time delays, with these constituents linked to the target geometry. A classifier applied to transient scattering data requires a statistical model for such fundamental constituents. A Markov model is employed to characterize the transient scattered fields – for a set of target-sensor orientation over which the transient scattering is stationary – utilizing a wavefront, resonance, time-delay "alphabet". The Markov model is utilized in a classifier developed for multi-aspect transient scattering data, with a hidden Markov model (HMM) employed to address the generally non-stationary nature of the multi-aspect waveforms. Each state of the HMM is characteristic of a set of target-sensor orientations for which the scattering statistics are stationary, the statistics of which are characterized via the aforementioned Markov model. The wavefront, resonance and time-delay features are extracted via a modified matching-pursuits algorithm.

1. INTRODUCTION

The classification of a target based on multi-aspect scattered waveforms constitutes a problem of long-standing interest. Scenarios for which this problem is relevant include target detection via synthetic aperture radar (SAR), in which the synthetic aperture implies the target is viewed from multiple orientations, or aspects. In the work presented here, we are interested in the detection and classification of a submerged (underwater) elastic target, based on acoustic scattering data, with the sensor sequentially positioned such that the target is viewed from multiple orientations [1]. The complication in these sensor problems is manifested in the fact that in practice both the target identity and orientation are unknown, necessitating the classifier to be robust to such uncertainty.

The hidden Markov model (HMM) has proven to be an effective tool for processing multi-aspect scattering data [1,2], with each HMM state corresponding to a generally contiguous set of target-sensor orientations over which the associated scattered fields are statistically stationary. When sensing a set of scattered waveforms from multiple orientations, we therefore implicitly sample waveforms from the different target states, with some states sampled more than once, and others not at all, depending on the target-sensor orientations considered. The sequence of sampled states is modeled via a Markov model, and since the target orientation (and identity) is unknown, the states being sampled are "hidden", necessitating a HMM.

The contribution of this paper involves the statistical characterization of the HMM states, for transient scattering data. The scattering data is parsed via a modified form of matching pursuits, employing a wavefront-resonance dictionary. The matching-pursuits parsing characterizes the transient scattered fields in terms of a sequence of wavefronts, resonances and time delays, sequentially arranged with increasing time (from left to right, as in speech). This implies that the scattered fields are characterized by three canonical members of an alphabet, with the sequence of such modeled via a Markov model, as applied in classical language models [3]. Each HMM state is characteristic of a distinct class of scattering phenomenology, and is therefore represented by a distinct Markov model, for the wavefront, resonance and time delay alphabet. This state-dependent model is closely connected with the underlying scattering physics, as compared with vector quantization and Gaussian-mixture models utilized previously [1,2]. We demonstrate that the HMM model with Markov state descriptor performs well in its role as a classifier. Moreover, by connecting the statistical model to the underlying scattering physics, one can address fundamental information-theoretic constructs such as the mutual information between different target models [4].

2. MODIFIED MATCHING PURSUITS

Assume we have a scattered waveform $f(t)$ that we wish to parse in terms of its underlying scattering mechanisms. Discrete scatterers on a target (edges, corners, etc.) generally give rise to temporally localized scattered fields, with these termed wavefronts. Wavefronts are typically a slightly modified form of the excitation pulse. When multiple wavefronts are closely spaced temporally, the set of wavefronts is represented more compactly in terms of a resonance. For example, the (resonant) ringing heard from a bell corresponds to a *sequence* of wavefronts, as the underlying acoustic wave circumnavigates the bell surface, shedding acoustic energy on every rotation. The temporal spacing between wavefronts and resonances is characteristic of the spatial separation between different target scatterers, this also containing important underlying information. We therefore seek to parse the transient scattered fields into their fundamental wavefront, resonance and time delay constituents. This is effected by employing a modified form of matching pursuits [5]. In particular, define a dictionary D_γ indexed by the parameters $\gamma_n = \{\alpha_n, \omega_n, \tau_n, \phi_n\}$, with dictionary elements defined parametrically as

$$e_{\gamma_n}(t) = K_{\gamma_n} \cos[\omega_n(t-\tau_n) + \varphi_n] \mathrm{e}^{-\alpha_n(t-\tau_n)} \mathrm{U}(t-\tau_n) \qquad (1)$$

where U(t) is the Heaviside step function (U(t)=0 for t<0 and U(t)=1 for t>0), and K_{γ_n} is a normalization constant. Note that this dictionary is capable of modeling both wavefronts (small temporal support, characterized by large damping α_n) and resonances (large temporal support, characterized by small damping α_n). The timing τ_n between consecutive extracted dictionary elements yields the aforementioned time delays.

Traditional matching pursuits (MP) is a greedy algorithm [5], selecting the "best" dictionary elements iteratively. The Relax algorithm has been widely applied to extracting wavefronts from high-resolution radar (HRR) data [6], and here we adapt it to matching-pursuits (yielding Relax-MP). As an example, on the first iteration of MP, assume dictionary element e_{γ_1} is selected, from which we have

$$R_0^{(1)} = < R_0^{(1)} \big| e_{\gamma_1} > e_{\gamma_1}(t) + R_1^{(1)} \qquad (2)$$

where $R_0^{(1)} \equiv f(t)$ and $< \cdot | \cdot >$ denotes inner product. On the second MP iteration we select e_{γ_2} as best matched to $R_1^{(1)}$ from which we have

$$f(t) = \sum_{n=1}^{2} < R_{n-1}^{(1)} \big| e_{\gamma_n} > e_{\gamma_n}(t) + R_2^{(1)} \qquad (3)$$

Note that despite $< e_{\gamma_1} \big| R_1^{(1)} >= 0$, in general $< e_{\gamma_1} \big| e_{\gamma_2} > \neq 0$. To ameliorate this, analogous to the Relax algorithm [6], we define $R_0^{(2)} \equiv f - < f \big| e_{\gamma_2} > e_{\gamma_2}$ and re-estimate e_{γ_1} as in (2). We then define $R_1^{(2)} \equiv f - < f \big| e_{\gamma_1} > e_{\gamma_1}$ from which we re-estimate e_{γ_2}. We iterate this process until convergence is achieved in $R_1^{(N)}$ and $R_2^{(N)}$, after which e_{γ_3} is computed, and we similarly iterate between e_{γ_1}, e_{γ_2}, and e_{γ_3}. After K such Relax-MP iterations, we determine the set of K elements of D_γ that "best" match the data. As we show below, Relax-MP often yields better performance on measured transient scattering data than does traditional MP.

3. HMM CLASSIFIER

Assume that M scattered waveforms are observed from a given target, with the target identity and orientation unknown. After performing Relax-MP feature parsing on each of the scattered waveforms, the sequence of M measurements is represented by the sequence of Relax-MP features $\mathbf{O}=\{\mathbf{o}_1, \mathbf{o}_2, \ldots, \mathbf{o}_M\}$, and we seek to associate **O** with the target with which they have the highest likelihood (maximum-likelihood classification). We therefore require the likelihood $p(\mathbf{O}|T_i)$ for target T_i. The complication of modeling $p(\mathbf{O}|T_i)$ is manifested by the fact that the scattered fields, and therefore $\mathbf{o}_m$, are typically a strong function of the (hidden) target-sensor orientation. Consequently, we define a set of states for target T_i, where a state is a generally contiguous set of target-sensor orientations for which the associated $\mathbf{o}_m$ are approximately stationary [1,2]. The sequence of observed **O** therefore correspond to sampling features from different target states, with the sequence of states well modeled via a Markov model (i.e., the probability of transitioning from one state to any other is dictated entirely by the current state occupied). While the Markovian model is clearly a simplification, it has been found appropriate for many practical scattering scenarios [1,2]. Since both the target identity and orientation are unknown, the particular states being sampled are unknown, or "hidden", necessitating the *hidden* Markov model (HMM).

Characterization of the HMM target model requires an initial-state probability vector and a state-transition matrix, with these closely linked and easily defined from the target geometry and state decomposition [1,2]. Techniques also exist for estimating the appropriate initial state decomposition, with all such parameters optimized during training [1,2]. What remains is characterization of the likelihoods $p(\mathbf{o}_m|q,T_i)$ where q represents a particular state of target T_i. In the past such likelihoods have been quantified through the use of vector quantization, yielding a discrete HMM. Alternatively, one can employ a state-dependent Gaussian-mixture model, from which we obtain a continuous HMM [1,2]. The principal contribution of this paper is development of a new state-dependent model for $p(\mathbf{o}_m|q,T_i)$, motivated by the models used to characterize another widely studied class of transient waveforms: speech. In particular, Shannon [3] introduced the concept of applying a Markov model to characterize the statistics of an observed sequence of alphabet elements, from which fundamental information-theoretic constructs can be developed. In the work reported here we extend the language-based Markov model to characterize the statistics of transient scattering, with the time-dependent scattered waveform model via a statistical sequence of wavefronts, resonances and time delays. As we demonstrate below, this model has been employed to successfully model the state-dependent likelihoods $p(\mathbf{o}_m|q,T_i)$ in the HMM, and in the future it can be employed in the context of fundamental information-theoretic metrics, such as mutual information and the Kullback-Leibler distance [4], from which fundamental performance bounds can be established. Such bounds will be the subject of future presentations, and here the we focus on applying the Markov model as an explicit component of the multi-aspect HMM classifier.

4. MARKOV MODEL

The state-dependent likelihood $p(\mathbf{o}_m|q,T_i)$ characterizes the statistics of an ensemble of transient scattered waveforms, with the underlying assumption that the waveforms within a given state are stationary (constituting a state of the HMM). The model employed is motivated by the Markov model of Shannon [3], but here our alphabet is composed of wavefronts, resonances and time delays. These constituents are defined as follows. As discussed above, wavefronts are characterized by narrow temporal support and resonances by more extended support. Consequently, considering the Relax-MP dictionary elements in (1), extracted dictionary elements with decay constant α larger than a prescribed value are termed wavefronts, and those with smaller α are termed resonances. Again in reference to (1), a time delay is defined by $\tau_i - \tau_{i-1}$, where τ_i and τ_{i-1} correspond to the ith and i-1 extracted elements, as ordered from left to right in increasing time (not necessarily in the order extracted via Relax-MP). There are therefore three Markov states, S_w, S_r and S_t, corresponding to wavefronts, resonances and time delays, respectively. In practice some time delays $\tau_i - \tau_{i-1}$ are small compared to the support of a wavefront; such delays are treated as a direct transition involving the S_w and S_r states (without an intervening transition to state S_t). The permitted state transitions are indicated in Fig. 1.

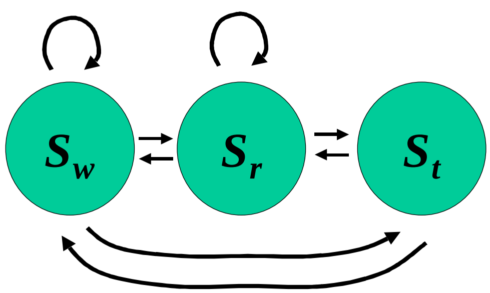

Fig. 1. State transitions for transient-signal Markov model.

The statistics of the associated parametric features for the Markov states are characterized by $p(\alpha,\omega|S_w)$, $p(\alpha,\omega|S_r)$ and $p(\tau_d|S_t)$, where τ_d represents the time delay. These Markov-state-dependent likelihoods characterize the wavefronts, resonances and time delays, as dictated by training data. To simplify the analysis, the likelihoods are characterized by vector quantization (VQ). In particular, a set of codes is used to characterize the variation of (α,ω) in states S_w and S_r (distinct codes for each), with separate codes used to characterize τ_d in S_t. For a given target, the same S_w, S_r and S_t codes are used for the Markov models used for all HMM states, with the HMM states distinguished by different probabilities of observing a given code. For example, let $\boldsymbol{C}=\{\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots, \boldsymbol{c}_L\}$ represent the codes for Markov state S_w. The statistics of S_w for the qth HMM state are characterized as $p(\alpha,\omega|q,S_w) = p(\mathbf{c}_l|q,S_w)$ for $l \in [1,L]$.

It is important to emphasize that here VQ is applied to the wavefront, resonance and time-delay states of the Markov model, while in previous work [1,2] VQ was applied directly on the feature vector $\boldsymbol{o}_m$. There are several advantages of the former approach. When VQ is applied on $\boldsymbol{o}_m$ directly, the size of the feature vector (Relax-MP iterations) must be constant for all target-sensor orientations, since the same codes are used for all HMM states. The Markov model is applicable to any sequence of wavefronts, resonances and time delays, and therefore the states of the HMM need not utilize the same number of Relax-MP iterations. This is important because the different HMM states represent different classes of wave scattering, and therefore in general the states may have different numbers of fundamental scattering features. Moreover, by tying the transient scattering to the fundamental physics (wavefronts, resonances and time delays) one can employ the statistical models to quantify fundamental information-theoretic bounds on classification performance, with such tied to the underlying sensor parameters. In the present study we focus on the Markov model as applied in the classifier, with future studies connected to performance bounds.

5. CLASSIFICATION RESULTS

Results are presented for acoustic scattering from five submerged (underwater) elastic targets. The shapes of the five targets are indicated in Fig. 2, where it is seen that the external shapes of the targets are almost identical, with variation only seen in the internal structure. Details on the targets and on the acoustic measurements can be found in [1].

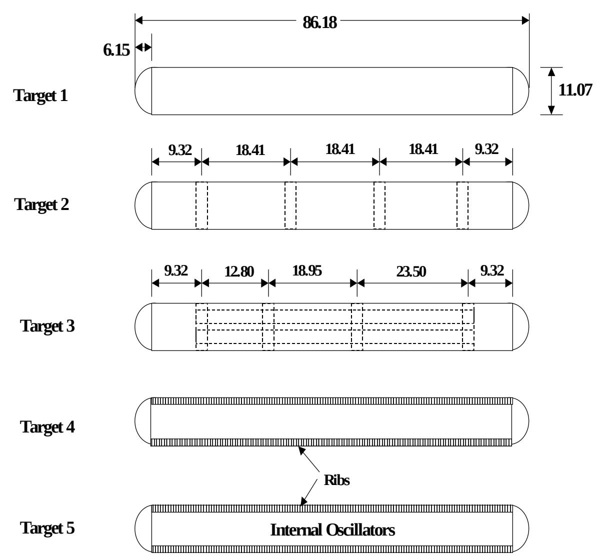

Fig. 2. Shapes of five targets, with all units in centimeters. The elastic targets only differ in their internal structure.

These five targets were selected as constituting a challenging classification problem. An HMM is designed independently for each of the five targets, with each HMM state characterized via a Markov model, in terms of a wavefront, resonance, time-delay alphabet, as discussed in Sec. 4. The bandwidth of the measurements was 7-50 KHz, and the scattering data was sampled in 1^{o} increments, over 360^{o}, in a plane bisecting the axis of the rotationally symmetric targets.

The first phase of the processing involves performing Relax-MP on the transient scattered waveform, as viewed from a particular target-sensor orientation. As an example of a typical transient scattered waveform, from one of the shell targets, and performance of MP and Relax-MP, in Fig. 3 we show the original scattered waveforms and its reconstruction via MP and Relax-MP, where in both cases K=5 iterations were used. The results in Fig. 3 are typical, and underscore the utility of Relax-MP in extracting the principal scattering mechanisms, in a small number of iterations.

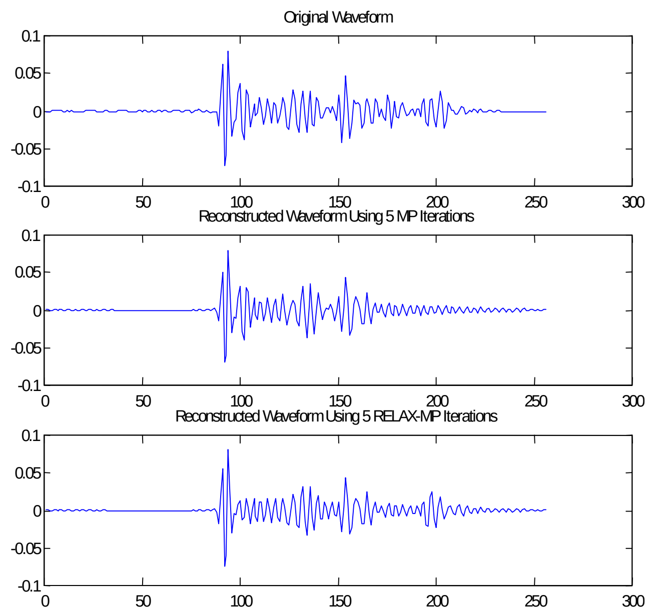

Fig. 3. Original scattered signal (top), MP (middle) and Relax-MP (bottom) reconstruction, using K=5 iterations.

For target classification a HMM is designed for each of the five targets in Fig. 2. A sequence of scattered waveforms is viewed from a set of target-sensor orientations, with the target identity and orientation unknown. Each scattered waveform is parsed with K=5 Relax-MP iterations, yielding the observation vectors $\mathbf{O}=\{\mathbf{o}_1, \mathbf{o}_2, \ldots, \mathbf{o}_M\}$ for M observation angles. The target is declared T_i if $p(\mathbf{O}|T_i) \geq p(\mathbf{O}|T_j)\ \forall\ T_j$. Each of the five targets typically had five states over a non-redundent 90^o set of scattering angles, and codebooks of size L=15 were used for Markov states S_w, S_r, and S_t. For the results presented here the angular sampling is 5^o and we consider M=10 observation angles (45^o aperture). For noise-free studies, the HMM training (with the Viterbi algorithm [1,2]) is performed using all sequences of length M=10 starting with odd initial angles, and testing is performed using all M=10 sequences with even angles. Note that the length of the training sequence need not be the same as the testing sequence [1,2]. For the noise-free case, the classification performance of the HMM was perfect. We therefore present results for noisy data.

The noise level is defined in terms of the peak signal energy over the noise variance, where here we employ additive colored noise, the latter computed by convolving white Gaussian noise with the incident-pulse shape (this noise therefore representative of clutter). Note that the target signature is a strong function of aspect (motivating the HMM), and therefore the peak signal strength is a function of orientation. The SNR is defined with respect to the peak signal strength, over all aspects and targets. Although space limitations our discussion here, at the conference we will detail how the SNR varies as a function of aspect and target type.

For the noisy results, training was performed using noise-free and 20 dB SNR data, while testing was performed using 15dB data (performance with noise-free and 20 dB data was essentially perfect). A confusion matrix for 15 dB SNR testing data, entirely independent from the training data. We see in Fig. 4 that the confusion-matrix results are very encouraging, despite the noise level and the similarity of the five targets.

	Target 1	Target 2	Target 3	Target 4	Target 5
Target 1	0.99	0.01	0.0	0.0	00.0
Target 2	0.04	0.92	0.03	0.0	0.0
Target 3	0.0	0.04	0.96	0.0	0.0
Target 4	0.0	0.0	0.0	1.0	0.0
Target 5	0.01	0.01	0.0	0.0	0.98

Fig. 4. Confusion matrix for 15 dB average SNR (colored noise)

6. CONCLUSIONS

A Markov model, employing a wavefront, resonance, time-delay alphabet, has been applied to the problem of target classification. The features for the Markov model are computed via Relax-MP feature parsing, and multi-aspect scattering is accounted for via an HMM. The classification performance, on measured data, is encouraging. Moreover, the fact that the hybrid HMM-Markov model is based on fundamental scattering physics, which will allow us to now study fundamental performance bounds, connected to the sensor parameters and target geometry.

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