FURTHER RESULTS ON BLIND ASYNCHRONOUS CDMA RECEIVERS USING CODE-CONSTRAINED INVERSE FILTER CRITERION

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ABSTRACT

A code-constrained inverse filter criterion (CC-IFC) based approach was presented recently in Tugnait and Li [7, 2000 ICASSP] for blind detection of asynchronous short-code DS-CDMA (direct sequence code division multiple access) signals in multipath channels. Only the spreading code of the desired user is assumed to be known; its transmission delay may be unknown. The equalizer was determined by maximizing the magnitude of the normalized fourth cumulant of inverse filtered (equalized) data with respect to the equalizer coefficients subject to the fact that the equalizer lies in a subspace associated with the desired user's code sequence. In this paper we analyze the identifiability properties of the approach of [7]. Global maxima and some of the local maxima of the cost function are investigated. These aspects were not discussed in [7]. More extensive simulation comparisons with existing approaches are also provided.

1. INTRODUCTION

Direct sequence code division multiple access (DS-CDMA) systems have been a subject of intense research interest in recent years. In CDMA systems multiple users transmit signals simultaneously leading to multiuser interference (MUI). In addition to MUI, presence of multipath propagation introduces intersymbol interference (ISI) causing distortion of the spreading code sequences. Moreover, in reverse links, unknown transmission delays (user asynchronism) also contribute to performance degradation.

In this paper we consider blind detection (i.e. no training sequence) of the desired user signal, given knowledge of its spreading code, in the presence of MUI, ISI and user asynchronism (lack of knowledge of user transmission delays, including that of the desired user). Past work on blind detection of DS-CDMA signals include [1]-[3], [5]-[7] and references therein. In this paper our focus is on extraction of a desired user's signal. Unlike [2],[3], [5] and [4], we do not assume synchronization with the desired user's signal. In [7] we investigated maximization of the normalized fourth cumulant magnitude of inverse filtered (equalized) data w.r.t. the equalizer coefficients subject to the equalizer lying in a subspace associated with the desired user's code sequence. Constrained maximization leads to extraction of the desired user's signal whereas unconstrained maximization leads to the extraction of any one of the existing users. In this paper we analyze the identifiability properties of the approach of [7]. Global maxima and some of the local maxima of the cost function are investigated. These aspects were not discussed in [7]. More extensive simulation comparisons with existing approaches are also provided.

2. SYSTEM MODEL

Consider an asynchronous short-code DS-CDMA system with M users and N chips per symbol with the j-th user's spreading code denoted by $\mathbf{c}_j = [c_j(0), \cdots, c_j(N-1)]^T$. Consider a baseband discrete-time model representation. Let $s_j(k)$ denote the j-th user's k-th symbol. The sequence $\{s_j(k)\}$ is zero-mean, independently and identically distributed (i.i.d.) either QAM $\forall j$ or binary $\forall j$. For different j's, $\{s_j(k)\}$'s are mutually independent. In the presence of a linear dispersive channe, let $g_j(n)$ denote the j-th user's effective channel impulse response (IR) assuming zero transmission delay, sampled at the chip interval T_c. Let

$$h_j(n) = \sum_{m=0}^{N-1} c_j(m) g_j(n-m), \tag{1}$$

where $h_j(n)$ represents the effective signature sequence of user j (i.e. code $c_j(n)$ "distorted" due to multipath etc.). Define a $[(d+1)N] \times [2N]$ code matrix

$$\mathbf{C}_j^{(d)} := \begin{bmatrix} c_j(0) & 0 & \cdots & 0 \\ c_j(1) & c_j(0) & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ c_j(N-1) & \ddots & \ddots & c_j(0) \\ 0 & c_j(N-1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & c_j(N-1) \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{2}$$

If we collect N chip-rate measurements of received signal (from all users) into N-vector $\mathbf{y}(k)$, then we obtain, at the symbol rate, the MIMO model (additive white Gaussian noise $\mathbf{w}(k)$ is defined in a manner similar to $\mathbf{y}(k)$):

$$\mathbf{y}(k) = \sum_{j=1}^{M} \sum_{l=0}^{L_j} \mathbf{h}_j(l) s_j(k-l) + \mathbf{w}(k) \tag{3}$$

where

$$\mathbf{h}_j(l) = [h_j(lN-d_j), \cdots, h_j(lN-d_j+N-1)]^T, \tag{4}$$

d_j $(0 \le d_j < N)$ is the (effective) transmission delay (mod N) of user j in chip intervals and L_j+1 is the length of the j-th user's vector IR. It follows that for any $d \ge 0$,

$$\mathbf{h}_j^{(d)} := \begin{bmatrix} \mathbf{h}_j^H(0) & \mathbf{h}_j^H(1) & \cdots & \mathbf{h}_j^H(d) \end{bmatrix}^H = \mathbf{C}_j^{(d)} \mathbf{g}_j \tag{5}$$

where the superscript H denotes the complex conjugate transpose (Hermitian) operation,

$$\mathbf{g}_j := \begin{bmatrix} g_j(-d_j) & g_j(-d_j+1) & \cdots & g_j(2N-d_j-1) \end{bmatrix}^T, \tag{6}$$

$\mathbf{h}_j^{(d)}$ is $(d+1)N$-vector, $\mathbf{g}_j$ is $2N$-vector and we assume that $g_j(l) = 0$ for $l > N$ (in addition to $g_j(l) = 0$ for $l < 0$), i.e. the multipath delays can be of at most one symbol duration (N chips). Not all elements in $\mathbf{g}_j$ are nonzero. It follows that $\mathbf{h}_j(l) = 0$ for $l \ge 3$.

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3. CODE-CONSTRAINED INVERSE FILTER CRITERION (CC-IFC)

3.1. Projection Approach to CC-IFC [7]

Consider an $N \times 1$ vector equalizer $\{\mathbf{f}(i)\}_{i=0}^{L_e-1}$ of length L_e symbols (NL_e chips) operating on the data $\mathbf{y}(n)$ (see (3)) to yield

$$e(n) = \sum_{i=0}^{L_e-1} \mathbf{f}^H(i)\mathbf{y}(n-i) \tag{7}$$

where $\mathbf{f}(i)$ is $N \times 1$. Define

$$\tilde{\mathbf{f}}^H := \begin{bmatrix} \mathbf{f}^H(0) & \mathbf{f}^H(1) & \cdots & \mathbf{f}^H(L_e-1) \end{bmatrix}. \tag{8}$$

Let $\text{cum}_4(e)$ denote the fourth-order cumulant of a complex-valued scalar zero-mean random variable e, defined as

$$\text{cum}_4(e) := E\{|e|^4\} - 2[E\{|e|^2\}]^2 - |E\{e^2\}|^2. \tag{9}$$

Following [7] consider maximization of the inverse filter cost

$$J_{42}(\tilde{\mathbf{f}}) := \frac{|\text{cum}_4(e(n))|}{[E\{|e(n)|^2\}]^2} \tag{10}$$

for designing the linear equalizer. It is shown in [4] that under certain mild sufficient conditions, when (10) is maximized w.r.t. $\{\mathbf{f}(i)\}_{i=0}^{L_e-1}$ using a stochastic gradient algorithm, then (12) reduces to

$$e(n) = \alpha s_{j_0}(n-n_0), \tag{11}$$

where complex $\alpha \neq 0$, $0 \leq n_0 \leq L_e - 1 + L_j$ is some integer, j_0 indexes some user out of the given M users, i.e., the equalizer output is a possibly scaled and shifted version of one of the users. The problem is that there is no control over which user is extracted.

It has been shown in [7] that in order to extract the desired user ($j_0 = 1$) with desired delay ($n_0 = d$), the linear equalizer should belong to the null space of a matrix $\mathcal{A}$ which is a function of the desired user's code matrix $\mathbf{C}_1^{(d)}$ and the data correlation matrix. It is a $[N(L_e-2)] \times [NL_e]$ matrix given by

$$\mathcal{A} = \mathcal{U}^{(1)H}\mathcal{T}\mathcal{R}_{yy} \tag{12}$$

where $\mathcal{R}_{yy}$ is the $[NL_e]\times[NL_e]$ data correlation matrix with ij-th block element $\mathbf{R}_{yy}(j-i) = E\{\mathbf{y}(k+j-i)\mathbf{y}^H(k)\}$,

$$\mathcal{T} := \begin{bmatrix} \mathcal{T}_d & 0 \\ 0 & I_{N(L_e-1-d)} \end{bmatrix} = [NL_e] \times [NL_e] \;\text{ matrix}, \tag{13}$$

I_K denotes a $K \times K$ identity matrix,

$$\mathcal{T}_d := \begin{bmatrix} 0 & \cdots & 0 & I_N \\ 0 & \cdots & I_N & 0 \\ \vdots & \ddots & \vdots & \vdots \\ I_N & \cdots & 0 & 0 \end{bmatrix} = [N(d+1)] \times [N(d+1)], \tag{14}$$

$$\mathcal{C}_1^{(d)} := \begin{bmatrix} \mathbf{C}_1^{(d)} \\ 0 \end{bmatrix} = [NL_e] \times [2N] \;\text{ matrix} \tag{15}$$

and columns of $\mathcal{U}^{(1)}$ denote an orthonormal basis for the orthogonal complement of $\mathcal{C}_1^{(d)}$. Since $\mathcal{C}_1^{(d)}$ is of full column rank, $\mathcal{U}^{(1)}$ is an $[NL_e] \times [NL_e - 2N]$ matrix (it can be obtained via an SVD (singular value decomposition) of $\mathcal{C}_1^{(d)}$). **Thus, the desired solution satisfies (16) in addition to maximizing (10) (in fact, in addition to being a stationary point of (10))** where

$$\mathcal{A}\tilde{\mathbf{f}} = 0. \tag{16}$$

By [4] and [7] there exists an equalizer that minimizes (10) as well satisfies (16).

Let $\Pi_{\mathcal{A}}^{\perp}$ denote the $[NL_e] \times [NL_e]$ projection matrix onto the null space of $\mathcal{A}$. The the following iterative, batch, projection stochastic gradient algorithm was used in [7] to obtain the desired equalizer. Let $\widehat{J}_{42}(\tilde{\mathbf{f}})$ denote the data-based cost (10) and let $\nabla_{\tilde{\mathbf{f}}^*}\widehat{J}_{42}(\tilde{\mathbf{f}}')$ denote its gradient (NL_e-column) w.r.t. $\tilde{\mathbf{f}}^*$ evaluated at $\tilde{\mathbf{f}}'$; (the symbol $*$ denotes the complex conjugation operation). Given the equalizer $\tilde{\mathbf{f}}^{(n)}$ at n-th iteration, the equalizer update at $n+1$st iteration is calculated as $\tilde{\mathbf{f}}^{(n+1)} = \tilde{\mathbf{f}}^{(n)} + \rho\Pi_{\mathcal{A}}^{\perp}\nabla_{\tilde{\mathbf{f}}^*}\widehat{J}_{42}(\tilde{\mathbf{f}}^{(n)})$, where ρ is a suitable step-size (see [7]). It is a projection algorithm since any changes in $\tilde{\mathbf{f}}^{(n)}$ are forced to lie in (projected onto) the null space of $\mathcal{A}$. Of course, we choose the initial guess $\tilde{\mathbf{f}}^{(0)}$ to satisfy (16) [7].

3.2. Constrained Global Maxima

We now consider investigate global maxima of (10) subject to (16). Assume no noise: $\mathbf{w}(k) \equiv 0$. When an equalizer is such that (11) is achieved, $J_{42}(\tilde{\mathbf{f}})$ is maximized [4]. It can be shown that

$$\max_{\tilde{\mathbf{f}}} J_{42}(\tilde{\mathbf{f}}) = \frac{|\text{cum}_4(s_j(n))|}{(E\{|s_j(n)|^2\})^2} =: |\overline{\gamma}_{4s}|. \tag{17}$$

Let $\tilde{\mathbf{f}}_{1o}$ be an equalizer for which $J_{42}(\tilde{\mathbf{f}}_{1o}) = |\overline{\gamma}_{4s}|$ with corresponding $e(n) = \alpha_1 s_1(n-d)$ where $\alpha_1 \neq 0$, i.e. $\tilde{\mathbf{f}}_{1o}$ leads to extraction of user 1 with delay d. Then, by construction, $\mathcal{A}\tilde{\mathbf{f}}_{1o} = 0$ [7]. It follows from the results of [4] that if $J_{42}(\tilde{\mathbf{f}}) \neq |\overline{\gamma}_{4s}|$, then (11) can not hold true (all stable local maxima of $J_{42}(\tilde{\mathbf{f}})$ lead to (11) for some j_0 and n_0 [4]). Therefore, constrained global maxima of $J_{42}(\tilde{\mathbf{f}})$ are given by those $\tilde{\mathbf{f}}$'s for which $J_{42}(\tilde{\mathbf{f}}) = |\overline{\gamma}_{4s}|$ and $\mathcal{A}\tilde{\mathbf{f}} = 0$, equivalently, for which (11) and (16) hold true. The equalizer that yields (11) satisfies [4]

$$\sum_{i=0}^{L_e-1} \mathbf{f}^H(i)\mathbf{h}_j(n-i) = \alpha\delta_{j,j_0}\delta_{n,n_0}, \;\; 1 \leq j \leq M, \;\; n \geq 0, \tag{18}$$

where $\delta_{j,i} = 1$ for $j = i$, 0 otherwise.

We now characterize the equalizer solutions that satisfy both (16) and (18). Define the $[NL_e]$-column vector, for $m = 0, \cdots, L_e - 1 + L_j$,

$$\tilde{\mathbf{h}}_j^{(m)} := \begin{bmatrix} \mathbf{h}_j^H(m) & \cdots & \mathbf{h}_j^H(0) & 0 & \cdots & 0 \end{bmatrix}^H. \tag{19}$$

Using (19) and results from [7] and Sec. 3.1, (18) can be rewritten as ($\mathcal{R}_{ss} = \mathcal{R}_{yy}$ under the no noise assumption)

$$\mathcal{T}\mathcal{R}_{yy}\tilde{\mathbf{f}} = \alpha\mathcal{T}\tilde{\mathbf{h}}_{j_0}^{(n_0)}, \; j_0 \in \{1, \cdots, M\}, \; n_0 \in \{0, \cdots, L_e-1+L_{j_0}\}. \tag{20}$$

Therefore, solutions satisfying (16) and (18) are equivalent to solutions satisfying (16) and (20). Consider

(C1) The $[NL_e] \times [2N+1]$ matrix $[\mathcal{C}_1^{(d)} \vdots \mathcal{T}\tilde{\mathbf{h}}_j^{(m)}]$ has full column rank for every $j \in \{2, 3, \cdots, M\}$ and every $m \in \{d-1, d\}$ where $L_e \geq d+1$ and $d \geq 2$.

Claim 1: Under (C1), any solution that satisfies (16) and (20) (i.e. any constrained global maximum of (10)), corresponds to $j_0 = 1$ and $n_0 \in \{d-1, d\}$ in (11), (18) and (20).

Proof: Suppose that for fixed j and m, $\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} \notin \mathrm{sp}\{\mathcal{C}_1^{(d)}\}$, where $\mathrm{sp}\{B\}$ denotes the linear subspace spanned by the columns of B. Then $\mathcal{U}^{(1)H}\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} \neq 0$ (i.e. if $\tilde{\mathbf{f}}$ satisfies (20) with $j_0 = j$ and $n_0 = m$, then it does not satisfy (16)), else $\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} \in \mathrm{sp}\{\mathcal{C}_1^{(d)}\}$. We now establish that $\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} \notin \mathrm{sp}\{\mathcal{C}_1^{(d)}\}$ for any $m > d$ irrespective of the nature of spreading codes and of multipaths. Recall that, by assumption, total delay spread is no more that $N+1$ chips for any user. We have $(m > d)$

$$\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} = \big[\mathbf{h}_j^H(m-d),\ \mathbf{h}_j^H(m-d+1),\ \cdots,\ \mathbf{h}_j^H(m)$$
$$\mathbf{h}_j^H(m-d-1),\ \mathbf{h}_j^H(m-d-2),\ \cdots,\ \mathbf{h}_j^H(0),\ 0,\ \cdots,\ 0\big]^H \tag{21}$$

Since $\mathbf{h}_j(0) \neq 0$, it follows that $\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} \notin \mathrm{sp}\{\mathcal{C}_1^{(d)}\}$ as the rows in $\mathcal{C}_1^{(d)}$ corresponding to the position of $\mathbf{h}_j(0)$ in $\mathcal{T}\tilde{\mathbf{h}}_1^{(m)}$ are all zeros. Therefore, $\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} \notin \mathrm{sp}\{\mathcal{C}_1^{(d)}\}$ for any $m > d$ and $\forall j$. We now turn to the case of $m \leq d$. In this case

$$\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} = [\ \mathbf{h}_j^H(m-d) \quad \mathbf{h}_j^H(m-d+1) \quad \cdots \quad \mathbf{h}_j^H(m)$$
$$0 \quad \cdots \quad 0\ ]^H \tag{22}$$

We have

$$\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} \in \mathrm{sp}\{\mathcal{C}_1^{(d)}\} \ \Leftrightarrow\ [\mathbf{h}_j^H(m-d), \cdots, \mathbf{h}_j^H(m)]^H \in \mathrm{sp}\{\mathbf{C}_1^{(d)}\}. \tag{23}$$

For (23) to be true, there must exist a $2N$-vector $\mathbf{g} \neq \mathbf{0}$ such that

$$[\mathbf{h}_j^H(m-d), \cdots, \mathbf{h}_j^H(m)]^H \stackrel{?}{=} \mathbf{C}_1^{(d)}\mathbf{g}. \tag{24}$$

If $d - m \geq 2$ (assuming that $d \geq 2$), then $\mathbf{h}_j(m-d) = \mathbf{h}_j(m-d+1) = \mathbf{0}$ leading to $\mathbf{g} = \mathbf{0}$ in (24): that is, (23) is never satisfied if $d - m \geq 2$; (since $m \geq 0$, this requires that $d \geq 2$.) Thus, $\mathcal{T}\tilde{\mathbf{h}}_j^{(m)} \notin \mathrm{sp}\{\mathcal{C}_1^{(d)}\}\ \forall j$ and any $m \notin \{d-1, d\}$ for any choice of spreading codes and multipaths. Finally, by construction, $\mathcal{U}^{(1)H}\mathcal{T}\tilde{\mathbf{h}}_1^{(d)} = 0$. This proves the desired result. •

If we pick $d \geq 2$, then the only possible convergence points from among (20) are $\mathcal{T}\tilde{\mathbf{h}}_j^{(m)}$ with $m = d$ or $m = d-1$ and $j = 1, 2, \cdots, M$. If $d = 3$, then both $\mathcal{T}\tilde{\mathbf{h}}_j^{(3)}$ and $\mathcal{T}\tilde{\mathbf{h}}_j^{(2)}$ contain the entire IR of the j-th user (recall that the IR is of maximum length $\overline{L} = 3$ symbols). If $d = 2$, then while $\mathcal{T}\tilde{\mathbf{h}}_j^{(2)}$ contains the entire IR, $\mathcal{T}\tilde{\mathbf{h}}_j^{(1)}$ may not since it does not contain $\mathbf{h}_j(2)$, which may (or may not) be nonzero. In order to better distinguish between two distinct users, it is therefore more prudent to use $d \geq 3$.

3.2.1. LOCAL MAXIMA:

Let us allow doubly infinite equalizers $\{\mathbf{f}(i)\}_{i=-\infty}^{\infty}$. Define the scalar composite channel-equalizer impulse response from the j-th user to the equalizer output as

$$r_j(n) := \sum_{l=-\infty}^{\infty} \mathbf{f}^H(l)\mathbf{h}_j(n-l), \tag{25}$$

$$\mathbf{r} := [\cdots, r_1(0), \cdots, r_M(0), r_1(1), \cdots, r_M(1), r_1(2), \cdots]^T. \tag{26}$$

It has been shown in [4, Appendix C] that the only stable local maxima of $\overline{J}_{42}(\mathbf{r}) (= J_{42}(\tilde{\mathbf{f}}))$ w.r.t. $\mathbf{r}$ are given by the solutions (18). In particular, let $\mathbf{r}_r$ and $\mathbf{r}_i$ denote the real and the imaginary parts, respectively, of $\mathbf{r}$. Let $\bar{\mathcal{J}}(\mathbf{r}')$ denote the the Hessian (second-order derivative) of $\overline{J}_{42}(\mathbf{r})$ w.r.t. $[\mathbf{r}_r^T\ \ \mathbf{r}_i^T]^T$ evaluated at $\mathbf{r} = \mathbf{r}'$. Let $\mathbf{r}^{(n_0,j_0)}$ denote the vector $\mathbf{r}$ specified in (26) with all zero entries except for the one corresponding to $r_{j_0}(n_0)$ (see (25)) which equals α (cf. (11) and (18)). Then by [4, Appendix C], $\bar{\mathcal{J}}(\mathbf{r}^{(n_0,j_0)})$ is negative definite on the set $\mathcal{F}_r = \{\mathbf{u} : \mathbf{u} = \mathbf{r} - \mathbf{r}^{(n_0,j_0)},\ \mathbf{u} \neq (\beta - 1)\mathbf{r}^{(n_0,j_0)}\ \forall\beta\}$, i.e. $[\mathbf{u}_r^T\ \ \mathbf{u}_i^T]\bar{\mathcal{J}}(\mathbf{r}^{(n_0,j_0)})[\mathbf{u}_r^T\ \ \mathbf{u}_i^T]^T < 0$ for any $\mathbf{u} \in \mathcal{F}_r$, $\mathbf{u} \neq 0$, and it is negative semidefinite in general. Since any perturbation in α alone in (18) leaves the cost unchanged (i.e. $\overline{J}_{42}(\beta\mathbf{r}^{(n_0,j_0)}) = \overline{J}_{42}(\mathbf{r}^{(n_0,j_0)})$), it follows that $[\mathbf{u}_r^T\ \ \mathbf{u}_i^T]\bar{\mathcal{J}}(\mathbf{r}^{(n_0,j_0)})[\mathbf{u}_r^T\ \ \mathbf{u}_i^T]^T = 0$ for $\mathbf{u} = (\beta-1)\mathbf{r}^{(n_0,j_0)}$. Thus, $\bar{\mathcal{J}}(\mathbf{r})$ has a strict local maximum at $\mathbf{r}^{(n_0,j_0)}$ for $\mathbf{r} \in (\{\mathbf{r}^{(n_0,j_0)}\} \cup \mathcal{F}_r')$ where $\mathcal{F}_r' := \{\mathbf{r} : \mathbf{r} = \mathbf{u} + \mathbf{r}^{(n_0,j_0)},\ \mathbf{u} \in \mathcal{F}_r\}$, and $\bar{\mathcal{J}}(\mathbf{r})$ has a local maximum at $\mathbf{r}^{(n_0,j_0)}$. Let $\mathcal{J}(\tilde{\mathbf{f}}')$ denote the Hessian of $J_{42}(\tilde{\mathbf{f}})$ w.r.t. the real and the imaginary parts $\tilde{\mathbf{f}}_r$ and $\tilde{\mathbf{f}}_i$, respectively, of $\tilde{\mathbf{f}}$, evaluated at $\tilde{\mathbf{f}} = \tilde{\mathbf{f}}'$. By (25), (26) and the definition of $\tilde{\mathbf{f}}$, it follows that there exists a complex-valued matrix $\mathbf{B} = \mathbf{B}_r + j\mathbf{B}_i$, a function only of the MIMO channel IR (3), such that

$$\mathbf{r} = \mathbf{B}\tilde{\mathbf{f}} \ \Rightarrow\ \begin{bmatrix} \mathbf{r}_r \\ \mathbf{r}_i \end{bmatrix} = \begin{bmatrix} \mathbf{B}_r & -\mathbf{B}_i \\ \mathbf{B}_i & \mathbf{B}_r \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{f}}_r \\ \tilde{\mathbf{f}}_i \end{bmatrix} =: \mathcal{B} \begin{bmatrix} \tilde{\mathbf{f}}_r \\ \tilde{\mathbf{f}}_i \end{bmatrix}. \tag{27}$$

Therefore, we have $\mathcal{J}(\tilde{\mathbf{f}}) = \mathcal{B}^T\bar{\mathcal{J}}(\mathbf{r})\mathcal{B}$ for any $\mathbf{r} = \mathbf{B}\tilde{\mathbf{f}}$. Let $\tilde{\mathbf{f}}^{(n_0,j_0)}$ denote an equalizer (not necessarily unique) corresponding to the composite channel-equalizer response $\mathbf{r}^{(n_0,j_0)}$, i.e. $\mathbf{B}\tilde{\mathbf{f}}^{(n_0,j_0)} = \mathbf{r}^{(n_0,j_0)}$. Then it follows that $\mathcal{J}(\tilde{\mathbf{f}}^{(n_0,j_0)})$ is negative definite on the set $\mathcal{F}_f := \{\mathbf{u} : \tilde{\mathbf{f}} - \tilde{\mathbf{f}}^{(n_0,j_0)},\ \mathbf{B}\mathbf{u} \neq (\beta-1)\mathbf{r}^{(n_0,j_0)}\ \forall\beta\}$, and negative semidefinite in general. Note that any perturbations in $\tilde{\mathbf{f}}^{(n_0,j_0)}$ that leave $\mathbf{r}^{(n_0,j_0)}$ unperturbed, do not change the IFC cost.

The above discussion pertains to the unconstrained cost. We now turn to the constrained case where we seek to maximize (10) subject to (16). The possible solutions to this problem are the stationary points of the Lagrangian (28) w.r.t. $\tilde{\mathbf{f}}$ and λ [9]

$$J_{42}(\tilde{\mathbf{f}}) + \Re\{\lambda^T V_1^H \tilde{\mathbf{f}}\} \tag{28}$$

where $\lambda = [\lambda_1\ \lambda_2\ \cdots\ \lambda_r]^T$, $r = \mathrm{rank}\{\mathcal{A}\} \leq N(L_e - 2)$, $r \times [NL_e]$ matrix V_1^H of rank r (see [7]) has the same null space as that of $\mathcal{A}$ (so that $\mathcal{A}\tilde{\mathbf{f}} = 0$ is equivalent to $V_1^H\tilde{\mathbf{f}} = 0$), λ_i's are the (complex) Lagrange multipliers and $\Re\{x\}$ denotes the real part of the complex scalar x. These stationary points satisfy (16) ($\equiv V_1^H\tilde{\mathbf{f}} = 0$) and

$$(I_{NL_e} - V_1(V_1^H V_1)^{-1}V_1^H)\nabla_{\tilde{\mathbf{f}}^*} J_{42}(\tilde{\mathbf{f}}) = 0. \tag{29}$$

Note that $V_1^H\tilde{\mathbf{f}} = 0$ and (29) is true when $\tilde{\mathbf{f}} = \tilde{\mathbf{f}}_{1o}$ where $\tilde{\mathbf{f}}_{1o}$ has been defined after (17). More generally, the constrained global maxima of (10) (see Claim 1) are stationary points of the Lagrangian (28) since $\nabla_{\tilde{\mathbf{f}}^*} J_{42}(\tilde{\mathbf{f}}) = 0$ at these points ($\Rightarrow$ (29)), and they also satisfy (16). The Hessian of the Lagrangian (28) w.r.t. the real and the imaginary parts of $\tilde{\mathbf{f}}$ evaluated at $\tilde{\mathbf{f}}'$, denoted by $\mathcal{L}(\tilde{\mathbf{f}}')$, clearly equals $\mathcal{J}(\tilde{\mathbf{f}}')$, the Hessian of $J_{42}(\tilde{\mathbf{f}})$ evaluated at $\tilde{\mathbf{f}}'$. Therefore, $\mathcal{L}(\tilde{\mathbf{f}}^{(n_0,j_0)})$ is negative definite on the set $\mathcal{F}_f$ defined earlier. Hence, by [8, p. 226], a constrained global maximum $\tilde{\mathbf{f}}^{(n_0,j_0)}$ is also a strict local constrained maximum of $J_{42}(\tilde{\mathbf{f}})$ on the set $(\{\tilde{\mathbf{f}}^{(n_0,j_0)}\} \cup$

$\mathcal{F}'_f$) where $\mathcal{F}'_f := \{\tilde{\mathbf{f}} : \tilde{\mathbf{f}} = \mathbf{u} + \tilde{\mathbf{f}}^{(n_0, j_0)},\ \mathbf{u} \in \mathcal{F}_f\}$, and it is a local constrained maximum of $J_{42}(\tilde{\mathbf{f}})$, in general.

In summary, we have shown that from among the stable stationary points of the unconstrained cost (10), only the solutions that also satisfy (16) are also the stable stationary points of the Lagrangian (28). Existence and characterization of stable stationary points of (28) that are not the stable stationary points of (10), is an open problem.

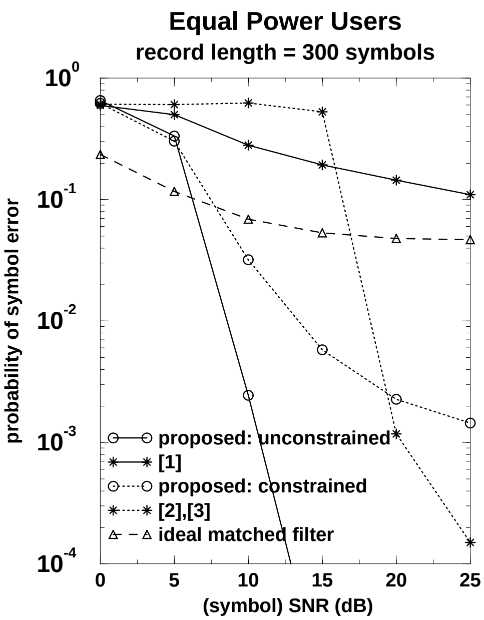

Fig. 1. Probability of symbol error for user 1: 8 chips/symbol, 3 users, equal power users, 4-QAM signals, $L_e = 5$, $d = 3$ for proposed methods, $d = 2$ for others, 100 Monte Carlo runs.

4. SIMULATION EXAMPLE

We consider the case of 3 users, each transmitting 4-QAM signals, and short-codes with 8 chips per symbol. The spreading codes were randomly generated binary (± 1, with equal probability) sequences. The multipath channels for each user have 4 paths with transmission delays uniformly distributed over one symbol interval, and the remaining 3 multipaths having mutually independent delays (w.r.t. the first arrival) uniformly distributed over one symbol interval. All four multipath amplitudes are complex Gaussian with zero-mean and identical variance. The channels for each user were randomly generated in each of the 100 Monte Carlo runs (i.e. they were different in different runs). Complex white zero-mean Gaussian noise was added to the received signal from the 3 users. The SNR refers to the symbol SNR of the desired user, which was user 1, and it equals the energy per symbol divided by N_0 (= one-sided power spectral density of noise $= 2E\{\|\mathbf{w}(k)\|^2\}/N$). In the equal-power case (0dB MUIs), all users have the same power; in the near-far case (10dB MUIs), the desired user power is 10 dB below that of other users.

Equalizer of length (L_e) 5 symbols and desired delay (lag) $d = 3$ was designed using the proposed approach. Two versions were considered: the constrained case refers to the approach outlined in [7]. The unconstrained case refers to further optimization of the cost (10) without any constraints, using the results of constrained optimization as initial guess. Initialization of the constrained version was done as in [7]. The approach of [3] (equivalent to that of [2]) was also simulated with a "smoothing factor" (m in [3]) of 5 ($=L_e$). The approach of [3] was used to estimate the desired user's channel IR which, in turn, was used in a MMSE equalizer with delay $d = 2$. We also applied the approach of [1] using equalizer of length 5 symbols and desired delay $d = 2$. We also simulate an ideal (clairvoyant) matched filter receiver which is matched to the true effective signature sequence $h_1(n - d_j)$ (or $\mathbf{h}_1(l)$) of user 1. After designing the equalizers based on the given data record, the designed equalizer was applied to an independent record of length 3000 symbols in order to calculate the probability of symbol detection error P_e, and the results were further averaged over 100 Monte Carlo runs. Figs. 1 and 2 show the results for various SNR's and approaches.

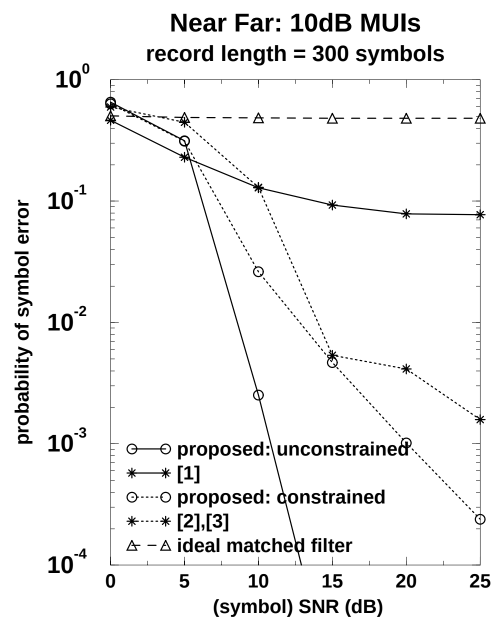

Fig. 2. Probability of symbol error for user 1: near-far case with the MUIs 10 dB stronger than the desired user, rest as for Fig. 1.

5. REFERENCES

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