EQUALIZATION AND BLOCK-SYNCHRONIZATION FOR OFDM SIGNALS

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ABSTRACT

In this paper, blind algorithms based on a deterministic frame-work are proposed for equalization of finite-memory Linear Time-Invariant (LTI) channels carrying OFDM signals. Unlike algorithms proposed to date, the channel memory is not restricted to be smaller than the cyclic prefix. It is shown that equalizers termed "synchronizer-equalizers" can be estimated blindly that always result in block-synchronized output, thus making separate synchronization unnecessary. These algorithms are based on a deterministic frame-work, and require short data records. Performance of the proposed algorithms is analyzed by computer simulations.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) systems have attracted a lot of attention in recent years because of the advantages they offer in dealing with channel impairments commonly found in many practical communication links (mobile communications and Digital Subscriber Loops). In these systems, the transmitter introduces redundancy R (in the form of a *cyclic prefix*). As a consequence, a one-tap equalizer is sufficient at the receiver, as is well known [1]. This is possible because (in the presence of redundancy) the receiver can convert a frequency-selective channel into a flat-fading one. However, this requires the Impulse Response (IR) spread to be no longer than the length of the cyclic prefix R (and the channel to satisfy some conditions [2]). When the IR spread is large, the redundancy required is very large, and considerably decreases transmission efficiency unless the IR spread is decreased by other techniques [3].

Most algorithms proposed for equalization of OFDM signals are restricted to the case when the IR spread is no longer than R [4][1]. In [5], an algorithm is proposed for identification of channels that relaxes this constraint. However, it is a statistical technique, and requires long data records. In this paper, a least-square algorithm is proposed for direct estimation of equalizer coefficients that does not place such restrictions on the impulse response length when oversampling/diversity is used. This allows us to increase transmission efficiency (decrease R) while having several advantages over least-squares algorithms proposed for traditional systems ($R = 0$) [6][7][8].

2. PROBLEM FORMULATION

In an OFDM system, some M transmitted symbols b_n in an M-length block $b_{nM+m}, m = 0, 1, \ldots, M-1$ are encoded using DFT as follows:

$$a_{nP+p} = \sum_{m=0}^{M-1} b_{nM+m} e^{j\frac{2\pi}{M}m(p-R)}, \quad p = 0, 1, \ldots, P-1 \qquad (1)$$

The resulting P *symbols* $a_{nP+p}, p = 0, 1, \ldots, P-1$ consist of a cyclic prefix (redundancy) of R symbols. Clearly, these are simply R repeated symbols. This repetition results in periodicity, and makes the transmitted signal periodic with a period of P symbols [5]. A composite impulse response $h(k)$ is assumed to contain the effects of transmitter and receive filters, besides the channel. The received signal is then given by:

$$r(k) = \sum_{n=-\infty}^{\infty} a_n h(k - nL) + n(k) \qquad (2)$$

where $n(k)$ is the additive noise, and L is the over-sampling rate used by the receiver. It is assumed in what follows that the IR of the channel is finite, and spans L_h symbol durations. The problems addressed in this paper are: 1) Blind estimation of equalizers, and 2) Estimation of blind block-Synchronizing equalizers.

3. EQUALIZATION OF OFDM SIGNALS

The receiver under-samples the received sequence by L to generate a vector sequence $\mathbf{r}(k)$ of size $L \times 1$:

$$\begin{aligned} \mathbf{r}(k) &= [r(kL)\, r(kL+1) \ldots\ldots r(kL+L-1)]^T \\ &= \textstyle\sum_{n=0}^{L_h} a_{k-n}\mathbf{h}(nL) + \mathbf{n}(k) \end{aligned} \qquad (3)$$

where L_h is the symbol spread of the impulse response, $\mathbf{n}(k) = [n(kL), \ldots, n(kL+L-1)]^T$ is the vector of noise samples, $\mathbf{h}(nL) = [h(nL), \ldots, h(nL+L-1)]^T$ are the $L \times 1$ size vector impulse response samples. Note that diversity of any other type can be used in lieu of (or with) over-sampling. If additional diversity of D is used, then the received vector is of size $LD \times 1$, and so is $\mathbf{h}(nL)$. Diversity is often obtained in wireless communications by use of an antenna array [9]. It is assumed in this paper that symbol synchronization information is available. However block synchronization information is not always assumed. Received vectors in N symbols are grouped into a vector $\mathbf{r}_N(k)$ given by:

$$\begin{aligned} \mathbf{r}_N(k) &= [\mathbf{r}^T(k-N+1), \ldots, \mathbf{r}^T(k)]^T \\ &= \mathcal{H}_N \mathbf{a}_{N,k} + \mathbf{n}_N(k) = \mathbf{y}_N(k) + \mathbf{n}_N(k) \end{aligned} \qquad (4)$$

where $\mathbf{n}_N(k)$ is the $NLD \times 1$ size vector of noise samples, $\mathbf{a}_{N,k} = [a_{k-N+1-L_h}, \ldots, a_k]^T$ is an $N + L_h$ sized vector of transmitted symbols, and $\mathcal{H}$ is a $NLD \times (N + L_h)$ size block Hankel matrix of impulse response samples.

The received vector in (4) without the noise is denoted by $\mathbf{y}_N(k)$. In what follows we use vectors separated by P samples:

$$\mathbf{y}_{N,r}(kP) = \mathbf{y}_N(kP + r) = \mathcal{H}_N \mathbf{a}_{N,kP+r} \quad (5)$$

In the above equation, r denotes the block- synchronization point. If $r = 0$, the most recent symbol used by the channel (last row of $\mathbf{a}_{N,kP}$) is a_{kP} which is the first symbol of the k^{th} block. In this section, we assume that block-synchronization information is available, so that r can be fixed as desired. We collect K such vectors (from as many blocks) in a matrix of size $NDL \times K$:

$$\begin{aligned} \mathbf{Y}_{N,r}(kP) &= [\mathbf{y}_{N,r}(kP) \ldots \mathbf{y}_{N,r}((k + K - 1)P)] \\ &= \mathcal{H}_N \mathbf{A}_{N,kP+r} \end{aligned} \quad (6)$$

where $\mathbf{A}_{N,kP+r} = [\mathbf{a}_{N,kP+r}, \ldots, \mathbf{a}_{N,(k+K-1)P+r}]$ is the $N + L_h \times K$ size matrix of transmitted symbols. We assume **(AS1)** that $\mathcal{H}_N$ is made tall by choosing $N \geq L_h$ with $LD \geq 2$ and that it is of full column rank (a standard assumption). It therefore has a pseudo-inverse denoted by $\mathcal{H}_N^{\dagger}$ whose rows can be used as equalizers of various delays. Our problem here is to estimate these equalizers.

Several algorithms have been proposed for equalization of single-tone systems based on a similar frame-work. However, the matrix $\mathbf{A}_{N,kP+r}$ is not of full row-rank in multi-tone systems because it will have repeated rows (when cyclic prefix is used).

We first assume that a zero-prefix of R symbols is used. This amounts to using a guard-band between blocks of data. This implies that $a_{kP+j} = 0$ for $j = 0 \ldots R - 1$. From the definition of $\mathbf{A}_{N,kP+r}$, it follows that every $r+1+s+M+lP^{th}$ row of the matrix is a zero row where s is an integer between 0 and $R-1$, and l is an integer such that $0 \leq r + 1 + s + M + lP \leq N + L_h - 1$. Depending on the block synchronization point r, the number of consecutive zero rows in $\mathbf{A}_{N,kP+r}$ can vary from 1 to R. We make the following observation:

Observation: The number $N_{l,r}$ of R consecutive zero rows in $\mathbf{A}_{N,kP+r}$ is given by the number of l that simultaneously satisfy the following equations:

$$\begin{aligned} r + 1 + M + lP &\geq 0 \\ r + 1 + M + R - 1 + lP &\leq N + L_h - 1 \end{aligned} \quad (7)$$

Arguments: As discussed already, every $r + 1 + s + M + lp^{th}$ row is a zero row, for $s = 0, 1, \ldots, R - 1$, and $0 \leq r + 1 + M + s + lp \leq N + L_h - 1$. For R consecutive zero rows to occur, the first inequality has to be satisfied for $s = 0$ and the second for $s = R - 1$, which results in (7).

We now state the following theorem:

Theorem 1 *The equalizer $\mathbf{g}_m^T$ for $m = r + 1 + M + R - 1 + lP$ for l satisfying (7) lies in the (left) null-space of $[\mathbf{Y}_{N,r}(kP), \ldots, \mathbf{Y}_{N,r+R-1}(kP)]$ that is,*

$$\mathbf{g}_m^T [\mathbf{Y}_{N,r}(kP), \ldots, \mathbf{Y}_{N,r+R-1}(kP)] = \mathbf{0} \quad (8)$$

Further when $N = P$, and $L_h \leq r \leq P - 1$, the only equalizer satisfying (8) that is not *in the left null-space of $\mathcal{H}_N$ is $\mathbf{g}_r^T$ (implying that equalizers $\mathbf{g}_{L_h}^T$ to $\mathbf{g}_{P-1}^T$ can be estimated). Clearly, channels with memory $L_h \leq P - 1$ can be perfectly equalized (using $\mathbf{g}_{P-1}^T$) with $N = P$ when block synchronization information is available and r is set to $P - 1$.*

Arguments: Since $\mathbf{Y}_{N,r}(kP) = \mathcal{H}_N \mathbf{A}_{N,kP+r}$, the equalizers $\mathbf{g}_m^T$ that point to the zero rows of $\mathbf{A}_{N,kP+r}$ lie in the left null-space of $\mathbf{Y}_{N,r}(kP)$. It follows from its Toeplitz nature that if the $r + 1 + M + lP^{th}$ row to the $r + 1 + M + lP + R - 1^{th}$ row of $\mathbf{A}_{N,kP+r}$ are zero rows, then the $r + 1 + M + lP + R - 1^{th}$ to the $r + 1 + M + lP + 2R - 2^{th}$ rows of $\mathbf{A}_{N,kP+R-1}$ are all zero. This implies that the zero-row(s) common to both these matrices is (are) the $r + 1 + M + R - 1 + lP^{th}$ row(s). It follows that $\mathbf{g}_m^T$ with m as defined above will be in the null-space of $[\mathbf{Y}_{N,r}(kP) \ldots \mathbf{Y}_{N,r+R-1}(kP)]$. Further, $L_h < P - 1$, $N_{l,r} = 1$ for $L_h \leq r \leq P - 1$. This implies that (in these cases) there is only one vector that is in the left null-space of $\mathbf{Y}_{N,r}(kP)$ that is not in the left null-space of $\mathcal{H}_N$. It can be shown that the minimum K required is such that $KR \geq N + L_h$.

We discuss the case of $L_h > P - 1$ later in this section. The above theorem shows that perfect zero-forcing equalization is possible for channels with memory upto $P - 1$ whenever block synchronization information is available. Equalization algorithms proposed to date limit the memory to be less than R. As noted in [5], this implies that transmission efficiency can be improved by using R much smaller than the channel memory. To estimate the equalizer $\mathbf{g}_m^T$ uniquely using the theorem, it will be necessary to ensure that the solution obtained for $\mathbf{g}_m^T$ does not belong to the left null-space of $\mathcal{H}_N$. We may place this additional constraint in many different ways, and some of these are described briefly below.

Semi-blind algorithms: In several practical communication links, some of the bits transmitted may be known at the receiver. This happens for example when user identification bits, training bits, or block/frame synchronization bits are used. Consider the case when a sequence of K_N known bits $\tilde{\mathbf{a}}$ are transmitted. We can then place the following constraint on theorem 1:

$$\mathbf{g}_m^T [\mathbf{y}_{N,r}(kP), \ldots, \mathbf{y}_{N,r+K_N-1}(kP)] = \tilde{\mathbf{a}} \quad (9)$$

Unlike traditional methods that exploit the training sequence to estimate the equalizer coefficients, the above method only uses the known bits to ensure that the solution obtained is non-trivial. It can therefore be expected to work with very short data records. The second advantage is that the constraint placed above is linear, so the linear equations (9) and (8) simply need to be solved together.

Quadratic constraint: We first rewrite (8) in the form:

$$\min_{\mathbf{g}_m^T} \left[\sum_{j=0}^{R-1} \| \mathbf{g}_m^T \mathbf{Y}_{N,r+j}(kP) \|_2^2 \right] \quad (10)$$

and place the following quadratic constraint:

$$\max_{\mathbf{g}_m^T} \left[\sum_{j=1}^{Q} \| \mathbf{g}_m^T \mathbf{Y}_{N,r-j}(kP) \|_2^2 \right] \quad (11)$$

where $Q = 1, \ldots, M$. Clearly, when $Q = M$, all samples within the block are utilized in the estimation procedure. We can alternatively write the above constraint equation as an equality. In either case, this is a standard least-squares problem with quadratic constraint and can be solved with SVD using methods discussed in [10].

Case of cyclic prefix: When a cyclic prefix is used, certain R symbols in a blocks are repeated. It can be easily shown that:

$$\mathbf{g}_m^T \mathbf{Y}_{N,r+s}(kP) \quad = \quad \mathbf{g}_m^T \mathbf{Y}_{N,r+M+s}(kP)$$

where $s = 0, 1, \ldots, R-1$. We call the above the 'redundancy equation'. Given a matrix $\mathbf{Y}_{N,r}(kP)$ of received vectors of an OFDM signal using a cyclic prefix, it can be seen that $\mathbf{Y}_{N,r}(kP) - \mathbf{Y}_{N,r+M}(kP)$ is the model for another OFDM signal transmitting data over the same channel, but with zero-prefix and modified data. It can be shown (proof is not presented here) that under some conditions $[\mathbf{Y}_{N,r+s}(kP) - \mathbf{Y}_{N,r+M+s}(kP)]$ can be used in lieu of $\mathbf{Y}_{N,r+s}(kP)$ in Theorem 1. We do not study the case of non-zero prefix separately in this paper.

Estimation of other Equalizers: When $N = P$, Theorem 1 allows us to estimate the equalizers of delays $\mathbf{g}_r^T$ where $r = L_h, \ldots, P-1$. Equalizers of delays between 0 and $L_h - 1$ and those between P and $P + L_h - 1$ can be estimated indirectly using estimates of other equalizers. It might sometimes be desirable to estimate equalizers of desired delay independent of others directly from the received sequence. Also, equalization of channels with $L_h \geq P$ is of interest. We establish in what follows that by using a set of so called Toeplitz equations along with (8), the remaining equalizers can be estimated (when $L_h < P$), and channels with larger memories can be equalized.

Theorem 2 *When* **AS1** *is satisfied, and* $0 \leq r \leq L_h - 1$, *equalizers* $\mathbf{g}_m^T$ *and* $\mathbf{g}_{m+C}^T$ *that satisfy the following Toeplitz equations:*

$$\mathbf{g}_m^T \mathbf{Y}_{N,r}(kP) \quad = \quad \mathbf{g}_{m+C}^T \mathbf{Y}_{N,r+C}(kP) \qquad (12)$$

along with (8) are $\mathbf{g}_r^T$ *and* $\mathbf{g}_{r+C}^T$ *provided they are not in the left null-space of* $\mathcal{H}_N$ *and* C *is chosen so that* $N + L_h - P - r \leq C \leq N + L_h - r - 1$.

Arguments: Only an outline of the arguments is presented here. If $\mathbf{g}_m^T$ satisfies (8), then it lies in the left null space of $[\mathbf{Y}_{N,r}(kP), \ldots, \mathbf{Y}_{N,r+R-1}(kP)]$. If we ignore vectors that lie in the null space of $\mathcal{H}_N$, the solutions are $\mathbf{g}_m^T$ where $m = r + 1 + M + R - 1 + lP$ where l satisfies (7). When $N = P$ and $L_h < P$, it can be shown that the equalizers $\mathbf{g}_r^T$ and $\mathbf{g}_{r+P}^T$ satisfy (8) for $0 \leq r \leq L_h - 1$. The Toeplitz equation is clearly satisfied by pairs of equalizers $\mathbf{g}_r^T$ and $\mathbf{g}_{r+C}^T$. If C is so chosen that $C \geq L_h - r$, then $\mathbf{g}_{r+P}^T$ cannot clearly be a solution to the Toeplitz equation. These equations therefore ensure that the lower delay equalizer is always chosen. Since the maximum delay is $P + L_h - 1$, C has to satisfy the inequality: $L_h - r \leq C \leq P + L_h - r - 1$. Similar arguments can be presented for the case when $L_h > P$ ($N > P$).

We note the following:
1) Note that while $\mathbf{g}_r^T$ for $L_h \leq r \leq P - 1$ were estimated independent of others, equalizers of small delays ($0 \leq r \leq L_h - 1$) and those of large delays ($P \leq r \leq P + L_h - 1$) are estimated together in pairs when $C = P$ is used.
2) When $N > P$, it is not always possible to directly estimate equalizers of all delays. Clearly, P equalizers of smallest delays and P of largest delays are estimated.
3) The Toeplitz equations by themselves have been used to estimate the equalizers $\mathbf{g}_0^T$ and $\mathbf{g}_{N+L_h-1}^T$ uniquely in the case of single-tone systems [8][7]. However, these methods are known to be sensitive to estimates of the channel memory L_h. In the above theorem, the Toeplitz equations are used merely to eliminate ambiguity in the estimates obtained by (8). It can therefore be expected to result in relatively robust algorithms.
4) Note that the value of C is allowed to vary over a wide range of P values. If the channel memory is underestimated by l, the lower limit for C no longer holds for $r = 0 \ldots l$. Note that at other synchronization points, the above theorem still holds. If the memory L_h is overestimated, then choosing the lower limit for C always ensures L_h has to be overestimated by P (which is extremely unlikely) before the limits on C is violated.
5) It is emphasized that the algorithm resulting from the above theorem (unlike algorithms proposed to date for multi-tone systems) does not place any restrictions on the IR spread relative to the block-size P or the redundancy R. It is only required that N is chosen such that $N \geq L_h$.

4. BLOCK SYNCHRONIZATION OF OFDM SIGNALS

In the previous section, algorithms designed for estimation of equalizers assumed that block-synchronization information was available. Because of the manner in which it is defined, $\mathbf{g}_r^T$ when applied on $\mathbf{Y}_{N,r}(kP)$ always results in the first symbol of the most-recently transmitted block of symbols. For the case when $N = P$, application of theorem 1 results in $\mathbf{g}_r^T$ when $\mathbf{Y}_{N,r}(kP)$ was used in the analysis. Note that explicit knowledge of r (block-synchronization information) was not required, provided r was in the range $L_h \leq r \leq P - 1$. Theorem 2 used the Toeplitz equations to estimate equalizers of delays r and $r + P$ in pairs where $0 \leq r \leq L_h - 1$. Once again, explicit knowledge of r is not necessary provided it is in the range $0 \leq r \leq L_h - 1$ and C is chosen to be P, as noted earlier. It is interesting to investigate therefore whether the ideas behind the two algorithms can be combined so that knowledge of r is avoided. Such an algorithm can therefore automatically estimate equalizers that always point to the first symbol of the most recent block of transmitted data. We establish in the following theorem that such equalizers (termed synchronizer-equalizers) can be designed simply by choosing C in theorem 2 carefully.

Theorem 3 *When* $N = P$, *and* $L_h < P$, *the only vector* $\mathbf{g}_m^T$ *that is in the left null-space of* $[\mathbf{Y}_{N,r}(kP), \ldots, \mathbf{Y}_{N,r+R-1}(kP)]$ *but not in the null-space of* $\mathcal{H}_N$, *and also satisfies the Toeplitz equations (12) for* $C = L_h$ *is the equalizer* $\mathbf{g}_r^T$ *irrespective of the block-synchronization point* r. *This implies that synchronizer- equalizers can be designed that do not require any knowledge of* r.

Arguments: Theorem 2 estimates $\mathbf{g}_r^T$ and $\mathbf{g}_{r+C}^T$. If $0 \leq r \leq P - 1$, then C cannot exceed L_h. But this is the minimum value of C for $r = 0$. This implies that $C = L_h$ is the only choice possible.

Remark: Unlike other equalization algorithms proposed here, the algorithm for estimation of the synchronizer-equalizer is critically dependent on correct estimate of L_h. It is noted that this feature is shared by many subspace algorithms like [8] and [7] that estimate equalizers for channels carrying single-tone signals.

5. PERFORMANCE ANALYSIS

We study performance of the equalization algorithm assuming that space diversity built with a two-sensor array is available. The two impulse responses were assumed to be the same as those used in [4]: $\mathbf{h}_1 = [0.5956, -0.3843 - j0.5020, -0.3145 - j0.1178, 0.1838 + j0.3148]$ and $\mathbf{h}_2 = [-0.7619 - j0.1887, 0.4166 + j0.0618, 0.1797 + j0.3201, -0.1815 - j0.1970]$. The SNR was fixed at $25dB$. We choose P, M, R, r, Q, and K to be 8,6, 2, L_h, 1 and 8 respectively, and use a zero-prefix. Note that the length of the zero-prefix is insufficient for equalization by standard equalization techniques proposed for OFDM. The input was assumed to be of 16-QAM type. The algorithm of this paper was used to estimate the equalizer coefficients. The resulting eye diagram is depicted before and after equalization in figure 1. Note that a larger Q can give better performance under noisy conditions (so can larger R). The MSE of the equalizer output is plotted in the figure versus number of blocks used in the estimation. When $\mathbf{a}_r$ is the actual data, and $\tilde{\mathbf{a}}_r$ is the recovered data, the MSE is defined as:

$$MSE \quad = \frac{1}{R}\sum_{r=0}^{R-1}\frac{||\tilde{\mathbf{a}}_r - \mathbf{a}_r||^2}{||\mathbf{a}_r||^2} \qquad (13)$$

where R is the number of realizations used (100 here). It can be seen that a small number of blocks K suffice for estimation of the equalizer coefficients.

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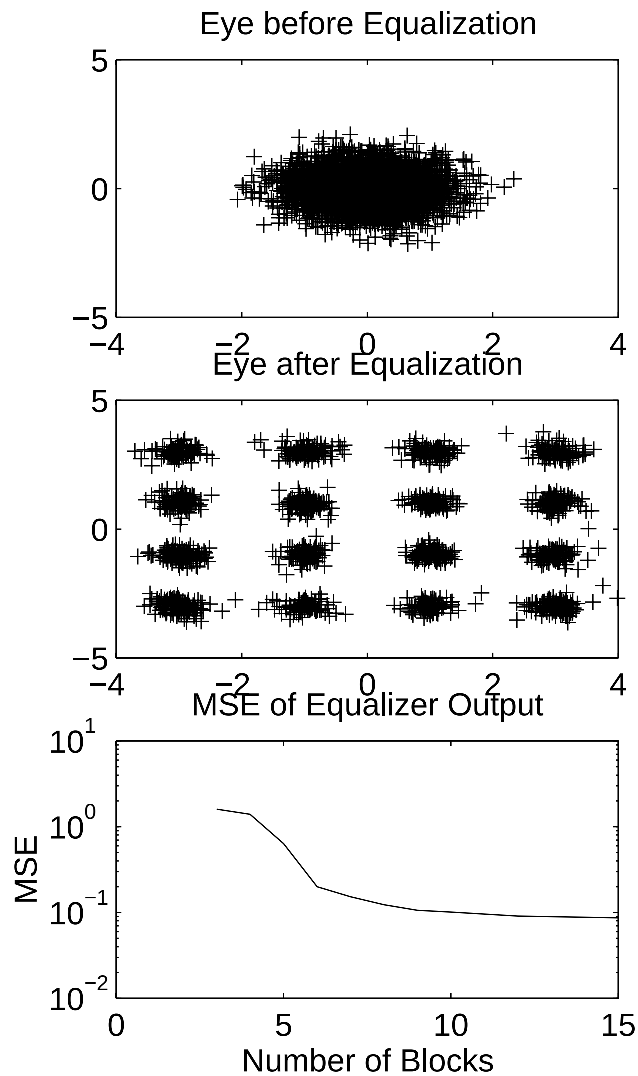

Fig. 1. Eye diagram before (top) and after equalization (middle). MSE of equalizer output versus number of blocks used K (bottom).