

# AN ATTRACTOR SPACE APPROACH TO BLIND IMAGE DECONVOLUTION

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## ABSTRACT

In this paper, we present a new approach to adaptive blind image deconvolution based on computational reinforced learning in attractor-embedded solution space. A new subspace optimization technique is developed to restore the image and identify the blur. Conjugate gradient optimization is employed to provide an adaptive image restoration while a new evolutionary scheme is devised to generate the high-performance blur estimates. The new technique is flexible as it does not suffer from various image or blur constraints imposed by most traditional blind methods. Experimental results show that the new algorithm is effective in blind deconvolution of images degraded under different blur structures and noise levels.

## 1. INTRODUCTION

Blind image restoration is a process of recovering the visual clarity from the degraded image without the prior knowledge of the blur. Its wide applications range from photography deblurring, medical imaging, remote sensing, multimedia processing, among others. Linear image degradation process is commonly modeled by [1]

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \mathbf{n} \quad (1)$$

where  $\mathbf{g}$ ,  $\mathbf{f}$  and  $\mathbf{n}$  are the lexicographically ordered degraded image, original image and additive white Gaussian noise (AWGN) respectively.  $\mathbf{H}$  is the linear distortion operator determined by the point spread function (PSF),  $\mathbf{h}$ . Blind image deconvolution is an inverse problem of inferring the best estimates,  $\hat{\mathbf{f}}$  and  $\hat{\mathbf{h}}$  to the original image and the actual blur based on linear degradation model. It is a difficult, ill-posed problem as the stability and uniqueness of the solution is not guaranteed.

Over the years, various techniques have been proposed to address blind image deconvolution. These include a priori blur identifications, auto regressive moving average (ARMA) methods, iterative support constraint techniques, and symmetrical double regularization approaches. A priori blur identifications are inflexible as they require the parametric structures of the blur to be known exactly, and are tailored specifically for the targeted blur type [2], [3]. In addition, they are ineffectual in identifying blurs that do not exhibit prominent frequency nulls. ARMA methods

require both the AR and MA support dimensions to be small to have a manageable computational load [4], [5]. This inadvertently undermines their modeling effectiveness. The ARMA image stationarity constraint is inconsistent with some real-life images consisting of inhomogeneous smooth, textured, and edge regions.

Iterative support constraint techniques alternate between the spatial and frequency domains, imposing constraints onto the image and blur estimates repeatedly [6], [7]. These approaches require the image object to have a known support dimension lying in a uniform background. This is clearly restrictive as most applications do not satisfy the precondition. The symmetrical double regularization approaches extend the blind problem into two symmetrical processes of image restoration and blur identification [8]. They do not take into account different characteristics of the image and blur, thereby ignoring the priority and knowledge in each domain.

In view of this, we propose a new approach to blind image deconvolution based on computational reinforced learning in attractor-embedded solution space. An extended evolutionary scheme that integrates priority-based subspace deconvolution is developed. The new technique incorporates the blur knowledge by embedding them as dynamic attractors in the solution space. A maximum a posteriori (MAP) estimator is employed to predict these attractors, and their relevance is assessed.

We develop a novel reinforced learning scheme that combines stochastic search and pattern acquisition throughout the blur identification. It enhances the algorithmic convergence and reduces the computational cost significantly. In addition, it alleviates the formulation dilemma encountered by other methods, namely integrating the information of well-known blurs without compromising their flexibility. Unlike most iterative schemes where the restoration results are highly dependent on the previous estimated solutions, the new scheme provides a multithreaded restoration that is robust towards divergence and poor local minima trapping.

## 2. PROBLEM FORMULATION

Blind image deconvolution is commonly formulated as the minimization of a multimodal cost function  $J(\hat{\mathbf{f}}, \hat{\mathbf{h}} | \mathbf{g})$ . Due to the distinctive characteristics of the image and blur domains, we project the cost function into their respective subspaces as:

$$J(\hat{f}/g, \hat{h}) = P_{\hat{f}} \{J(\hat{f}, \hat{h}|g)\} \quad (2)$$

$$J(\hat{h}/g, \hat{f}) = P_{\hat{h}} \{J(\hat{f}, \hat{h}|g)\} \quad (3)$$

where  $J(\hat{f}/g, \hat{h})$  and  $J(\hat{h}/g, \hat{f})$  are the subspace cost functions, and  $P_{\hat{f}}$  and  $P_{\hat{h}}$  are the projection operators with respect to the image and blur domains. A recursive scheme can be developed to restore the image and identify the blur by minimizing the cost functions in (2) and (3) iteratively. However, the alternating minimization procedure experiences intersubspace dependency, and tends to converge poorly. To address this difficulty, we extend the minimization procedure into an evolutionary scheme.

The mathematical formulation of the new algorithm is given as:

- (i) Initialize  $\Phi_0$
- (ii) For  $i$ -th generation, determine the dynamic image and blur solution spaces,  $\Omega_i$  and  $\Phi_i$  :

$$\Omega_i = \left\{ \hat{f}_i \mid \hat{f}_i = \arg \min_{\hat{f}} J(\hat{f}/g, \hat{h}_i \in \Phi_i) \right\} \quad (4)$$

$$\Phi_i = \left\{ \hat{h}_i \mid \hat{h}_i = L \circ R \circ S \circ F(\hat{f}_{i-1}/\hat{f}_{i-1} \in \Omega_{i-1}) \right\} \quad (5)$$

- (iii) Stop when convergence or the maximum number of generation is reached

The blur solution space  $\Phi_i$  is generated based on concatenation of the performance evaluation operator  $F$ , the candidate selection operator  $S$ , the recombination operator  $R$ , and the reinforced learning operator  $L$ . The new technique preserves the algorithmic simplicity of the projection-based deconvolutions by performing image restoration in (4) and blur identification in (5). Moreover, it exploits the virtue of the evolutionary scheme to alleviate interdomain dependency and poor convergence, thereby enhancing the robustness of the deconvolution scheme.

### 3. BLUR IDENTIFICATION IN ATTRACTOR SPACE

#### 3.1 Dynamic Attractor Estimation

It is well documented that most real-life PSF satisfy up to a certain degree of parametric structure [1-3]. Therefore, it is most appropriate that we incorporate blur knowledge as dynamic attractors in our restoration scheme. The MAP estimator for the soft parametric blur can be expressed as:

$$\bar{h} = \arg \max_{\tilde{h} \in \tilde{\mathbf{H}}} \log p(\tilde{h}/\hat{h}) \quad (6)$$

where  $\tilde{\mathbf{H}}$  is the parametric solution space, and  $p(\tilde{h}/\hat{h})$  is the conditional probability density function of  $\tilde{h}$  given the observation  $\hat{h}$ .

Assuming that  $n = \tilde{h} - \hat{h}$  follows an *iid.* multivariate Gaussian distribution, we can rewrite (6) in terms of its covariance matrix,  $\Sigma_{nn} = \sigma_n^2 \mathbf{I}$  and support size  $M \times N$ :

$$\bar{h} = \arg \max_{\tilde{h} \in \tilde{\mathbf{H}}} \left\{ -\frac{1}{2} MN \log(2\pi) - \frac{1}{2} \log(\sigma_n^2 MN) - \frac{1}{2\sigma_n^2} (\tilde{h} - \hat{h})^T (\tilde{h} - \hat{h}) \right\} \quad (7)$$

The estimator attempts to provide the unbiased parametric estimate to the evolved blur from  $\tilde{\mathbf{H}}$ . The solution space can be tailored to encompass various parametric structures such as uniform and Gaussian blurs. The parametric estimates will function as the attractors in  $\Phi_i$ , thereby induce reinforce learning towards them.

#### 3.2 Reinforced Learning

Conventional evolutionary schemes perform mutation to provide a random search through the solution spaces [9]-[11]. The major disadvantage lies in its slow convergence, particularly for high dimensional problem such as image restoration. To alleviate this difficulty and enhance the convergence of the algorithm, we introduce a novel attractor-based reinforced learning to perform the blur estimation.

The reinforced learning operator involves the functional mapping of  $L: \mathcal{R}^{M \times N} \rightarrow \mathcal{R}^{M \times N}$  given by:

$$\hat{h}_k^{i+1} = L(\hat{h}_k^i) \quad (8)$$

$$= \hat{h}_k^i + (1-\alpha) \Delta \hat{h}_k^i - \alpha \frac{\partial L(\hat{h}_k^i, \tilde{h}(\tilde{\theta}_k^i))}{\partial \hat{h}_k^i} \quad (9)$$

where  $\hat{h}_k^i$  and  $\hat{h}_k^{i+1}$  are the  $k$ -th blur estimate for  $i$ -th and  $(i+1)$ -th generations,  $\Delta \hat{h}_k^i$  is the stochastic perturbation,  $\tilde{\theta}_k^i$  is the reinforced parametric vector,  $\tilde{h}(\tilde{\theta}_k^i)$  is the predictor for dynamic attractor,  $\alpha$  is the learning rate, and  $L(\hat{h}_k^i, \tilde{h}(\tilde{\theta}_k^i))$  is the blur-domain cost function.

The reinforced learning paradigm offers a compromise between the stochastic search and pattern acquisition towards the dynamic attractors. It is observed from (9) that the stochastic search and steepest descent learning form a complementary pair. If the evolved blur matches the attractors closely, it will boost the confidence that the actual blur follows a parametric structure. Therefore, learning towards the soft estimate is emphasized, and the stochastic search is reduced. The reverse applies when a poor proximity occurs between the evolved blurs and the soft estimates.

The learning rate,  $\alpha$  functions as a confidence measure between the evolved blur and the attractors. The terms  $\alpha$  and  $L(\hat{h}_k^i, \tilde{h}(\tilde{\theta}_k^i))$  can be further expressed as:

$$\alpha = e^{-\beta \|\hat{h}_k^i - \tilde{h}_k^i\|^2} \quad (10)$$

$$L(\hat{\mathbf{h}}_k^i, \tilde{\mathbf{h}}(\tilde{\theta}_k^i)) = \frac{1}{2} \|\tilde{\mathbf{h}}(\tilde{\theta}_k^i) - \hat{\mathbf{h}}_k^i\|^2 \quad (11)$$

where  $\beta$  is the field strength factor that determines the extent of reinforced learning.

### 3.3 Recombination

The recombination operator  $\mathbf{R}: \mathfrak{R}^{M \times N} \rightarrow \mathfrak{R}^{M \times N}$  involves the selection of random candidates, and the application of global intermediary recombination. If the candidates have different support dimensions, either a random candidate is chosen or a scaling process among the candidates is performed.

## 4. IMAGE RESTORATION

The image-domain solution space  $\Omega_i$  given in (4) is generated based on the evolved blur population  $\Phi_i$ :

$$\begin{aligned} \Omega_i &= \left\{ \hat{\mathbf{f}}_i \mid \hat{\mathbf{f}}_i = \arg \min_{\hat{\mathbf{f}}} J(\hat{\mathbf{f}}/\mathbf{g}, \hat{\mathbf{h}}_i \in \Phi_i) \right\} \\ &= \left\{ \hat{\mathbf{f}}_i \mid \hat{\mathbf{f}}_i = \arg \min_{\hat{\mathbf{f}}} \left[ \frac{1}{2} \|\mathbf{g} - \hat{\mathbf{H}}_k \hat{\mathbf{f}}\|^2 + \frac{1}{2} \hat{\mathbf{f}}^T \mathbf{\Lambda} \mathbf{D}^T \mathbf{D} \hat{\mathbf{f}} + C_f \right] \right\} \quad (12) \end{aligned}$$

where  $\mathbf{\Lambda}$  is the new image-domain regularization matrix,  $\mathbf{D}$  is the Laplacian high-pass operator, and  $C_f$  is the projection constant. The cost function in (12) consists of data fidelity measure, regularization term and the projection constant. The data fidelity criterion functions as a restoration measure. However, it is susceptible to ringing and noise amplification in the smooth image background. The regularization term is introduced to suppress these undesirable effects. The regularization matrix,  $\mathbf{\Lambda}$  provides an adaptive restoration by preserving the fine details at the textured and edge regions while suppressing noise and ringing in the smooth backgrounds. Conjugate gradient optimization is employed due to its computational efficiency and robustness.

## 5. PERFORMANCE EVALUATION

Evolutionary algorithms are often employed to solve difficult optimization problems where most traditional path and volume-oriented methods fail to address adequately. Its main advantages lie in its robustness and flexibility, enabling the most tangible performance measure to be adopted. In view of this, we propose a novel entropy-based objective function  $F: \mathfrak{R}^{P \times Q + M \times N} \rightarrow \mathfrak{R}$  that functions as the restoration performance indicator:

$$F(\hat{\mathbf{h}}, \hat{\mathbf{f}}) = \frac{1}{2} \|\mathbf{g} - \hat{\mathbf{H}} \hat{\mathbf{f}}\|^2 + \frac{1}{2} \mathbf{u}^T \mathbf{W}_u \mathbf{u} + \frac{1}{2} \mathbf{v}^T \mathbf{W}_v \mathbf{v} \quad (13)$$

where  $\mathbf{u}$  and  $\mathbf{v}$  are the entropy and inverse-entropy vectors, and  $\mathbf{W}_u$  and  $\mathbf{W}_v$  are the corresponding weight matrices. The new objective

function utilizes the intuitive local entropy statistics to assess the performance of the restored images. The entropy and inverse-entropy vectors are the lexicographically ordered visual activity measure for the local image neighborhood. The weight matrix  $\mathbf{W}_u$  has large values in the smooth backgrounds and small values at the textured regions. This is combined with entropy vector to ensure ringing and noise amplifications are penalized in the smooth backgrounds. The reverse argument applies for weight matrix  $\mathbf{W}_v$ .

A combination of deterministic and stochastic selection process  $\mathbf{S}: \mathfrak{R} \rightarrow \mathfrak{R}^{P \times Q + M \times N}$  is adopted to choose the offspring for the future generation. Due to the multithreaded nature of evolutionary scheme, the algorithm is more robust towards poor local minima trapping and does not rely heavily on the previous estimate. The scheme continues until either the convergence or the maximum number of iteration is reached.

## 6. EXPERIMENTAL RESULTS

We illustrate the effectiveness of our proposed algorithm by using the well-known ‘‘Lena’’ image in Fig. 1. The original image shown in Fig. 1(a) was degraded by the 5×5 Gaussian blur given in Fig. 1(f), coupled with 40dB additive noise to form the degraded image in Fig. 1(b). We applied the algorithm with  $\mu=10$  parents,  $\nu=10$  offspring,  $\rho=2$  ancestors, and  $\beta=150$  field strength factor. The final restored image after the convergence is reached is given in Fig. 1(c).

It is observed that the restored image achieves very good restoration by recovering the visual clarity and sharpness of the image. The algorithm is effective in preserving the fine details near the feather texture regions of the hat as well as the edges. There is no visible ringing and noise amplification in the smooth backgrounds. The initial and final blur estimates after 6 generations are given in Figs. 1(d) and 1(e) respectively. They are the estimates with the best fitness function from their respective population pools. We notice that the blur estimate evolves from the initial random pattern to the final structure that closely resembles the actual Gaussian blur shown in Fig. 1(f). This illustrates the advantages of employing reinforced learning in the attractor-based solution space to achieve good deconvolution results.

## 7. CONCLUSIONS

We present a new approach to adaptive blind image deconvolution based on computational reinforced learning in attractor space. The new technique formulates the problem into the image subspace optimization and blur solution space construction. Conjugate gradient optimization is adopted to provide an adaptive restoration. An evolutionary scheme is devised to generate the high-performance blur estimates. The information of well-known blurs is incorporated into the scheme by embedding the best-fit parametric estimates as the attractors in the solution space. Experimental results show that the new technique is effective in restoring the degraded image without the prior knowledge of the blur.

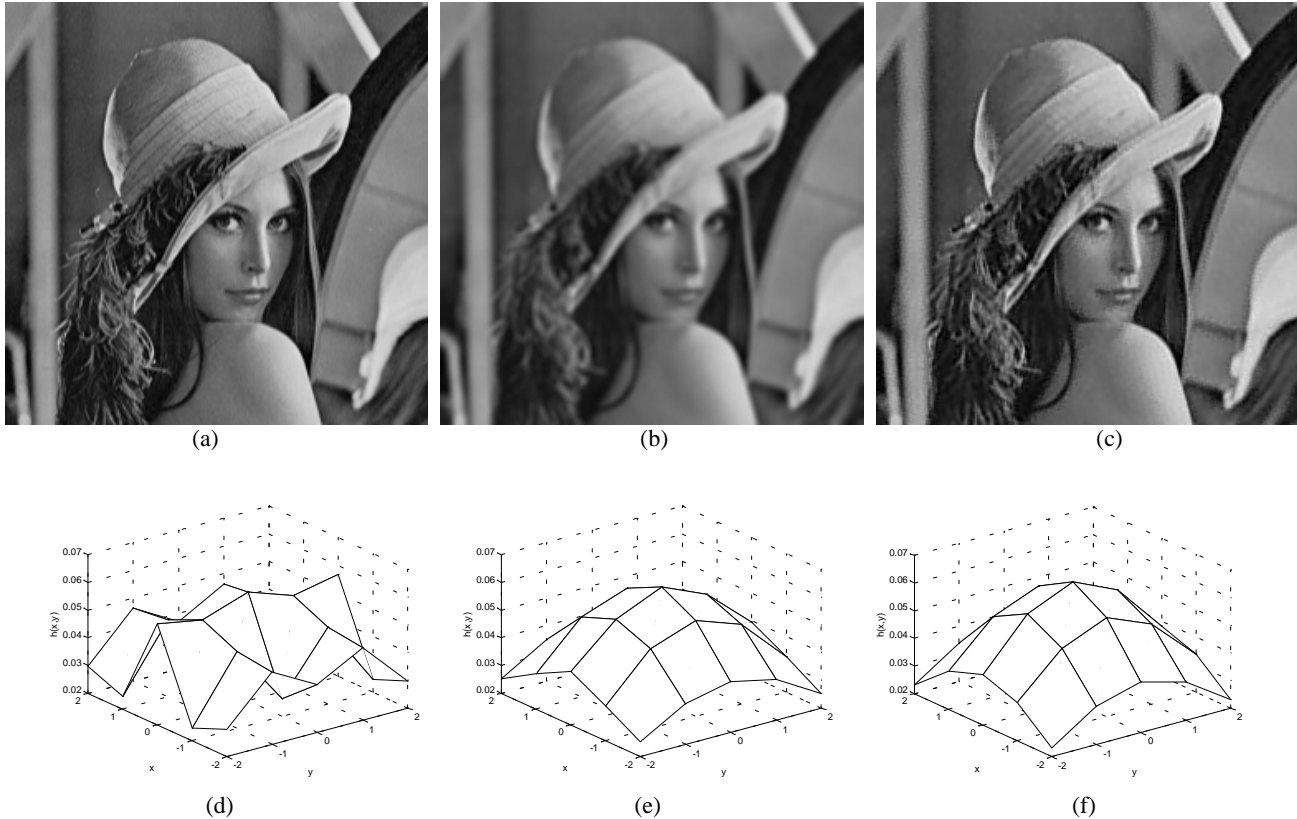


Figure 1. Blind image deconvolution of an image degraded by Gaussian blur  
 (a) original image, (b) degraded image, (c) restored image, (d) initial random blur, (e) identified blur, (f) actual blur

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