A MULTIRATE PILOT-SYMBOL-ASSISTED CHANNEL ESTIMATOR FOR OFDM TRANSMITTER DIVERSITY SYSTEMS

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) transmitter diversity techniques have been shown to be efficient means of achieving near optimal diversity gain in frequency-selective fading channels. For these systems, knowledge of the channel parameters is required at the receivers for diversity combining and decoding. In this paper, we propose a low complexity, bandwidth efficient, pilot-symbol-assisted channel estimator for multiple transmitter OFDM systems. The pilot symbols are constructed to be non-overlapping in frequency to allow for the simultaneous sounding of the multiple channels. The time-varying channel responses are tracked by interpolating a set of estimates obtained through periodically transmitted pilot symbols. The effectiveness and limitations of the proposed estimator are verified by simulations.

1. INTRODUCTION

A number of space-time and space-frequency coded OFDM transmitter diversity techniques have recently been proposed for frequency-selective fading channels [1–3]. These techniques are capable of achieving near optimal diversity gain when the receivers have perfect knowledge of the channels. In practice, the channel parameters have to be estimated at the receivers. Channel estimation for transmitter diversity systems is complicated by the fact that the received signal is the superposition of signals transmitted simultaneously from multiple transmitters. In [4], a minimum mean square error (MMSE) channel estimator for OFDM transmitter diversity systems was proposed. The main drawback of the MMSE channel estimation approach is the high computational complexity required to update the channel estimates during the data transmission mode. In this paper, we investigate a low complexity channel estimation technique for multiple transmitter OFDM systems. The proposed technique uses bandwidth efficient pilot symbols to facilitate temporal estimation of the multiple channel responses. Simple interpolation filters are then used to update the estimates during the data transmission mode.

2. OFDM TRANSMITTER DIVERSITY SYSTEMS

A block diagram of a two-branch OFDM transmitter diversity system is shown in Fig. 1. Let $X(m)$ denote the input serial data symbols with symbol duration T_S. The serial to parallel converter collects K serial data symbols into a data vector $\mathbf{X}(n)$ that has a block duration of KT_S. The transmitter diversity encoder codes $\mathbf{X}(n)$ into two vectors $\mathbf{X}_1(n)$ and $\mathbf{X}_2(n)$ according to an appropriate coding scheme as in [1–3]. The coded vector $\mathbf{X}_1(n)$ is modulated by an inverse discrete Fourier transform (IDFT) into an OFDM symbol sequence. A length G cyclic extension is added to the OFDM symbol sequence, and the resulting signal is transmitted from the first transmit antenna. Similarly, vector $\mathbf{X}_2(n)$ is modulated by an IDFT, cyclically extended, and transmitted from the second transmit antenna. Let $\mathbf{h}_1(n)$ denote the impulse response of the channel between the first transmit antenna and the receiver and $\mathbf{h}_2(n)$ denote the impulse response of the channel between the second transmit antenna and the receiver. The length of the cyclic extension is chosen to be greater than or equal to L, the order of the channel impulse responses, i.e., $G \geq L$. At the receiver, the received signal vector first has the cyclic prefix removed and is then demodulated by a discrete Fourier transform (DFT) to yield the demodulated signal vector $\mathbf{Y}(n)$. Assuming the channel impulse responses remain constant during the entire block interval, it can be easily shown that the demodulated signal is given by

$$\mathbf{Y}(n) = \mathbf{\Lambda}_1(n)\mathbf{X}_1(n) + \mathbf{\Lambda}_2(n)\mathbf{X}_2(n) + \mathbf{Z}(n), \quad (1)$$

where $\mathbf{\Lambda}_1(n)$ and $\mathbf{\Lambda}_2(n)$ are two diagonal matrices whose elements are the DFTs of the respective channel impulse responses, $\mathbf{h}_1(n)$ and $\mathbf{h}_2(n)$, and $\mathbf{Z}(n)$ is the DFT of the

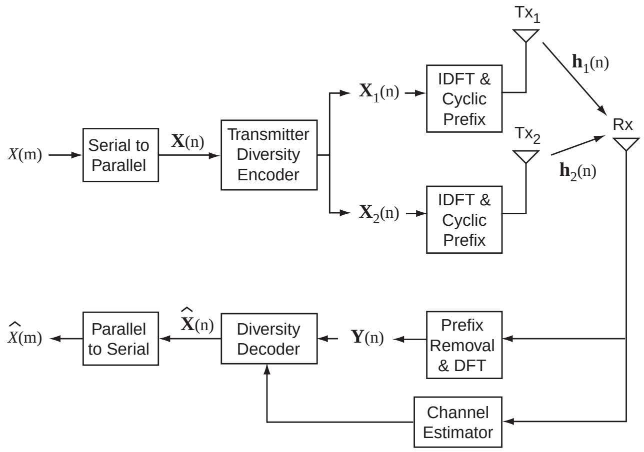

Fig. 1. Block diagram of a two-branch OFDM transmitter diversity system.

channel noise. Clearly, the demodulated signal vector $\mathbf{Y}(n)$ is the superposition of the two encoded vectors $\mathbf{X}_1(n)$ and $\mathbf{X}_2(n)$, which makes the estimation of the channel parameters; i.e., $\mathbf{h}_1(n)$ and $\mathbf{h}_2(n)$ or, equivalently, $\mathbf{\Lambda}_1(n)$ and $\mathbf{\Lambda}_2(n)$; from $\mathbf{Y}(n)$ challenging, especially during the data transmission mode.

3. PILOT SYMBOLS FOR MULTIPLE TRANSMITTER OFDM SYSTEMS

Pilot-symbol-assisted (PSA) channel estimation techniques for single transmitter systems have been proposed and are well studied [5,6]. However, there is little literature on PSA channel estimation techniques for multiple transmitter systems. In [7], an alternating PSA channel estimation scheme for transmitter diversity systems was suggested. With the alternating pilot symbol scheme, M times as many pilot symbols are needed to estimate all the channels in an M transmit antenna system as compared to that required in a single transmit antenna system. The expansion in pilot symbols is undesirable from the standpoint of data throughput and bandwidth efficiency. Here, we propose a multirate PSA channel estimation technique that does not require expansion in the number of pilot symbols for multiple transmitter OFDM systems.

Although the different signals from multiple transmitters in a transmitter diversity system tend to interfere with each other, training or pilot symbols can be constructed for multiple transmitter OFDM systems to avoid this form of interference. Thus, simplifying the task of channel estimation during the training mode. Notice that the subchannels in properly designed OFDM systems are decoupled from each other. Therefore, if the pilot symbols are constructed so that pilot symbols transmitted from different transmitters occupy different frequency bins, any individual symbol in the demodulated signal vector $\mathbf{Y}(n)$ will then contain the contribution from only one transmitter, and the complex channel gain for that particular subcarrier can be easily estimated. An obvious choice is to have the pilot symbols among the transmitters evenly distributed while non-overlapping in frequency. In theory, any pilot symbols that satisfy the non-overlapping conditions will be sufficient. In practice, the pilot symbols should be chosen to have other desirable OFDM properties as well. Chirp sequences are attractive for channel estimation in OFDM systems because they have a flat power spectrum and a low peak-to-average power ratio. Here, we propose the use of chirp sequences, with different phase offsets from antenna to antenna, as pilot symbols for multiple transmitter OFDM systems. Define a length K chirp sequence as

$$C\left(k\right) = e^{j\frac{\pi k^2}{K}}, \quad 0 \leq k \leq K-1\,. \qquad (2)$$

Let $PS_i(n,k)$ denote the $k-\mathrm{th}$ tone of the pilot symbol transmitted from the $i-\mathrm{th}$ transmit antenna during the block instant n. The pilot symbols are given by

$$PS_i\left(n,k+i\right) = \begin{cases} (-1)^i\sqrt{M}\,C\left(k+i\right)\,, & (k)_M = 0 \\ 0\,, & \text{otherwise} \end{cases}, \qquad (3)$$

where M is the number of transmitters, $(k)_M$ denotes k modulo M, and $0 \leq i \leq M-1$. Since the pilot symbols are known to the receiver, the $(k+i)-\mathrm{th}$ diagonal element of $\mathbf{\Lambda}_i(n)$, i.e., the complex gain of the $(k+i)-\mathrm{th}$ subcarrier from the $i-\mathrm{th}$ transmitter, can be estimated by

$$\widetilde{\Lambda}_i\left(n,k+i\right) = \begin{cases} \dfrac{Y\left(n,k+i\right)}{PS_i\left(n,k+i\right)}\,, & (k)_M = 0 \\ 0\,, & \text{otherwise} \end{cases}. \qquad (4)$$

The diagonal elements of $\widetilde{\mathbf{\Lambda}}_i(n)$ are, in effect, samples of the frequency response of the $i-\mathrm{th}$ channel. Let $\widetilde{\mathbf{h}}_i(n)$ be the IDFT of the diagonal of $\widetilde{\mathbf{\Lambda}}_i(n)$. In the absence of noise, $\widetilde{\mathbf{h}}_i(n)$ is related to $\mathbf{h}_i(n)$ by

$$\widetilde{h}_i(n,k) = \frac{1}{M}\sum_{l=0}^{M-1} h_i\left(n, \left(k+\frac{K}{M}l\right)_K\right)e^{j\frac{2\pi i}{M}l}\,. \qquad (5)$$

Notice that $\widetilde{\mathbf{h}}_i(n)$ is the sum of circularly shifted versions of $\mathbf{h}_i(n)$. To avoid aliasing in the time domain, the condition $K \geq M(L+1)$ must be satisfied. To improve the estimate of the channel impulse response (CIR) and to remove the images, $\widetilde{\mathbf{h}}_i(n)$ is passed through a length $L+1$ rectangular window of gain M to yield the temporal estimate $\widehat{\mathbf{h}}_i(n)$.

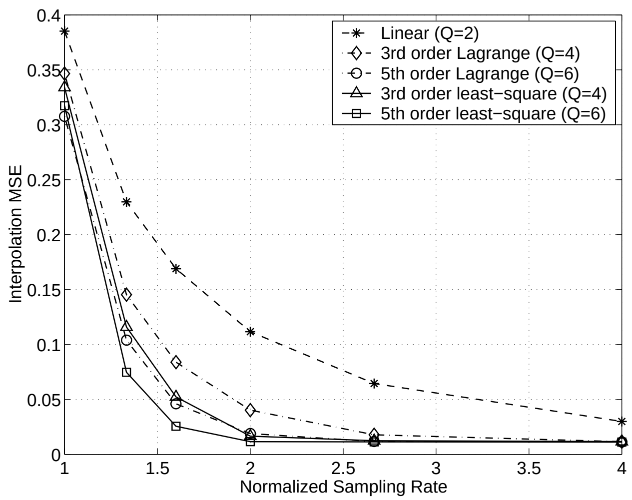

Fig. 2. Interpolation MSE as a function of normalized sampling rate.

4. INTERPOLATION OF CHANNEL PARAMETERS FOR MULTIPLE TRANSMITTER SYSTEMS

Instead of explicitly tracking the multiple time-varying CIRs using a continuous decision-directed algorithm that would require the MMSE solution as in [4], we propose to track the multiple CIRs by interpolating a set of estimated channel responses $\widehat{\mathbf{h}}_i(n)$ obtained from periodically transmitted pilot symbols $\mathbf{PS}_i(n)$, i.e., the insertion of one pilot symbol every N transmitted OFDM symbols. The interpolator takes Q consecutive channel estimates obtained from the pilot symbols at a rate of $1/\left(N\left(K+G\right)T_S\right)$, interpolates the estimates with a real-valued finite impulse response (FIR) digital filter, and generates $N-1$ interpolated CIR samples at the OFDM symbol rate of $1/\left(\left(K+G\right)T_S\right)$. To satisfy the Nyquist criteria, the sampling rate of the channel estimates must satisfy $f_s \geq 2f_{Dm}$, where f_s is the sampling frequency and f_{Dm} is the maximum Doppler frequency. The equivalent condition

$$N \leq \frac{1}{2f_{Dm}\left(K+G\right)T_S} \tag{6}$$

gives an upper bound on the pilot symbol spacing. It is well-known that the impulse response of the ideal interpolator for bandlimited signals is the sinc function, which has an infinite number of taps and is, therefore, unrealizable. A number of practical interpolators have been proposed in [5,8,9]. As shown in [8], even order interpolation filters, i.e., when Q is odd, do not have linear phase. The phase distortion can cause discontinuity in the envelope of the interpolated signal. Therefore, we will focus on odd order, linear phase interpolation filters.

In general, the interpolation process improves with increased sampling rates and with higher order interpolation filters. However, there is no analytical expression for the interpolation error of bandlimited signals using these interpolators. Therefore, the interpolation errors of a number of interpolators were simulated to provide a qualitative measure of how well these interpolators may track a frequency-selective fading channel. The interpolation performance criteria used is the mean square error (MSE) between the interpolated and the actual CIR. Assuming the pilot symbols are transmitted at block instants $n = 0, N, 2N$, etc., the interpolation MSE is defined as

$$\varepsilon = \frac{1}{N_I\left(N-1\right)L} \cdot \sum_{p=0}^{N_I-1} \sum_{n=pN+1}^{pN+N-1} \sum_{l=0}^{L-1} \left|\widehat{h}(n,l) - h(n,l)\right|^2, \tag{7}$$

where N_I is the number of interpolation intervals in the simulation. The COST207 six-ray typical urban channel power delay profile was used throughout the simulations. Simulation results of the interpolation MSE for the linear interpolator, third and fifth order Lagrange interpolators [8], and third and fifth order least-square interpolators ($\alpha = 0.5$) [9] are shown in Fig. 2. Simulations show that the linear interpolator has significant interpolation error until the sampling rate is well above 4 times the Nyquist rate. As expected, the higher order interpolators all have better performance than the linear interpolator. Interestingly, the interpolation errors of the fifth order Lagrange, and the third and fifth order least-square interpolators are very close to the error floor at only twice the Nyquist rate. From these results, the third order least-square interpolator operating with $N \approx 1/\left(4f_{Dm}\left(K+G\right)T_S\right)$ should achieve good interpolation performance at a reasonable complexity and delay. Hardware complexity of the interpolator can be further reduced by employing the polyphase filter structure as shown in [9].

5. CHANNEL ESTIMATION FOR OFDM TRANSMITTER DIVERSITY SYSTEMS

Channel estimators based on the pilot symbols and interpolators described in previous sections have been evaluated with the two-branch space-time coded OFDM (ST-OFDM) transmitter diversity system in [2]. For the simulations, the ST-OFDM system employed 128 subcarriers with 4-QAM modulation at a symbol rate of 2^{20} symbols per second on each subcarrier, i.e., $K = 128$ and $T_S = 2^{-20}$ seconds. The pilot symbol spacing is set at $N = 20$ so that the sampling frequency is near twice the Nyquist rate at a maximum Doppler frequency of 100Hz. Simulation results of the average bit error rate (BER) performance for a two-branch ST-OFDM system with ideal channel parame-

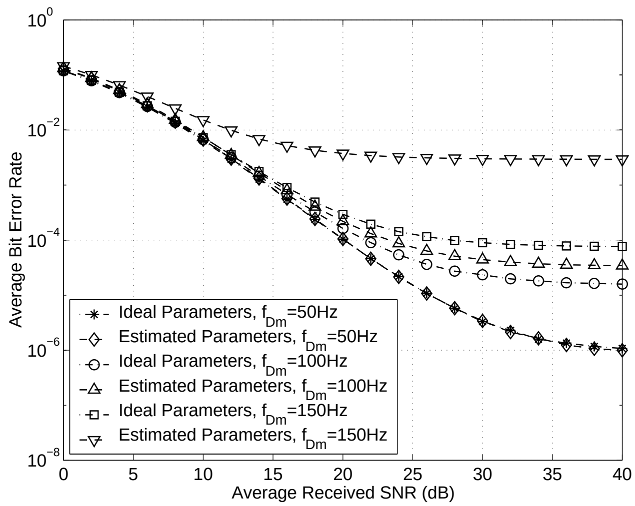

Fig. 3. Performance comparison of ST-OFDM systems with ideal channel parameters and channel parameters estimated by a 3rd order Lagrange interpolator.

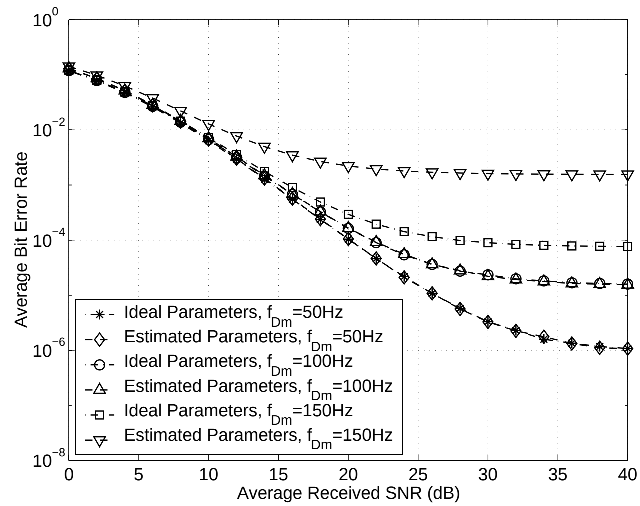

Fig. 4. Performance comparison of ST-OFDM systems with ideal channel parameters and channel parameters estimated by a 3rd order least-square interpolator.

ters and with channel parameters estimated by a third order Lagrange interpolator are shown in Fig. 3. Simulation results with a third order least-square interpolator are shown in Fig. 4. Simulation results confirm that at slow fading conditions, such as when $f_{Dm} = 50$Hz, both the third order Lagrange and third order least-square interpolators perform quite well. In fact, there is no noticeable BER degradation between the system using ideal channel parameters versus that using the estimated parameters. At $f_{Dm} = 100$Hz, which corresponds to sampling at about twice the Nyquist rate, the BER performance with the Lagrange interpolator is degraded slightly, while that with the least-square interpolator still shows no degradation. This is in agreement with the results in Fig. 2, where the third order least-square interpolator has a lower interpolation MSE than the third order Lagrange interpolator. At a faster fading condition of $f_{Dm} = 150$Hz, which corresponds to sampling at about 1.4 times the Nyquist rate, the BER performances of the systems with estimated channel parameters are severely degraded. Clearly, a sufficiently high sampling rate is crucial to the performance of the proposed channel estimator.

6. SUMMARY

A low complexity, bandwidth efficient, pilot-symbol-assisted channel estimator for OFDM transmitter diversity systems has been presented. Simulation results verify that the proposed technique is well suited for channel estimation in space-time coded OFDM transmitter diversity systems. Although not shown in this paper, the proposed channel estimator achieves similar performance in space-frequency coded OFDM transmitter diversity systems [3] as well.

7. REFERENCES

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