CHARACTERIZATION OF THE SCATTERING CENTERS OF A RADAR TARGET WITH POLARIZATION DIVERSITY USING POLYNOMIAL ROOTING

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ABSTRACT

In this paper, we propose a new high resolution method with polarization diversity for the characterization of the scattering centers of a radar target, using a stepped-frequency radar system. The proposed method is based on the polynomial rooting technique. It allows to optimally use the information contained in the polarization of the received wave while keeping the calculation cost moderate. Moreover, it can estimate the range and the polarization parameters of each scattering center in only one step. Simulation results are presented to show the performance of the algorithm.

1. INTRODUCTION

In this paper, the radar target is modeled by an array of discrete scattering centers. Each scattering center is characterized by its range and its complex amplitude. It is known that the characterization of the scattering centers is equivalent to the spectral analysis problem. In section 2, we briefly revise this theory.

The amplitude, the phase and the polarization are needed for a complete characterization of a radar target. The classical radar system works with a single polarization, but it is shown that the use of polarization diversity can considerably improve the system performance. Moreover, the wave polarization provides additional information about the radar target. The polarization characterization of a radar target is presented in section 3.

In section 4, we present a new high resolution method with polarization diversity. This method is based on the polynomial rooting technique. We show that this method allows a full characterization of the target by estimating the amplitude, the phase, the range and the polarization of the scattering centers of the radar target, and it provides also a better performance in terms of resolution power.

Finally, some simulation results and conclusions are presented.

2. DATA MODEL

Assuming that the radar target is modeled by m independent scattering centers, let q be the transmitted polarization, p the received polarization and υ the incident wave frequency. The received signal of the radar target can be expressed as :

$$s_{pq}(\upsilon) = \sum_{i=1}^{m} b_{pqi} e^{j\frac{4\pi}{c}R_i \upsilon} \tag{1}$$

where b_{pqi} is the complex amplitude, R_i the range of the i^{th} scattering center.

A stepped-frequency radar system is considered in this paper. The radar emits a series of bursts of narrow band pulses, where each burst consists of N pulses stepped in frequency from pulse to pulse by a fixed frequency step size $\partial\upsilon$. So for the n^{th} pulse, the emitted frequency is $\upsilon_n = \upsilon_0 + n\partial\upsilon$, with $n = 0, 1, \cdots, N-1$, and the received signal is given by :

$$s_{pq}(n) = \sum_{i=1}^{m} b_{pqi} e^{j\frac{4\pi}{c}R_i \upsilon_n} = \sum_{i=1}^{m} a_{pqi} e^{j2\pi\left(\frac{2\partial\upsilon}{c}R_i\right)n} \tag{2}$$

(2) can be rewritten in the following more convenient normalized manner :

$$s_{pq}(n) = \sum_{i=1}^{m} a_{pqi} e^{j2\pi\left(\frac{R_i}{R_a}\right)n} = \sum_{i=1}^{m} a_{pqi} e^{j2\pi f_i n} \tag{3}$$

where $R_a \equiv \frac{c}{2\partial\upsilon}$ is the well known unambiguous range.

This received signal model allows us to introduce the normalized frequency concept : $f_i = \frac{R_i}{R_a}$ $(i = 1, 2, \cdots, m)$. Normalizing the sampling frequency to 1, $s_{pq}(n)$ can be considered as the sample at instant n of a signal composed of the sum of m complex sinusoids with frequency equals to $f_i = \frac{R_i}{R_a}$.

Of course the received signal is corrupted by a noise, so the characterization of the scattering centers is equivalent to the estimation of the parameters of a signal composed of the sum of sinusoids embedded in noise.

3. POLARIZATION CHARACTERIZATION

The polarization properties are completely described by the scattering matrix which gives the relationship between the polarization of the scattered wave and that of the incident wave. This matrix can be obtained by estimating the polarization of the scattered waves for two incident waves with orthogonal polarizations. The most important point then, is the estimation of the polarization of the received wave, whatever the polarization of the incident wave. For estimating the received wave's polarization, it is necessary to have polarization diversity at the receiver allowing the reception simultaneously of the horizontally and vertically polarized waves. Let's denote the polarization vector of the incident wave by the following Jones vector :

$$\mathbf{q} = \begin{bmatrix} q_h \\ q_v \end{bmatrix} \tag{4}$$

In frequency domain, the received wave for an incident wave of polarization $\mathbf{q}$ can be written as :

$$\begin{bmatrix} s_{hr}(n) \\ s_{vr}(n) \end{bmatrix} = \begin{bmatrix} s_{hh}(n) & s_{hv}(n) \\ s_{vh}(n) & s_{vv}(n) \end{bmatrix} \begin{bmatrix} q_h \\ q_v \end{bmatrix} \tag{5}$$

Using (3), and (5) it is straightforward that the received signal vector can be expressed as :

$$\begin{bmatrix} s_{hr}(n) \\ s_{vr}(n) \end{bmatrix} = \sum_{i=1}^{m} \begin{bmatrix} a_{h_i} \\ a_{v_i} \end{bmatrix} e^{j2\pi f_i n} \tag{6}$$

where a_{h_i} and a_{v_i} contain the polarization information of the i^{th} scattering center.

The wave polarization is usually represented by the polarization ellipse, characterized by the ellipticity angle τ_i ($|\tau_i| \leq \pi/4$) and by the tilt angle ϕ_i $(0 \leq \phi_i < \pi)$. These two parameters can be calculated from a_{h_i} and a_{v_i} by the following equations [1] :

$$\begin{aligned}
\gamma_i &= \arctan\left[\frac{|a_{v_i}|}{|a_{h_i}|}\right] \\
\delta_i &= \arg(a_{v_i}) - \arg(a_{h_i}) \\
\tau_i &= \frac{1}{2}\arcsin\left[\sin(2\gamma_i)\sin(\delta_i)\right] \\
\phi_i &= \frac{1}{2}\arctan\left[\tan(2\gamma_i)\cos(\delta_i)\right] \\
\phi_i &= \begin{cases} \phi_i + \frac{\pi}{2} & si \quad \gamma_i > \frac{\pi}{4} \\ \phi_i + \pi & si \quad \gamma_i \leq \frac{\pi}{4}, \alpha_i < 0 \end{cases}
\end{aligned} \tag{7}$$

4. HIGH RESOLUTION METHOD

Recently, many high resolution methods have been proposed in order to overcome the limitations of the classical FFT method. While they have found successful application in the case of scalar signal waves where the polarization properties of received waves are ignored, very little application can be found in the presence of polarization diversity [1], [2]. The wave polarization diversity is however an additional information, which can be used to improve the performance of classical methods. We can note that the polarization properties have been used with success for estimating the directions of arrival of multiple plane waves with an array of antennas [3], [4], [5]. In the following, we present a new high resolution method with polarization diversity.

Assuming that we have N samples of the received signal vector (6), let's consider the two following matrices utilizing all the available data :

$$\mathbf{S}_h = \begin{bmatrix} s_{hr}(0) & s_{hr}(1) & \cdots & s_{hr}(N-p) \\ s_{hr}(1) & s_{hr}(2) & & \vdots \\ \vdots & & \ddots & \\ s_{hr}(p-1) & & & s_{hr}(N-1) \end{bmatrix} \tag{8}$$

$$\mathbf{S}_v = \begin{bmatrix} s_{vr}(0) & s_{vr}(1) & \cdots & s_{vr}(N-p) \\ s_{vr}(1) & s_{vr}(2) & & \vdots \\ \vdots & & \ddots & \\ s_{vr}(p-1) & & & s_{vr}(N-1) \end{bmatrix} \tag{9}$$

In order to exploit the information contained in the two matrices $\mathbf{S}_h$ et $\mathbf{S}_v$, we propose to use the matrix $\mathbf{S}$ constructed in the following manner :

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_h \\ \mathbf{S}_v \end{bmatrix} \tag{10}$$

According to the structure of the matrices $\mathbf{S}_h$ and $\mathbf{S}_v$ and that of the received signal (6), the matrix $\mathbf{S}$ can be written as :

$$\mathbf{S} = \sum_{i=1}^{m} \begin{bmatrix} a_{h_i} & \cdots & a_{h_i}e^{j2\pi f_i(N-p)} \\ \vdots & \ddots & \\ a_{h_i}e^{j2\pi f_i(p-1)} & & a_{h_i}e^{j2\pi f_i(N-1)} \\ a_{v_i} & \cdots & a_{v_i}e^{j2\pi f_i(N-p)} \\ \vdots & \ddots & \\ a_{v_i}e^{j2\pi f_i(p-1)} & & a_{v_i}e^{j2\pi f_i(N-1)} \end{bmatrix}$$

which can be factorized as :

$$\mathbf{S} = \sum_{i=1}^{m} \mathbf{g}_{2p}(a_{h_i}, a_{v_i}, f_i)\, \mathbf{d}_{n1}^T(f_i) \tag{11}$$

with $\mathbf{g}_{2p}(a_h, a_v, f)$, $\mathbf{d}_{n_1}(f)$ two vectors of dimension $2p$ and $n_1 = N - p + 1$ respectively :

$$\begin{aligned}
\mathbf{d}_{n_1}(f) &= \begin{bmatrix} 1 & e^{j2\pi f} & \cdots & e^{j2\pi(n_1-1)f} \end{bmatrix}^T \\
\mathbf{g}_{2p}(a_h, a_v, f) &= \begin{bmatrix} a_h\mathbf{d}_p^T(f) & a_v\mathbf{d}_p^T(f) \end{bmatrix}^T
\end{aligned}$$

It can be shown that if $p > m$ and $n_1 > m$, the matrix S is rank deficient. Moreover if $f_i \neq f_j, \forall i \neq j$, for $i, j = 1, 2, \cdots m$, the rank of the data matrix $\mathbf{S}$ is equal to the number of scattering centers m. Consequently, the high resolution method principle can be applied to the matrix $\mathbf{S}$.

Let $\mathbf{U}_1$ be the matrix of dimension $(2p, m)$ formed by the m left singular vectors associated with the m largest singular values ; and $\mathbf{U}_2$ the matrix of dimension $(2p, 2p - m)$ formed by the remaining $(2p - m)$ left singular vectors of the matrix $\mathbf{S}$. Also let $\Pi_b = \mathbf{U}_2 \mathbf{U}_2^H$. The frequency (range) and the polarization parameters can be estimated by the following minimization problem :

$$\min_{a_h, a_v, f} \frac{\mathbf{g}_{2p}^H (a_h, a_v, f) \Pi_b \mathbf{g}_{2p} (a_h, a_v, f)}{\mathbf{g}_{2p}^H (a_h, a_v, f) \mathbf{g}_{2p} (a_h, a_v, f)} \tag{12}$$

This minimization requires a three dimensional search, a computationally intensive procedure, which is difficult to realize practically. Fortunately, the structure of the vector $\mathbf{g}_{2p} (a_h, a_v, f)$ allows us to considerably simplify this minimization problem.

The vector $\mathbf{g}_{2p} (a_h, a_v, f)$ can be rewritten as :

$$\mathbf{g}_{2p} = \begin{bmatrix} a_h \mathbf{d}_p (f) \\ a_v \mathbf{d}_p (f) \end{bmatrix} = \begin{bmatrix} \mathbf{d}_p & \mathbf{0}_p \\ \mathbf{0}_p & \mathbf{d}_p \end{bmatrix} \begin{bmatrix} a_h \\ a_v \end{bmatrix} = \mathbf{D}\mathbf{a} \tag{13}$$

with $\mathbf{D}$ a matrix of dimension $(2p, 2)$ containing frequency information and $\mathbf{a}$, a 2 dimensional vector, containing the polarization information. Consequently, this factorization of the vector $\mathbf{g}_{2p}$ allows us to separate the frequency contribution and that of the polarization.

As $\mathbf{D}^H \mathbf{D} = p \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$, the minimization (12) can be reformulated as :

$$\min_{f, \mathbf{a}} \frac{\mathbf{a}^H \mathbf{D}^H \Pi_b \mathbf{D} \mathbf{a}}{\mathbf{a}^H \mathbf{D}^H \mathbf{D} \mathbf{a}} = \min_{f, \mathbf{a}} \frac{\mathbf{a}^H \mathbf{D}^H \Pi_b \mathbf{D} \mathbf{a}}{p \mathbf{a}^H \mathbf{a}} \tag{14}$$

which can be done by first minimizing the cost function relative to the frequency f, and then to the vector $\mathbf{a}$:

$$\min_{\mathbf{a}} \left[\min_{f} \frac{\mathbf{a}^H \mathbf{D}^H \Pi_b \mathbf{D} \mathbf{a}}{p \mathbf{a}^H \mathbf{a}} \right] \tag{15}$$

The minimum of (15) is given by the smallest eigenvalue of the matrix $\mathbf{D}^H \Pi_b \mathbf{D}$, a Hermitian nonnegative definite matrix. Its minimum is zero if f and $\mathbf{a}$ correspond to the true parameters of a scattering center. So the frequencies f can be estimated, in the ideal situation, by the frequencies such that :

$$\det \left[\mathbf{D}^H \Pi_b \mathbf{D} \right] = 0 \tag{16}$$

To simplify this formula, define

$$\Pi_b = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{12}^H & \mathbf{R}_{22} \end{bmatrix} \tag{17}$$

so that we have

$$\mathbf{D}^H \Pi_b \mathbf{D} = \begin{bmatrix} \mathbf{d}_p^H \mathbf{R}_{11} \mathbf{d}_p & \mathbf{d}_p^H \mathbf{R}_{12} \mathbf{d}_p \\ \mathbf{d}_p^H \mathbf{R}_{12}^H \mathbf{d}_p & \mathbf{d}_p^H \mathbf{R}_{22} \mathbf{d}_p \end{bmatrix} \tag{18}$$

and

$$\begin{aligned} \det \left[\mathbf{D}^H \Pi_b \mathbf{D} \right] &= \left(\mathbf{d}_p^H \mathbf{R}_{11} \mathbf{d}_p \right) \left(\mathbf{d}_p^H \mathbf{R}_{22} \mathbf{d}_p \right) \\ &\quad - \left(\mathbf{d}_p^H \mathbf{R}_{12}^H \mathbf{d}_p \right) \left(\mathbf{d}_p^H \mathbf{R}_{12} \mathbf{d}_p \right) \end{aligned} \tag{19}$$

Let's define the complex variable $z = \exp(j2\pi f)$, so that the vector $\mathbf{d}_p (f)$ becomes :

$$\mathbf{d}_p (z) = \begin{bmatrix} 1 & z & \cdots & z^{p-1} \end{bmatrix}^T \tag{20}$$

Now the determinant (19) is a function of z and can be expressed as :

$$\begin{aligned} &\left(\mathbf{d}_p^T \left(z^{-1} \right) \mathbf{R}_{11} \mathbf{d}_p (z) \right) \left(\mathbf{d}_p^T \left(z^{-1} \right) \mathbf{R}_{22} \mathbf{d}_p (z) \right) \\ &- \left(\mathbf{d}_p^T \left(z^{-1} \right) \mathbf{R}_{12} \mathbf{d}_p (z) \right) \left(\mathbf{d}_p^T \left(z^{-1} \right) \mathbf{R}_{12} \mathbf{d}_p (z) \right) \end{aligned} \tag{21}$$

The frequencies can then be estimated by the roots of the polynomial (21).

Note that $\mathbf{d}_p^T \left(z^{-1} \right) \mathbf{R}_{mn} \mathbf{d}_p (z)$ $(m, n = 1, 2)$ can be written as :

$$\mathbf{d}_p^T \left(z^{-1} \right) \mathbf{R}_{mn} \mathbf{d}_p (z) = \mathbf{a}^T (z) \mathbf{c}_{mn} \tag{22}$$

with

$$\begin{aligned} \mathbf{a}(z) &= \left[z^{-(p-1)}, z^{-p+2}, \ldots, z^{p-1} \right]^T \\ \mathbf{c}_{mn} &= \left[\alpha_{-(p-1)}, \alpha_{-p+2}, \ldots, \alpha_{p-1} \right]^T \\ \alpha_l &= \sum_{i=\max(1,1-l)}^{\min(p,p-l)} \mathbf{R}_{mn}(i, l+i) \end{aligned} \tag{23}$$

Now, the polynomial (21) becomes :

$$\mathbf{a}^T (z) \left(\mathbf{c}_{11} \mathbf{c}_{22}^T - \mathbf{c}_{12} \mathbf{c}_{21}^T \right) \mathbf{a} (z) = \mathbf{a}^T (z) \mathbf{F} \mathbf{a} (z) \tag{24}$$

so the final expression of the polynomial is given by :

$$P(z) = \mathbf{a}^T (z) \mathbf{F} \mathbf{a} (z) = \mathbf{k}^T \mathbf{b} (z) \tag{25}$$

with $\mathbf{b}(z)$ and $\mathbf{k}$ of dimension $4p - 3$, defined by :

$$\begin{aligned} \mathbf{b}(z) &= \left[z^{-2(p-1)}, \ldots, z^{2(p-1)} \right]^T \\ \mathbf{k} &= \left[\alpha_1, \ldots, \alpha_{4p-3} \right]^T \\ \alpha_l &= \sum_{i=\max(1,l-2p+2)}^{\min(2p-1,l)} \mathbf{F}(i, l+1-i) \end{aligned} \tag{26}$$

Considering only the roots inside the unit circle, the frequencies are estimated by the roots of the polynomial

$\mathbf{k}^T\mathbf{b}(z)$ nearest to the unit circle. Once the frequencies have been estimated, the associated amplitudes can be estimated by the corresponding eigenvectors given by :

$$\begin{bmatrix} a_{hi} \\ a_{vi} \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ \frac{e_{11}-e_{22}+\sqrt{(e_{11}-e_{22})^2+4|e_{12}|^2}}{e_{12}} \end{bmatrix} \qquad (27)$$

where

$$e_{mn} = \mathbf{a}^T(\widehat{z}_i)\,\mathbf{c}_{mn} \quad et \quad \widehat{z}_i = \exp(j2\pi\widehat{f}_i) \qquad (28)$$

and the polarization parameters can be estimated with the formulas (7) .

5. SIMULATION

The model proposed in [1] is adopted here for simulation. The target is modeled by 4 scattering centers. The distances of these scattering centers with respect to a reference point are: 8 cm, 12 cm, 13,3 cm et 14 cm, and the corresponding scattering matrices are : $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $10\begin{bmatrix} \cos^2 10 & \frac{\sin 20}{2} \\ \frac{\sin 20}{2} & \sin^2 10 \end{bmatrix}$, $2\begin{bmatrix} \cos^2 10 & \frac{\sin 20}{2} \\ \frac{\sin 20}{2} & \sin^2 10 \end{bmatrix}$, $3\begin{bmatrix} \sin^2 10 & -\frac{\sin 20}{2} \\ -\frac{\sin 20}{2} & \cos^2 10 \end{bmatrix}$.

The incident wave is left circularly polarized. The received signal vector for each frequency between 2 and 18 GHz in 50MHz steps are generated. This corresponds to an unambiguous range of 3m, and 320 samples. In order to keep the unambiguous range near the target size, the scattering data is decimated by a factor of 10 before processing. So the new unambiguous range is 30cm, the sample number is 32 and the four normalized frequencies are 0.2667, 0.4, 0.4433, 0.4667.

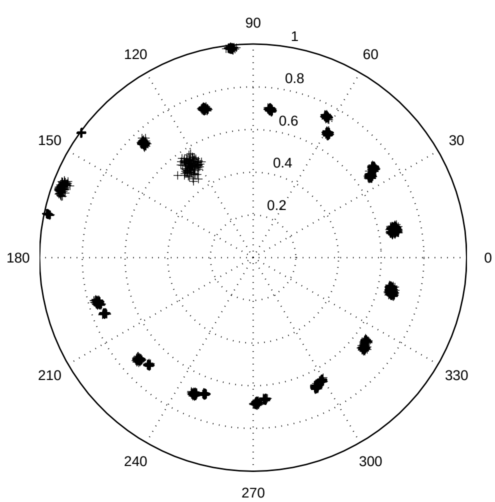

Fig. 1. Roots of polynomial $P(z)$ for 100 trials.

The proposed algorithm is processed for 100 independent trials at a SNR of 5 dB. Fig. 1 shows the roots situated inside the unit circle of the polynomial $P(z)$ (25) obtained by the simulation. In Fig. 1, four roots appear clearly near the unit circle corresponding to the 4 scattering centers. The worst case is given by the scattering center with range of 13.3 cm. For this scattering center, we give in Table 1 the root-mean-square error of both the distance range and the polarization parameters estimates. The estimation of the parameters of the others scattering centers will be better.

Table 1 : Simulation Results

	d(cm)	τ(deg)	ϕ(deg)
Actual values	13.3	0	10
Mean values	13.30	-0.39	9.02
Standard deviation	0.09	3.98	3.36

6. CONCLUSION

In this paper, we have presented a new high resolution method for the characterization of a radar target modeled by scattering centers, using a stepped-frequency radar system. The proposed method allows to optimally use the information contained in the polarization of the received wave while keeping the calculation cost moderate. Moreover, it can estimate the frequencies and the polarization parameters in only one step, contrary to the method proposed in [1] where these two parameters are estimated in two separate procedures.

7. REFERENCES

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