CRAMER-RAO BOUND FOR LOCATION ESTIMATION OF A MOBILE IN ASYNCHRONOUS DS-CDMA SYSTEMS

Cyril Botteron, Anders Høst-Madsen, Michel Fattouche

University of Calgary, Department of Electrical and Computer Engineering
2500 - University Drive N.W., Calgary, Alberta, T2N 1N4, Canada

ABSTRACT

Commercial applications for the location of subscribers of wireless services continue to expand. Consequently, finding the Cramer-Rao bound (CRB), which serves as an optimality criterion for the location estimation problem, is of interest. In this article, we derive the CRB for the estimation of channel parameters and mobile position in an asynchronous direct sequence code division multiple access (DS-CDMA) system operating over fading channels. It is assumed that the location estimates are obtained from the bearings and/or propagation delays estimated at one or more cluster(s) of antenna array of arbitrary geometry. Among other applications, the CRB on the positioning accuracy may serve as a design tool to find an optimum antenna placement, or to evaluate the practicability of a legal demand for emergency location.

1. INTRODUCTION

Wireless location has received considerable attention over the past few years. A major motivation is personal safety, such as in the location-based emergency service (E-911), which is a requirement for the wireless carriers in the USA [1]. Other applications include traffic management, accident detection, and other emerging services [2].

This paper focuses on network-based cellular geolocation techniques, where the estimated time-delays (TD) and/or angle of arrivals (AOA) of the received CDMA signals are used to find the mobile's position. Cellular geolocation has many advantages over other methods since it relies on the existing infrastructure of cellular base stations, and support E-911 implementation that is compatible with any existing phone.

Despite its importance in practice, the problem of deriving the CRBs for the location of a mobile and for the bearings and time-delays of multipath propagation signals in a K-user asynchronous CDMA system using clusters of antennas seems not to have been addressed in the literature. Some simpler although similar problems were addressed, such as the derivation of the CRBs for the (joint) time-delays and directions of arrival estimation in non-CDMA systems [3] - [5], or for the time-delays (without angle estimation) in CDMA systems [6] - [7].

This paper has been organized as follows. Section 2 describes the asynchronous CDMA system model. Section 3 presents the CRB for the joint estimation of the time-delays and directions of arrival. The CRB for the location estimation problem is derived in Section 4. An application example is presented in Section 5. Finally, the conclusions and further research possibilities are discussed in Section 6.

This work was supported by Cell-Loc Inc., Calgary, Alberta, Canada (company's internet site: www.cell-loc.com).

2. ASYNCHRONOUS DS-CDMA SYSTEM MODEL

The system model is partially adopted from [6], an asynchronous K-user DS-CDMA system operating in a fading environment. The modulation scheme is binary phase shift keying (BPSK). The transmitted symbols duration is T and the chip duration is $T_c = T/N$, where N is an integer. The spreading code have unit amplitude and are assumed to be rectangular and periodic with period T. The kth user baseband signal is formed by pulse amplitude modulating the data stream, $d_k(m) \in \{+1, -1\}$, with a period of the code waveform $b_k(t)$ of user k, i.e. $s_k(t) = \sum_{m=-\infty}^{\infty} d_k(m) b_k(t - mT)$. The transmitted signal is formed by multiplying $s_k(t)$ with the carrier $\sqrt{2\mathrm{P}_k}\cos(w_c t + \varphi_k')$, where P_k is the transmitted power and φ_k' is the random carrier phase uniformly distributed in $[0, 2\pi)$.

Considering C clusters of antennas each composed of P_c antennas with arbitrary location and arbitrary directional characteristics, the channel for the kth user as seen from the pth antenna belonging to the cth cluster is modeled as a time-varying filter with impulse response $h_{c,p,k}(\tau, t)$ [1]

$$h_{c,p,k}(\tau, t) = \sum_{r=1}^{R_{c,k}} w_{c,p}(\theta_{c,k,r}) \gamma_{c,k,r}(t) \delta_K(\tau - \tau_{c,k,r}) \qquad (1)$$

where $R_{c,k}$ is the number of paths (or rays) from user k impinging on the cth cluster, $w_{c,p}(\theta_{c,k,r})$ is the pth element of the steering vector $\mathbf{w}_c(\theta_{c,k,r})$ for the cth array toward the direction $\theta_{c,k,r}$, $\gamma_{c,k,r}(t)$ is the product of the path gain $\alpha_{c,k,r}(t)$ with the fading process $\zeta_{c,k,r}(t)$, and $\tau_{c,k,r}$ is the propagation delay. It is assumed that the channel is slowly time-varying, such that the number of paths and propagation delays remain constant over the observation time. Furthermore, the path gains and fading processes are assumed wide-sense stationary and to vary slowly in time compared to the symbol time, i.e. $\gamma_{c,k,r}(t) = \gamma_{c,k,r}(mT)$ for $t \in [mT, (m+1)T)$.

The received signal for the pth antenna of the cth cluster can be written as

$$r_{c,p}(t) = \sum_{k=1}^{K} \sum_{r=1}^{R_{c,k}} w_{c,p}(\theta_{c,k,r}) \gamma_{c,k,r}(t) s_k(t - \tau_{c,k,r}) \times \sqrt{2\mathrm{P}_k}\cos(w_c t + \varphi_{c,k,r}) + n_{c,p}(t) \qquad (2)$$

[1] $\delta_K(t)$ denotes the Dirac delta function , i.e. $\delta_K(t) = 1$ if $t = 0$ and $\delta_K(t) = 0$ otherwise.

where $\varphi_{c,k,r} = \varphi'_k - w_c\tau_{c,k,r}$ and $n_{c,p}(t)$ is a white Gaussian noise waveform with two-sided power spectral density $N_0/2$.

The signal $r_{c,p}(t)$ is downconverted to baseband after multiplication with $\sqrt{2}\cos(w_c t)$ and $-\sqrt{2}\sin(w_c t)$, and passed through an integrate-and-dump filter, with integration time $T_i = T_c/Q$, where Q is called the oversampling factor. Ignoring double frequency terms, the equivalent complex received sequence may be written as

$$r_{c,p}(l) = \sum_{k=1}^{K}\sum_{r=1}^{R_{c,k}} w_{c,p}(\theta_{c,k,r})\beta_{c,k,r}(\lfloor (l-1)/QN \rfloor) \times \frac{1}{T_i}\int_{(l-1)T_i}^{lT_i} s_k(t-\tau_{c,k,r})dt + n_{c,p}(l) \quad (3)$$

where $\beta_{c,k,r}(m) = \gamma_{c,k,r}(mT)\sqrt{\mathrm{P}_k}exp(j\varphi_{c,k,r})$, $n_{c,p}(l)$ is a white Gaussian sequence with variance $\sigma^2 = E[|n_{c,p}(l)|^2] = N_0/T_i = N_0 QN/T$, and $\lfloor x \rfloor$ denotes the largest integer y such that $y \leq x$. After some straightforward calculations, we can formulate the received vector for the pth antenna of the cth cluster, $\mathbf{r}_{c,p}(m) \in \mathbb{C}^{QN}$, as

$$\mathbf{r}_{c,p}(m) = \mathbf{A}_c \mathbf{s}_{c,p}(m) + \mathbf{n}_{c,p}(m) \quad (4)$$

where

$$\begin{aligned}
\mathbf{r}_{c,p}(m) &= [r_{c,p}(mQN+1)\cdots r_{c,p}(mQN+QN)]^T \\
\mathbf{n}_{c,p}(m) &= [n_{c,p}(mQN+1)\cdots n_{c,p}(mQN+QN)]^T \\
\mathbf{A}_c &= [\mathbf{A}_{c,1}, \mathbf{A}_{c,2}\cdots \mathbf{A}_{c,K}] \\
\mathbf{A}_{c,k} &= [\mathbf{a}_{c,k,1}, \mathbf{a}_{c,k,2}\cdots \mathbf{a}_{c,k,2R_{c,k}}] \\
\mathbf{s}_{c,p}(m) &= [\mathbf{s}_{c,p,1}^T(m)\cdots \mathbf{s}_{c,p,K}^T(m)]^T \\
\mathbf{s}_{c,p,k}(m) &= [s_{c,p,k,1}(m)\cdots s_{c,p,k,2R_{c,k}}(m)]^T.
\end{aligned}$$

The columns of $\mathbf{A}_{c,k} \in \mathbb{R}^{QN\times 2R_{c,k}}$ represent the shifted signal sequences for every path coming from the kth user and impinging on the cth cluster. They are defined as

$$\mathbf{a}_{c,k,2r-1} = \left[\frac{\delta_{c,k,r}}{T_i}\mathbf{D}_1^{p_{c,k,r}+1} + (1-\frac{\delta_{c,k,r}}{T_i})\mathbf{D}_1^{p_{c,k,r}}\right]\mathbf{c}_k$$

$$\mathbf{a}_{c,k,2r} = \left[\frac{\delta_{c,k,r}}{T_i}\mathbf{D}_{-1}^{p_{c,k,r}+1} + (1-\frac{\delta_{c,k,r}}{T_i})\mathbf{D}_{-1}^{p_{c,k,r}}\right]\mathbf{c}_k$$

where $\tau_{c,k,r} = p_{c,k,r}T_i + \delta_{c,k,r}$, such that $p_{c,k,r}$ is an integer and $\delta_{c,k,r} \in [0, T_i)$. The vector $\mathbf{c}_k \in \mathbb{R}^{QN}$ and the permutation matrix $\mathbf{D}_s^p \in \mathbb{R}^{QN\times QN}$ are defined as

$$\mathbf{c}_k = \left[\frac{1}{T_i}\int_0^{T_i} b_k(t)dt \cdots \frac{1}{T_i}\int_{(QN-1)T_i}^{QNT_i} b_k(t)dt\right] \quad (5)$$

$$\mathbf{D}_s^p = \begin{bmatrix} \mathbf{0} & s\mathbf{I}_p \\ \mathbf{I}_{QN-p} & \mathbf{0} \end{bmatrix} \quad (6)$$

where $\mathbf{I}_x$ denotes the $x \times x$ identity matrix. The elements of $\mathbf{s}_{c,p,k}(m)$ are

$$\begin{aligned}
s_{c,p,k,2r-1} &= w_{c,p}(\theta_{c,k,r})\beta_{c,k,r}(m) z_{2k-1}(m) \\
s_{c,p,k,2r} &= w_{c,p}(\theta_{c,k,r})\beta_{c,k,r}(m) z_{2k}(m)
\end{aligned}$$

where $z_{2k-1}(m) = \frac{d_k(m)+d_k(m+1)}{2}$, $z_{2k}(m) = \frac{d_k(m)-d_k(m+1)}{2}$.

Finally, we can write the total received vector $\mathbf{r}(m)$ for all the clusters of antennas as

$$\mathbf{r}(m) = \mathbf{\Lambda}\mathbf{s}(m) + \mathbf{n}(m) \quad (7)$$

where [2, 3]

$$\begin{aligned}
\mathbf{\Lambda} &= \mathrm{diag}(\mathbf{I}_{P_1}\otimes \mathbf{A}_1 \cdots \mathbf{I}_{P_C}\otimes \mathbf{A}_C) \\
\mathbf{r(m)} &= [\mathbf{r}_{1,1}^T(m)\cdots \mathbf{r}_{1,P_1}^T(m)\cdots \mathbf{r}_{C,1}^T(m)\cdots \mathbf{r}_{C,P_C}^T(m)]^T.
\end{aligned}$$

and $\mathbf{s(m)}$, $\mathbf{n(m)}$ are defined similarly to $\mathbf{r(m)}$.

3. CRB FOR THE BEARINGS AND TIME-DELAYS ESTIMATION

In this section, we present the CRB, which is a lower bound on the covariance matrix of any unbiased estimator, for the parameters vector $\boldsymbol{\eta} = [\boldsymbol{\theta}^T, \boldsymbol{\tau}^T]^T$. The vector $\boldsymbol{\theta}$ is a vector containing the bearings for all the paths, i.e.

$$\boldsymbol{\theta} = [\boldsymbol{\theta}_{1,1}^T \cdots \boldsymbol{\theta}_{1,K}^T \cdots \boldsymbol{\theta}_{C,1}^T \cdots \boldsymbol{\theta}_{C,K}^T]^T$$

where $\boldsymbol{\theta}_{c,k} = [\theta_{c,k,1}\cdots\theta_{c,k,R_{c,k}}]^T$, and the vector $\boldsymbol{\tau}$, similarly defined, is a vector containing the time-delays. In order to make the problem tractable, we condition the likelihood function on the transmitted bits and treat the fading processes as unknown deterministic parameters. We also also assume that the estimated parameters at one cluster are jointly estimated using only the received signals at that particular cluster. The resulting CRB can be interpreted as the CRB conditioned on the fading processes for estimators that are given knowledge of $z(m)$.

Theorem 1: Under the assumptions stated, the CRB for the parameters vector $\boldsymbol{\eta} = [\boldsymbol{\theta}^T, \boldsymbol{\tau}^T]^T$ can be written as

$$\mathbf{CRB}(\boldsymbol{\eta}) = \frac{\sigma^2}{2}\Big\{\sum_{m-1}^{M}\mathrm{Re}[\mathbf{E}^H(m)\mathbf{F}^H \times[\mathbf{I}_{PQN} - \mathbf{\Lambda}\mathbf{Z}(m)\mathbf{\Xi}^{-1}(m)\mathbf{Z}^H(m)\mathbf{\Lambda}^H]\mathbf{F}\mathbf{E}(m)]\Big\}^{-1} \quad (8)$$

where the matrices $\mathbf{\Xi}(m)$, $\mathbf{E}(m)$, $\mathbf{F}$ and $\mathbf{Z}(m)$ are defined as

$$\begin{aligned}
\mathbf{\Xi}(m) &= \mathbf{Z}^H(m)\mathbf{\Lambda}^H\mathbf{\Lambda}\mathbf{Z}(m) && (9)\\
\mathbf{E}(m) &= \mathrm{diag}\big(\mathbf{D}(m), \mathbf{\Gamma}(m)\big) && (10)\\
\mathbf{F} &= \mathbf{\Lambda}\Psi && (11)\\
\mathbf{Z}(m) &= \mathrm{diag}(\mathbf{Z}_1(m)\cdots\mathbf{Z}_C(m)) && (12)\\
\mathbf{Z}_c(m) &= [\mathbf{Z}_{c,1}^T(m)\cdots\mathbf{Z}_{c,P_c}^T(m)]^T \\
\mathbf{Z}_{c,p}(m) &= \mathrm{diag}(\mathbf{Z}_{c,p,1}(m)\cdots\mathbf{Z}_{c,p,K}(m))
\end{aligned}$$

and the matrices $\mathbf{D}(m)$ and $\mathbf{\Gamma}(m)$ are defined similarly to $\mathbf{Z}(m)$. The elements of $\mathbf{Z}_{c,p,k}(m)$, $\mathbf{D}_{c,p,k}(m)$ and $\mathbf{\Gamma}_{c,p,k}(m)$ are defined as

$$(\mathbf{Z}_{c,p,k}(m))(i,j) = \begin{cases} w_{c,p}(\theta_{c,k,j})z_{2k-1}(m)) & ,i=2j-1 \\ w_{c,p}(\theta_{c,k,j})z_{2k}(m)) & ,i=2j \\ 0 & ,\text{otherwise} \end{cases}$$

$$(\mathbf{D}_{c,p,k}(m))(i,j) = \begin{cases} \frac{\partial w_{c,p}}{\partial\theta_{c,k,j}}\beta_{c,k,j}z_{2k-1}(m)) & ,i=2j-1 \\ \frac{\partial w_{c,p}}{\partial\theta_{c,k,j}}\beta_{c,k,j}z_{2k}(m)) & ,i=2j \\ 0 & ,\text{otherwise} \end{cases}$$

$$(\mathbf{\Gamma}_{c,p,k}(m))(i,j) = \begin{cases} s_{c,p,k,i}(m) & ,i\in\{2j-1,2j\} \\ 0 & ,\text{otherwise} \end{cases}$$

[2] $\mathrm{diag}(\mathbf{X}_1\cdots\mathbf{X}_K)$ denotes the $K\times K$ block diagonal matrix whose kth block element is $\mathbf{X}_k$. On the other hand, diag($\mathbf{X}$) is a diagonal matrix of same dimension and diagonal as $\mathbf{X}$, i.e. $(\mathrm{diag}(\mathbf{X}))(k,k) = \mathbf{X}(k,k)$.

[3] $\mathbf{A}\otimes\mathbf{B}$ denotes the Kronecker matrix product, i.e. $\mathbf{A}\otimes\mathbf{B}$ is a block matrix so that the (i,j)th block of $\mathbf{A}\otimes\mathbf{B}$ is $A_{ij}\mathbf{B}$.

and Ψ is defined as

$$\begin{aligned}\Psi &= \mathrm{diag}(\mathbf{I}_{P_1}\otimes\boldsymbol{\psi}_1\cdots\mathbf{I}_{P_C}\otimes\boldsymbol{\psi}_C) \qquad (13)\\ \boldsymbol{\psi}_c &= [\boldsymbol{\psi}_{c,1,1}\cdots\boldsymbol{\psi}_{c,1,2R_{c,1}}\cdots\boldsymbol{\psi}_{c,K,1}\cdots\boldsymbol{\psi}_{c,K,2R_{c,K}}]\\ \boldsymbol{\psi}_{c,k,2r-1} &= \frac{\partial \mathbf{a}_{c,k,2r-1}}{\partial\tau_{c.k.r}} = \frac{1}{T_i}\left[\mathbf{D}_1^{p_{c,k,r}+1}-\mathbf{D}_1^{p_{c,k,r}}\right]\mathbf{c}_k\\ \boldsymbol{\psi}_{c,k,2r} &= \frac{\partial \mathbf{a}_{c,k,2r}}{\partial\tau_{c.k.r}} = \frac{1}{T_i}\left[\mathbf{D}_{-1}^{p_{c,k,r}+1}-\mathbf{D}_{-1}^{p_{c,k,r}}\right]\mathbf{c}_k.\end{aligned}$$

Proof: See [8].

It is interesting to note that the CRB for $\boldsymbol{\eta}$ is independent of the near-far problem, i.e. the CRB for user k does not depend on the received powers of the $K-1$ other users. This fact can be easily verified using the same procedure as in [7, Appendix A].

4. CRB FOR THE LOCATION ESTIMATION

We define the position parameter vector $\mathbf{p}$ as

$$\mathbf{p} = [\mathbf{b}_1^T\cdots\mathbf{b}_C^T,\boldsymbol{\rho}^T]^T \qquad (14)$$

where

$$\begin{aligned}\boldsymbol{\rho} &= [\mathbf{u}_1^T\cdots\mathbf{u}_K^T,\mathbf{v}_1^T\cdots\mathbf{v}_C^T]^T\\ \mathbf{v}_c &= [\mathbf{v}_{c,1,2}^T\cdots\mathbf{v}_{c,1,R_{c,1}}^T,\cdots,\mathbf{v}_{c,K,2}^T\cdots\mathbf{v}_{c,K,R_{c,K}}^T]^T\end{aligned}$$

and $\mathbf{b}_c = [b_{c,x},b_{c,y}]^T$ denotes the *known* two-dimensional position in Cartesian coordinates for the cth cluster (or cth base station) and $\mathbf{u}_k = [u_{k,x},u_{k,y}]^T$ and $\mathbf{v}_{c,k,r} = [v_{c,k,r,x},v_{c,k,r,y}]^T$ denote respectively the *unknown* two-dimensional position for the kth user, and for the reflector of the rth path emanating from the kth user and impinging on the cth cluster. Without loss of generality, we assume that the path numbered $r=1$, is always a line of sight (LOS) path. We note that we can express the bearing and time-delay elements of the vector $\boldsymbol{\eta}$ as a function of the elements of the position parameter vector $\boldsymbol{\rho}$ as

$$\theta_{c,k,r} = \begin{cases}\tan^{-1}\left(\frac{u_{k,y}-b_{c,y}}{u_{k,x}-b_{c,x}}\right) & ,r=1\\ \tan^{-1}\left(\frac{v_{c,k,r,y}-b_{c,y}}{v_{c,k,r,x}-b_{c,x}}\right) & ,r\neq 1\end{cases} \qquad (15)$$

$$\tau_{c,k,r} = \begin{cases}\frac{\|\mathbf{u}_k-\mathbf{b}_c\|}{\upsilon} & ,r=1\\ \frac{\|\mathbf{u}_k-\mathbf{v}_{c,k,r}\|+\|\mathbf{v}_{c,k,r}-\mathbf{b}_c\|}{\upsilon} & ,r\neq 1\end{cases} \qquad (16)$$

where υ denotes the speed of light and $\|\mathbf{x}\| = (\mathbf{x^Tx})^{\mathbf{1/2}}$ denotes the Euclidean norm of $\mathbf{x}$.

Assuming that the positioning system uses the subset $\breve{\boldsymbol{\eta}}\subseteq\boldsymbol{\eta}$ of estimated AOA and TD parameters to estimate the users' positions, we can express the CRB for $\boldsymbol{\rho}$ as a function of the CRB for $\boldsymbol{\eta}$, as (see [8, Proposition 1])

$$\mathbf{CRB}(\boldsymbol{\rho}) = \left(\mathbf{H}\{\mathbf{C\breve{R}B}(\breve{\boldsymbol{\eta}})\}^{-1}\mathbf{H}^T\right)^{-1} \qquad (17)$$

where $\mathbf{C\breve{R}B}(\breve{\boldsymbol{\eta}})$ is the matrix obtained by deleting the rows and columns of (8) that correspond to the subset of parameters that are not used by the positioning system to estimate $\boldsymbol{\rho}$, i.e. $\boldsymbol{\omega} = \{\boldsymbol{\eta}\}-\{\breve{\boldsymbol{\eta}}\}$, and $\mathbf{H}$ is a transformation matrix whose elements are defined as

$$(\mathbf{H})(i,j) = \{\partial\breve{\eta}_j/\partial\rho_i\}\,. \qquad (18)$$

Because we are limited in space, we will only express the matrix $\mathbf{H}$ for the special case when there is no multipath propagations (i.e. when $R_{c,k}=1$ and $\mathbf{v}=\emptyset$), and when all estimated LOS signals parameters are used to estimate the positions (i.e. when $\breve{\boldsymbol{\eta}}=\boldsymbol{\eta}$). Under that assumption, we can formulate the matrix $\mathbf{H}$ (after some straightforward differentiations) as

$$\mathbf{H} = [\mathbf{M}_1\cdots\mathbf{M}_C,\mathbf{N}_1\cdots\mathbf{N}_C] \qquad (19)$$

where the elements of $\mathbf{M}_c\in\mathbb{R}^{2K\times K}$ and $\mathbf{N}_c\in\mathbb{R}^{2K\times K}$ are defined as

$$(\mathbf{M}_c)(i,j) = \begin{cases}\frac{b_{c,y}-u_{j,y}}{\|\mathbf{u}_j-\mathbf{b}_c\|^2} & ,i=2j-1\\ \frac{u_{j,x}-b_{c,x}}{\|\mathbf{u}_j-\mathbf{b}_c\|^2} & ,i=2j\\ 0 & ,\text{otherwise}\end{cases} \qquad (20)$$

$$(\mathbf{N}_c)(i,j) = \begin{cases}\frac{1}{\upsilon}\frac{u_{j,x}-b_{c,x}}{\|\mathbf{u}_j-\mathbf{b}_c\|} & ,i=2j-1\\ \frac{1}{\upsilon}\frac{u_{j,y}-b_{c,y}}{\|\mathbf{u}_j-\mathbf{b}_c\|} & ,i=2j\\ 0 & ,\text{otherwise}\end{cases} \qquad (21)$$

Note that the transformation matrix $\mathbf{H}$ when only a subset $\breve{\boldsymbol{\eta}}$ of the LOS parameters $\boldsymbol{\eta}$ is used is readily obtained by deleting the columns of (19) corresponding to the unused parameters of $\boldsymbol{\eta}$.

5. APPLICATION EXAMPLE

As an application example, we will assess the achievable positioning accuracy for a 4-user asynchronous DS-CDMA cellular system with 2 adjacent hexagonal cells of radius $R=2887$ meters. Each cell receives the signals from 2 in-cell and 2 out-of-cell users, and each user is assigned a different Gold code sequence of length 31. Perfect power control is assumed, i.e. the received power from the users is unity at the serving base stations, and the signal-to-noise ratio (SNR), defined as the received symbol energy over the two-sided power spectral density of noise $N_o/2$, is 15 dB at the serving base stations. For simplicity, we did not simulate multipath propagations and oversampling was not used, i.e. the sampling rate was equal to the chip rate .

Because the CRB for the time-delay and bearing estimation depends on the users' locations and assumes perfectly known symbol sequences and unknown deterministic fading processes, 2×10^3 independent Monte-Carlo trials were conducted to average the CRB for the locations over the positions, data, and fading processes. In order to generate randomly the users' positions, the hexagonal cells were approximated with circles of radius $R=$ 2887m, and we assumed a uniform density of users within the circular cells. The fading processes $\{\zeta_{c,k,r}(mT)\}$, normalized to have unit amplitude, were independent standard Raleigh fading processes with a Doppler frequency of 83.2 Hz (corresponding to a carrier frequency of 900 MHz, data rate of 32 kHz, and a transmitter-receiver relative speed of 100km/h). Since the time-delay $\tau_{c,k,r}$ is proportional to the distance traveled by the rth ray coming from the kth user and impinging on the cth cluster, we calculated the received amplitude for the kth user serviced by the dth cell as

$$\alpha_{c,k,r}\sqrt{\mathrm{P}_k} = \left(\frac{\tau_{d,k,1}}{\tau_{c,k,r}}\right)^{n/2} \qquad (22)$$

where $n=4$ is a path-loss exponent corresponding to a shadowed urban cellular radio environment. The received vector $\mathbf{r}(m)$ was observed for M = 10 symbols. We conducted this experiment for clusters (or base stations) equipped with either a single

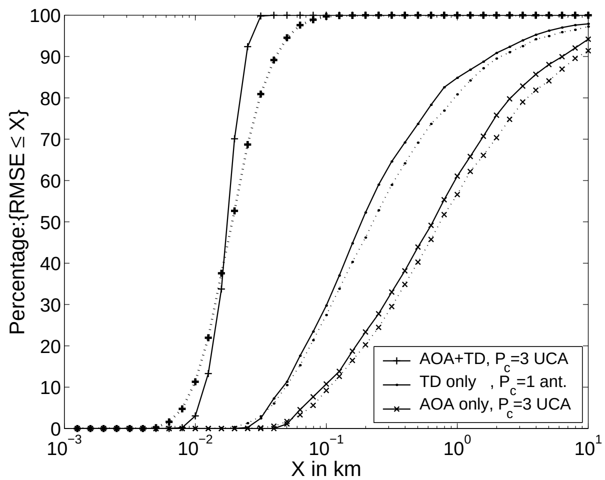
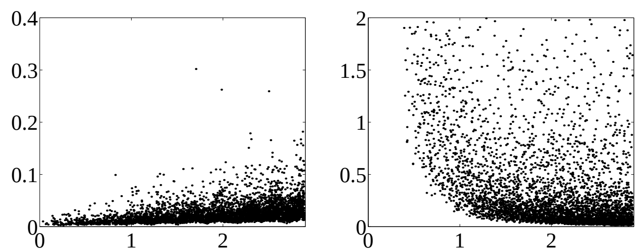


Fig. 1. percentage of events $\{$position RMSE $\leq$ threshold $X\}$ for 3 different methods with fading (dashed line) and without fading (continuous line).



(a) AOA+TD, with fading. (b) TD only, with fading.

Fig. 2. Position RMSE outcomes plotted versus the separation distance between the mobile and its serving base station (in km).

omni-directional antenna, or with a 3-element uniform circular array (UCA) having a diameter of 2λ, and with and without path fading simulations.

Fig.1 shows the cumulative distribution function (CDF) for the position root-mean-square-error (RMSE) events obtained for 3 different positioning methods. The RMSE was calculated as RMSE $= \sqrt{\sigma_x^2 + \sigma_y^2}$, where σ_x^2 and σ_y^2 are the location error variances calculated using (17). Clearly, a hybrid technique using both the bearing and the time-delay measurements gives the lowest positioning error among the three methods considered. For the two other methods, we note that a smaller RMSE can be achieved using a single antenna to measure the time-delays rather than using a 3-element UCA to measure the bearings.

On Fig.2, we plotted the position RMSE outcomes versus their corresponding distances between mobile and serving base station. Except for the scaling, the plot for the AOA method was similar to Fig.2(b), which clearly illustrates the near-far effect. This effect arises because the users' power is turned down to the minimum level by their serving base stations in order to maximize capacity, resulting in an insufficient signal to interference ratio (SIR) at the neighboring base stations for accurate time-delay or bearing estimation. Fig.2(a) shows how that near-far effect can be effectively combated using antenna arrays, allowing the positioning system to estimate the mobile's position using only the signals received at the serving base station. Also apparent on Fig.2 is the spreading of the position RMSE outcomes due to the fading processes.

6. CONCLUSION

In this paper, we have developed the CRB for the location estimation of wireless subscribers in an asynchronous DS-CDMA system operating over fading channels, using the signals that are already transmitted by the cell phones and received at one or more base station(s). This CRB, which does not assume any particular trilateration model, can be used to assess the achievable positioning accuracy for a given cellular system, or as a design tool to find an optimum antenna placement. Another possible application is to evaluate if a given cellular system geometry can fulfill positioning requirements, such as for the E-911 services.

One limitation of this paper is that we assumed (for mathematical tractability) a DS-CDMA system using symbol-periodic spreading codes. A second limitation is the assumption that the receiver or positioning system uses bearings and/or time-delays (or round-trip delays) estimation. In practice, time difference of arrival (TDOA) measurements in between different base stations (or antenna clusters) are often employed to eliminate the need for accurate time reference at the mobile. Our future work will relieve the last limitation and also investigate other system geometries.

Finally, we note that the same approach could be used to find the CRB on the positioning accuracy for other cellular systems or standards than CDMA, using the appropriate CRB for time-delay and bearing estimation.

7. REFERENCES

[1] "FCC adopts rules to implement Enhanced 911 for wireless services," *FCC News*, CC docket No. 94-102, June 12,1996.

[2] H. Koshima, J. Hoshen,"Personal Locator Services Emerge," *IEEE Spectrum Mag.*, vol. 37, no. 2, pp. 41–48, Feb. 2000.

[3] P.Stoica, and A. Nehorai, "MUSIC, Maximum Likelihood, and Cramer-Rao Bound," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, no. 5, pp. 720–741, May 1989.

[4] M. Wax, A. Leshem, "Joint Estimation of Time Delays and Directions of Arrival of Multiple Reflections of a known Signal," *IEEE Trans. Signal Processing*, vol. 45, no. 10, pp. 2477–2484, Oct. 1997.

[5] M. C. Vanderveen, A. van der Veen, A. Paulraj, "Estimation of Multipath Parameters in Wireless Communications," *IEEE Trans. Signal Processing*, vol. 46, no. 3, pp. 682–690, Mar. 1998.

[6] E. G. Ström, S. Parkvall, S. L. Miller, and B. E. Ottersten, "DS-CDMA Synchronization in Time-Varying Fading Channels," *IEEE Journal on Selected Areas in Commu.*, vol. 14, no. 8, pp. 1636–1642, Oct. 1996.

[7] E. G. Ström, and F. Malmsten, "A Maximum Likelihood Approach for estimating DS-CDMA Multipath Fading Channels," *IEEE Journal on Selected Areas in Commu.*, vol. 18, no. 1, pp. 132–140, Jan. 2000.

[8] C. Botteron, A. Høst-Madsen, M. Fattouche, "Cramer-Rao Bound for the Estimation of Multipath Parameters and Mobiles' Positions in Asynchronous DS-CDMA Systems," *IEEE Trans. Signal Processing*, submitted Jan. 2001.