IRREDUCIBLE FORM FOR AP ALGORITHM FOR DETECTING THE NUMBER OF COHERENT SIGNALS BASED ON THE MDL PRINCIPLE

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ABSTRACT

This paper presents an improvement of the Alternating Projection (AP) algorithm for detecting the number of coherent signals based on the Minimum Description Length (MDL) principle using a uniform linear array of sensors. The criterion of the AP algorithm for the detection becomes indefinite, when estimated bearings more than one approach to the identical value. This paper derives an irreducible form of the AP criterion for the detection, which never get indefinite. The irreducible form is represented as a rational function and real-valued version of FFT can be exploited efficiently. The proposed algorithm reduces the order of the amount of arithmetic operations. Finally, simulation results are shown to demonstrate the validity of the proposed algorithm.

1. INTRODUCTION

In many scenes of passive sensor array processing, the detection of the number of signals impinging on the array is one of the most important problems, especially in the the super resolution techniques for bearing estimation such as, MUSIC [1] and ML [2, 3, 4]

Wax and Kailath [5] have proposed an approach to this problem, based on the information theoretic criteria such as the AIC and the MDL (Minimum Description Length). This approach is applicable to the case of incoherent signals, but not to the case of the coherent signals. Wax and Ziskind [6] have proposed another approach for detecting the number of coherent signals based on the MDL principle. The signal model of this approach includes bearing parameter to be determined so that the MDL cost function should be minimized. This minimizing process takes huge computational cost because of multivariate optimization problem involved. Suzuki, et al. [7] have proposed an efficient algorithm for the approach in [6]. We shall call it the Alternating Projection (AP) algorithm since it efficiently utilizes a projection operation as well as the AP algorithm for maximum bearing estimation [2].

This paper presents an improvement of the AP algorithm for detecting the number of coherent signals based on the MDL principle using a uniform linear array of sensors. The criterion of the AP algorithm for the detection becomes indefinite, when estimated bearings more than one approach to the identical value. This paper derives an irreducible form of the AP criterion, which never get indefinite. The irreducible form is represented as a rational function and real-valued version of FFT can be exploited efficiently. The proposed algorithm reduces the order of the amount of arithmetic operations. Finally, simulation results are shown to demonstrate the validity of the proposed improvement.

2. FORMULATION

Consider an array composed of p sensors with arbitrary known locations and arbitrary known directional characteristics, and assume that q narrow-band signals, centered around a known frequency, ω_0, impinge on the array from locations θ_1, θ_2, ..., θ_q.

The complex-valued p-dimensional vector at the n-th snapshot of the complex envelopes for the observation from the array are expressed as

$$\boldsymbol{x}_n = \sum_{i=1}^{q} \boldsymbol{a}(\theta_i) s_{i,n} + \boldsymbol{n}_n, \qquad n = 1, 2, ..., N \tag{1}$$

where $\boldsymbol{a}(\theta)$ is the “steering vector” determined by the array configuration. $s_{i,n}$ is a received signal from the i-th source at the reference point, i.e., the position of the first sensor, and $\boldsymbol{n}_n$ is the complex-valued p-dimensional vector consisting of additive noises at sensors.

The detection problem can be stated as the problem to select the number q form the possible variation { 0, 1, ..., $p-1$ } when given the N snapshots $\{\boldsymbol{x}_n\}$.

To solve the problem, we make the following assumptions.

A1) Any subset of p steering vectors from the array manifold is linearly independent. This required to guarantee the uniqueness of the solution of bearings.

A2) the noises $\boldsymbol{n}_n$ are independent and identically distributed complex zero-mean, Gaussian vectors with variance matrix $\sigma^2\boldsymbol{I}$, where σ is unknown scalar and $\boldsymbol{I}$ is an identity matrix,

The MDL of the model with the signal number q for the observed data $\boldsymbol{x}_n$, $n = 1$, 2, ..., N, is given as [6]

$$\mathrm{MDL}(q) = N(p-q)\ln L^{(p,q)}(\hat{\Theta}^{(q)}) + \nu(p,q,N) \tag{2}$$

where

$$\nu(p,q,N) = \frac{1}{2} q(2p-q+1)\log N \tag{3}$$

$$\Theta^{(q)} = \{ \ \theta_1 \ \ \theta_2 \ \ \cdots \ \ \theta_q \ \} \tag{4}$$

$$\hat{\Theta}^{(q)} = \arg\min_{\Theta^{(q)}} L^{(p,q)}(\Theta^{(q)}) \tag{5}$$

$$L^{(p,q)}(\Theta^{(q)}) = \frac{\dfrac{1}{p-q}\displaystyle\sum_{k=1}^{p-q}\lambda_k}{\left(\displaystyle\prod_{k=1}^{p-q}\lambda_k\right)^{1/(p-q)}} \tag{6}$$

λ_k, $k = 1, 2, \cdots, p-q$ are non-zero eigenvalues of the matrix

$$\boldsymbol{P}_A^{\perp}(\Theta^{(q)})\boldsymbol{R}\boldsymbol{P}_A^{\perp}(\Theta^{(q)}) \tag{7}$$

where

$$\boldsymbol{R} = \sum_{n=1}^{N} \boldsymbol{x}_n \boldsymbol{x}_n^H \tag{8}$$

$$\boldsymbol{A}(\Theta^{(q)}) = [\ \boldsymbol{a}(\theta_1)\ \ \boldsymbol{a}(\theta_2)\ \ \cdots\ \ \boldsymbol{a}(\theta_q)\ ] \tag{9}$$

$$\boldsymbol{P}_A(\Theta^{(q)}) = \boldsymbol{A}(\Theta^{(q)})\{\boldsymbol{A}^H(\Theta^{(q)})\boldsymbol{A}(\Theta^{(q)})\}^{-1}\boldsymbol{A}^H(\Theta^{(q)}) \tag{10}$$

$$\boldsymbol{P}_A^{\perp}(\Theta^{(q)}) = \boldsymbol{I} - \boldsymbol{P}_A(\Theta^{(q)}) \tag{11}$$

where the superscript H indicates the Herimitian transpose of a matrix.

The number of signals is determined as the value of q which minimizes MDL(q) in (2).

3. ALGORITHMS

The alternating minimization (AM) algorithm is a simple iterative technique for multidimensional minimization. At every iteration a minimization is carried out with respect to a single parameter while all the other parameters are held fixed. The bearing parameters $\Theta^{(q)}$ minimizing $L^{(p,q)}(\Theta)$ may be obtained as follows.

[Initialization Phase]

First assuming a single source, $q = 1$, find θ_1 minimizing $L^{(p,1)}(\Theta^{(1)})$. Next, assuming two sources, $q = 2$, and fixing θ_1 at the value obtained for the single source, find θ_2 minimizing $L^{(p,2)}(\Theta^{(2)})$. Continue in this fashion until all the initial values for θ_k, $k = 1, 2, ..., q$ are computed.

[Convergence Phase]

Repeat the following updating process until all parameters are converged. At each updating process, find the value of one parameter, say θ_k, which minimizes the criterion $L^{(p,q)}(\Theta^{(q)})$ while all other parameters are held fixed, and then change the index k of the parameter to be updated into $(k \mod q) + 1$.

From now on, we concentrate on the determination of bearing parameters $\Theta^{(q)}$ which minimize $L^{(p,q)}(\Theta^{(q)})$ in (6) and the superscripts (p, q) and (q) will be omitted.

3.1. Alternating Minimization (AM) Algorithm

Let $\boldsymbol{V}_N(\Theta)$ be a $p\times(p-q)$ matrix of which $(p-q)$ columns make up an orthonormal system of the noise subspace that is the orthogonal complement of the signal subspace span$\{\boldsymbol{A}(\Theta)\}$ = span$\{\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_q)\}$. $\boldsymbol{V}_N(\Theta)$ can be evaluated by applying Schmidt's orthogonalization to the columns of $\boldsymbol{P}_A^{\perp}(\Theta)$.

$$L(\Theta) = \frac{\dfrac{1}{p-q}\mathrm{tr}\{\boldsymbol{R}_N(\Theta)\}}{(\det\{\boldsymbol{R}_N(\Theta)\})^{1/(p-q)}} \tag{12}$$

$$\boldsymbol{R}_N(\Theta) = \boldsymbol{V}_N^H(\Theta)\boldsymbol{R}\boldsymbol{V}_N(\Theta) \tag{13}$$

At each step in each iteration with respect to a single parameter $L(\Theta)$ in (12) has to be evaluated. It requires $O((p-q)p^2)$ multiplications.

3.2. Alternating Projection (AP) Algorithm

The Alternating Projection (AP) algorithm [7] for the problem of minimizing $L(\Theta)$ in (12) is summarized as follows. Define

$$\Theta_k = \{\ \theta_1\ \ \theta_2\ \ \cdots\ \ \theta_{k-1}\ \ \theta_{k+1} \cdots\ \ \theta_q\ \} \tag{14}$$

$$\boldsymbol{A}_k = [\ \boldsymbol{a}(\theta_1)\ \cdots \boldsymbol{a}(\theta_{k-1})\ \boldsymbol{a}(\theta_{k+1})\ \cdots\ \boldsymbol{a}(\theta_q)\ ] \tag{15}$$

$$\boldsymbol{P}_{A_k} = \boldsymbol{A}_k\{\boldsymbol{A}_k^H\boldsymbol{A}_k\}^{-1}\boldsymbol{A}_k^H \tag{16}$$

$$\boldsymbol{P}_{A_k}^{\perp} = \boldsymbol{I} - \boldsymbol{P}_{A_k} \tag{17}$$

Entries in Θ_k are fixed parameters and θ_k is variable. Let $\boldsymbol{V}_{N_k}$ be a $p \times (p-q+1)$ matrix of which $(p-q+1)$ columns compose an orthonormal system of the orthogonal complement of the subspace span$\{\boldsymbol{A}_k\}$. $\boldsymbol{V}_{N_k}$ can be evaluated by applying Schmidt's orthogonalization to the columns of $\boldsymbol{P}_{A_k}$. Define

$$\boldsymbol{R}_{N_k} = \boldsymbol{V}_{N_k}^H\boldsymbol{R}\boldsymbol{V}_{N_k} \tag{18}$$

Then we have

$$\begin{aligned} D_k(\theta_k) &= \det\{\boldsymbol{R}_N(\Theta)\} \\ &= \det\{\boldsymbol{R}_{N_k}\}\frac{\boldsymbol{a}^H(\theta_k)\boldsymbol{V}_{N_k}\boldsymbol{R}_{N_k}^{-1}\boldsymbol{V}_{N_k}^H\boldsymbol{a}(\theta_k)}{\boldsymbol{a}^H(\theta_k)\boldsymbol{V}_{N_k}\boldsymbol{V}_{N_k}^H\boldsymbol{a}(\theta_k)} \end{aligned} \tag{19}$$

$$\begin{aligned} T_k(\theta_k) &= \mathrm{tr}\{\boldsymbol{R}_N(\Theta)\} \\ &= \mathrm{tr}\{\boldsymbol{R}_{N_k}\} - \frac{\boldsymbol{a}^H(\theta_k)\boldsymbol{V}_{N_k}\boldsymbol{R}_{N_k}\boldsymbol{V}_{N_k}^H\boldsymbol{a}(\theta_k)}{\boldsymbol{a}^H(\theta_k)\boldsymbol{V}_{N_k}\boldsymbol{V}_{N_k}^H\boldsymbol{a}(\theta_k)} \end{aligned} \tag{20}$$

$$L_k(\theta_k) = \frac{\dfrac{1}{p-q}T_k(\theta_k)}{D_k^{1/(p-q)}(\theta_k)} \tag{21}$$

$\boldsymbol{V}_{N_k}$, $\boldsymbol{R}_{N_k}$, $\boldsymbol{R}_{N_k}^{-1}$, det$\{\boldsymbol{R}_{N_k}\}$ and tr$\{\boldsymbol{R}_{N_k}\}$ are evaluated before starting the iteration with respect to the single parameter, θ_k. Therefore at each step in each iteration, only the Hermitian forms in (19) and (20) are evaluated. It requires $O((p-q+1)^2)$ multiplications.

3.3. Irreducible Form of AP Algorithm (API)

When θ_k in (19) approaches to one of the values of the fixed parameters θ_i, $i \neq k$, both of the numerator and the denominator of $D_k(\theta_k)$ vanish, and then $D_k(\theta_k)$ becomes indefinite. The same thing occurs for $T_k(\theta_k)$. This kind of numerical difficulties must be avoid.

The steering vector of a uniform linear array is represented as

$$\boldsymbol{a}(\theta) = [1\ \ e^{-j\theta}\ \ e^{-2j\theta}\ \ ...\ \ e^{-j(p-1)\theta}]^T \tag{22}$$

where the superscripts T indicates the transpose of a matrix. θ is the difference in phase between adjacent sensors.

Define the polynomial $W_k(z)$ having zeros at $z = e^{j\theta_i}$, $i = 1, ..., k-1, k+1, ..., q$

$$W_k(z) = \prod_{\substack{i=1 \\ i\neq k}}^{q}(z - e^{j\theta_i}) = w_0 + w_1 z + \cdots + w_{q-1}z^{q-1} \tag{23}$$

and also define the following matrix using the coefficients of $W_k(z)$ as

$$\boldsymbol{W}_k = \begin{array}{c} \overbrace{\hphantom{w_{q-1} \quad w_{q-2} \quad \cdots \quad w_{q-1}}}^{p-q+1} \\ \left.\begin{bmatrix} w_0 & 0 & \cdots & 0 \\ w_1 & w_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ w_{q-1} & w_{q-2} & \ddots & w_0 \\ 0 & w_{q-1} & \ddots & w_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{q-1} \end{bmatrix}\right\} p \end{array} \tag{24}$$

Then we have

$$\begin{aligned} \boldsymbol{a}^H(\theta)\boldsymbol{W}_k &= [\, W_k(z)\ zW_k(z)\ \cdots\ x^{p-q}W_k(z)\,]\big|_{z=e^{j\theta}} \\ &= W_k(z)\boldsymbol{\zeta}_{p-q}(z)\big|_{z=e^{j\theta}} \end{aligned} \tag{25}$$

$$\boldsymbol{\zeta}_{p-q}(z) = [\ 1\ \ z\ \ \cdots\ \ z^{p-q}\ ] \tag{26}$$

and

$$\boldsymbol{a}^H(\theta_i)\boldsymbol{W}_k = \boldsymbol{0} \quad \text{for } i = 1, 2, \cdots, k-1, k+1, \cdots, q \tag{27}$$

Eqn (27) indicates that all column vectors of $\boldsymbol{W}_k$ are orthogonal to $\boldsymbol{a}(\theta_i)$ for $i = 1, 2, \cdots, k-1, k+1, \cdots, q$. Therefore the orthonormal system of the orthogonal complement of the subspace span$\{\boldsymbol{A}_k\}$ can be obtained by applying Schmidt's orthogonalization to column vectors of $\boldsymbol{W}_k$.

$$\boldsymbol{W}_k = \boldsymbol{V}_{N_k}\boldsymbol{B}_k \tag{28}$$

where $\boldsymbol{B}_k$ is a lower or upper triangular matrix. The inversion $\boldsymbol{B}_k^{-1}$ can be computed efficiently by the forward substitution or the backward substitution.

$$\boldsymbol{V}_{N_k} = \boldsymbol{W}_k\boldsymbol{B}_k^{-1} \tag{29}$$

Substituting (29) into (19) and (20), a common factor, $W_k(z)W_k^*(1/z^*)$, appears in the numerators and the denominators, where z^* is the complex-conjugate of z. By canceling the common factor, irreducible forms of $D_k(\theta_k)$ and $T_k(\theta_k)$ are derived as

$$D_k(\theta) = \det\{\boldsymbol{R}_{N_k}\}\frac{\boldsymbol{\zeta}_{p-q*}(z)\boldsymbol{N}_k^{(det)}\boldsymbol{\zeta}_{p-q}(z)}{\boldsymbol{\zeta}_{p-q*}(z)\boldsymbol{D}_k\boldsymbol{\zeta}_{p-q}(z)}\Bigg|_{z=e^{j\theta}} \tag{30}$$

$$T_k(\theta) = \mathrm{tr}\{\boldsymbol{R}_{N_k}\} - \frac{\boldsymbol{\zeta}_{p-q*}(z)\boldsymbol{N}_k^{(tr)}\boldsymbol{\zeta}_{p-q}(z)}{\boldsymbol{\zeta}_{p-q*}(z)\boldsymbol{D}_k\boldsymbol{\zeta}_{p-q}(z)}\Bigg|_{z=e^{j\theta}} \tag{31}$$

where

$$\boldsymbol{N}_k^{(det)} = \boldsymbol{B}_k^{-1}\boldsymbol{R}_{N_k}^{-1}\boldsymbol{B}_k^{-1,H} = \{\nu_{i,j}^{(det)}\}_{i,j=0}^{p-q} \tag{32}$$

$$\boldsymbol{N}_k^{(tr)} = \boldsymbol{B}_k^{-1}\boldsymbol{R}_{N_k}\boldsymbol{B}_k^{-1,H} = \{\nu_{i,j}^{(tr)}\}_{i,j=0}^{p-q} \tag{33}$$

$$\boldsymbol{D}_k = \boldsymbol{B}_k^{-1}\boldsymbol{B}_k^{-1,H} = \{\delta_{i,j}\}_{i,j=0}^{p-q} \tag{34}$$

and $\boldsymbol{\zeta}_{p-q*}(z) = \boldsymbol{\zeta}_{p-q}^H(1/z^*)$.

Eqns.(30) and (31) can be written in the following forms.

$$D_k(\theta) = \det\{\boldsymbol{R}_{N_k}\}\frac{\mathcal{R}\{n_k^{(det)}(z)\}}{\mathcal{R}\{d_k(z)\}}\Bigg|_{z=e^{j\theta}} \tag{35}$$

$$T_k(\theta) = \mathrm{tr}\{\boldsymbol{R}_{N_k}\} - \frac{\mathcal{R}\{n_k^{(tr)}(z)\}}{\mathcal{R}\{d_k(z)\}}\Bigg|_{z=e^{j\theta}} \tag{36}$$

where $n_k^{(det)}(z)$, $n_k^{(tr)}(z)$ and $d_k(z)$ are polynomials given as

$$n_k^{(det)}(z) = n_0^{(det)} + n_1^{(det)}z + \cdots + n_{p-q}^{(det)}z^{p-q} \tag{37}$$

$$n_k^{(tr)}(z) = n_0^{(tr)} + n_1^{(tr)}z + \cdots + n_{p-q}^{(tr)}z^{p-q} \tag{38}$$

$$d_k(z) = d_0 + d_1 z + \cdots + d_{p-q}z^{p-q} \tag{39}$$

$$n_0^{(det)} = \sum_{i=0}^{p-q}\nu_{i,i}^{(det)} \quad n_m^{(det)} = 2\sum_{i=m}^{p-q}\nu_{i,i-m}^{(det)} \tag{40}$$

$$n_0^{(tr)} = \sum_{i=0}^{p-q}\nu_{i,i}^{(tr)} \quad n_m^{(tr)} = 2\sum_{i=m}^{p-q}\nu_{i,i-m}^{(tr)} \tag{41}$$

$$d_0 = \sum_{i=0}^{p-q}\delta_{i,i} \quad d_m = 2\sum_{i=m}^{p-q}\delta_{i,i-m} \tag{42}$$

$$\text{for } m = 1, 2, \cdots, p-q$$

and $\mathcal{R}\{\cdot\}$ indicates the real part of a complex number.

Since the irreducible form of the AP criterion represented as (21) using (35) and (36) does not have any indefinite point, the computation of the criterion using the irreducible form is always numerically stable.

Before starting the one-dimensional search with respect to the single parameter, θ_k, the polynomials, $n_k^{(det)}(z)$, $n_k^{(tr)}(z)$ and $d_k(z)$, are obtained. Its computational cost is not reduced compared with AP algorithm in 3.2. However, most of the computational cost in a whole algorithm is consumed for the iteration in the one-dimensional search. Using the irreducible form, each step in the one-dimensional search requires only the evaluation of the real parts of three polynomials and just few arithmetic operation. The amount of the arithmetic operations required is $O(p-q)$. The order of the computation cost is reduced.

Furthermore, in order to evaluate the real parts of the polynomials, the real-valued version of FFT can be applied. Let $\{z_n\}$ be a sequence of complex numbers

$$z_n = r_n + jx_n, \quad n = 0,\ 1,\ \ldots,\ M. \tag{43}$$

and let $\{Z_n\}$ be the Fourier coefficients obtained by applying the N-point FFT of $\{z_n\}$ filled with zero for $n = M+1, ..., N-1$,

$$Z_n = R_n + jX_n, \quad n = 0,\ 1,\ \ldots,\ N-1. \tag{44}$$

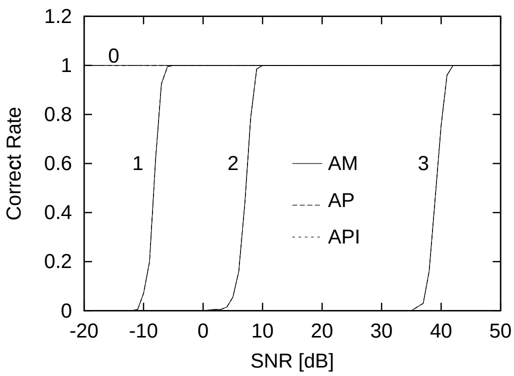
where $N = 2^r$ and $N > 2M$. Define a real-valued sequence $\{z_n^{(r)}\}$ as

$$z_0^{(r)} = r_0, \tag{45}$$

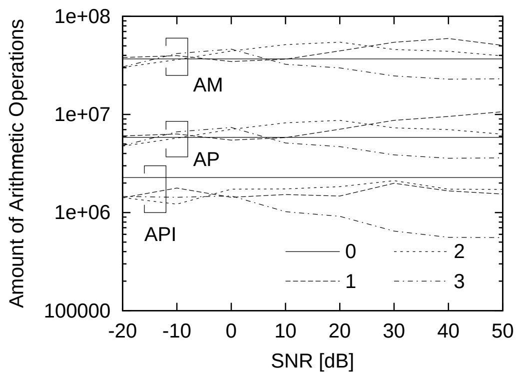
$$z_n^{(r)} = \frac{1}{2}\{r_n + x_n\}, \quad n = 1,\ 2,\ \ldots,\ M \tag{46}$$

$$z_{N-n}^{(r)} = \frac{1}{2}\{r_n - x_n\}, \quad n = 1,\ 2,\ \ldots,\ M \tag{47}$$

$$z_n^{(r)} = 0, \quad n = M+1,\ M+2,\ \ldots,\ N-M-1. \tag{48}$$



(a) Correct rate.



(b) Amount of arithmetic operations.

Fig. 1. Comparison of AM, AP and API algorithms.

Applying the real-valued version of FFT (RFFT) to $\{z_n^{(r)}\}$, we have the Fourier coefficients

$$Z_n^{(r)} = R_n^{(r)} + jX_n^{(r)}, \quad n = 0,\ 1,\ ...,\ N/2. \tag{49}$$

Then the real part $\{R_n\}$ can be obtained as

$$R_n = R_n^{(r)} - X_n^{(r)}, \quad n = 0,\ 1,\ ...,\ N/2, \tag{50}$$

$$R_{N-n} = R_n^{(r)} + X_n^{(r)}, \quad n = \ 1,\ ...,\ N/2-1. \tag{51}$$

To obtain $\{R_n\}$, $n = 0$, 1, ..., $N-1$ from $\{x_n\}$, $n = 0$, 1, ..., M, use of the N-point RFFT is more efficient than the ordinary complex-valued FFT. Furthermore, the N-point RFFT requires $N(\log_2 N - 3) + 4 + 2M$ real multiplications, while the direct evaluation of the real part of the polynomial of order M for the same N points requires $2NM$ real multiplications. The amount of multiplications in both of RFFT and the polynomial evaluation, the preparation of $e^{j2\pi t/M}$, $t = 0$, 1, ..., $M-1$ is excepted. The use of the RFFT has the advantage over the direct evaluation of polynomials, when M, i.e., $(p-q)$, is relatively large.

4. SIMULATIONS

Simulation results are shown in Fig. 1. The array is uniform and linear consisting of 4 sensors, $p = 4$, spaced half a wavelength apart. The k-th signal, $k = 1, 2, 3$, impinges from $8(k-1)^\circ$ on the sensor array. All signals are coherent. Simulations are performed when the signal number q is 0, 1, 2 and 3. When q is not 0, the first signal to q-th signal impinge on the sensor array.

200 snapshots are used for the detection of the signal number. In one-dimensional search, at first 512 points are searched for rough detection and then Fibonacci method is used until the precision achieves 1e-5° in phase. The correct rate of signal detection in (a) and the average of the amount of all arithmetic operations required for the detection in (b) are based on 200 independent trials. The Arabic numerals in Fig. 1 show the number of signals.

It is found From Fig. 1 (a) that AM, AP and API algorithm detect the number of coherent signals with the same correct rate. Fig. 1 (b) indicates that the proposed API algorithm requires less amount of arithmetic operations compared with AM and AP algorithms. It is less than about 1/20 of AM algorithm and 1/3 of AP algorithm.

5. CONCLUSIONS

This paper has presented an improvement of the AP algorithm for detecting the number of coherent signals based on the Minimum Description Length (MDL) principle using a uniform linear array of sensors. The criterion of the AP algorithm for the detection becomes indefinite, when estimated bearings more than one approach to the identical value. This paper has derived an irreducible form of the AP criterion to achieve numerical stability and to reduce the computational cost. Finally, simulation results have been shown to demonstrate the validity of the proposed improvement.

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