OPTIMIZED NEURAL NETWORKS FOR MODELING OF LOUDSPEAKER DIRECTIVITY DIAGRAMS

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ABSTRACT

For the electro-acoustical simulation of sound reinforcement systems, calculation and simulation of the sound field distribution requires measurement and storage of the frequency dependent directivity characteristics (level and phase) of the used loudspeaker models. In modern simulation programs, the spatial resolution can be less than five degrees in third – or even twelfth – octave frequency bands. Therefore, modeling of the directivity diagram of loudspeakers can reduce storage place and simulation time and may even increase the accuracy of the simulation.
Modeling – in the sense of mapping the resulting enormous amount of measured data – can be realized very efficiently and with small approximation error using second order neural networks. To reduce the model development time, we in addition created a new adaptation rule for feedforward neural networks with improved convergence behavior. This is achieved only by using the training data and the output error to analytically determine values for the learning parameters *momentum* and *learning rate* in each learning step.
We will show the advantages of using neural networks with optimized learning parameters by the example of modeling measured directional response patterns of two real loudspeakers. For measurement we used maximum length sequences (MLSSA).

1. NEURAL NETWORKS IN MODELING AND OPTIMIZATION

1.1. Introduction

Artificial neural networks are structures that are completely characterized by topology, activation function and adaptation rule. They are widely applied in research and sciences because of their interpolating or associative properties. Besides, increasing computing speed enables an effective use of the properties of higher order sigma pi neurons. They are not subject to the condition of linear separability, thus complex transformations can be realized by small network structures.
We use neural networks in the development of behavior models and use their interpolating properties. Here, we use a *multilayer feedforward network* to approximate multidimensional nonlinear continuous functions.
A loudspeakers directivity pattern, can be approximated very accurately by an appropriate network paradigm and adapted learning rule in a multilayer feedforward neural network with, for example, a backpropagation approach. This algorithm is based on the classical gradient–descent method. It approximates any continuous input–output mappings in a given tolerance range [1]. Although the directivity is measured at discrete angles and frequencies, the interpolating property of neural networks maps them into continuous functions. The mathematical description of the functions is then used during simulation to determine angle and frequency dependent levels and phases. The exactness of the approximation to the original terminal behavior is driven by the activation rule, the size of the neural network and the number of learning steps.

1.2. ACL – New adaptation rule for backpropagation based algorithms

We have improved the rate of convergence of the neural network adaptation by an evaluation of the learning parameters with respect to the output error.
Generalized delta–rule is applied to use interpolating and associative properties of multi–layered feedforward neural networks. Slow convergence is the major disadvantage of its direct application. Modification of the basic algorithm by using variable learning rate may lead to an improvement of the rate of convergence [3]. In some methods the rate of convergence depends on the initial values of the learning parameters, and in others they are simply estimated. The momentum of the adaptation rule is hereby either constant or not taken into consideration.

The fastest known learning rules which can be applied to Sigma–Pi Networks are SuperSAB, Quickprop and backpropagation based algorithms [6]. There, learning parameters depend on their initial value and are estimated at the beginning of the adaptation or in each learning step. An improvement of the convergence properties of the basic algorithm can only be achieved by a formal description of the learning dynamics. Their adaptation to the approximation error in each training step accelerates the gradient descent – and thus the rate of convergence. We have developed a new learning algorithm with improved convergence behavior, where the learning parameters are calculated in each learning

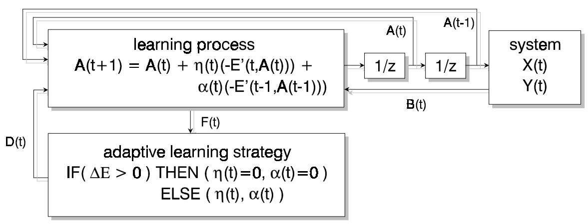

Figure 1: Strategy for the adaptive calculation of the learning parameters

and learning rate for each learning step for feedforward sigma–pi networks. This ACL (adaptive correction of learning parameters) backpropagation learning's convergence was compared to SuperSAB, Quickprop and extended backpropagation learning. Benchmark examples show a superiority of the ACL–BP concerning convergence behavior: a small overall error is yielded with far fewer iterations (factor 150 typically). The reason lies in a controlled increase of the learning parameter values with decreasing error, overstepping the interval $[0, 1]$.

The network learning dynamics can be described by the following equation [4]:

$$\begin{aligned}\mu_k(t)net_k^p(t) &= \mu_k(t)\sum_j w_{jk}(t)I_j^p(t)\\ &= \sum \mu_k(t)w_{jk}(t)I_j^p(t)\\ &= \sum_j z_{jk}(t)I_j^p(t)\end{aligned} \quad (1)$$

The net term of the activation function times the steepness parameters is written in the standard framework [4], where $\mathbf{I}^p$ is the input vector to the neuron k, and μ is the steepness parameter.

The change of z determines the network dynamics:

$$z_{jk}(t+1) = \mu_k(t+1)w_{jk}(t+1) \quad (2)$$

After some rearrangements we obtain

$$\begin{aligned}\Delta z_{jk}(t) = &\left(\frac{\partial E(t)}{\partial z_{jk}(t)}\right)^2 \eta^2(t)\mu_k(t)\frac{net_k(t)}{I_j(t)}\\ &+\left(-\frac{\partial E(t)}{\partial z_{jk}(t)}\right)\mu_k^2(t)\eta(t)\\ &+\left(-\frac{\partial E(t)}{\partial z_{jk}(t)}\right)\eta(t)\frac{net_k(t)}{I_j(t)}w_{jk}(t)\\ &+\left(-\frac{\partial E(t)}{\partial z_{jk}(t)}\right)\left(-\frac{\partial E(t-1)}{\partial z_{jk}(t-1)}\right)\\ &\qquad \eta(t)\alpha(t)\mu_k(t-1)\frac{net_k(t)}{I_j(t)}\\ &+\left(-\frac{\partial E(t-1)}{\partial z_{jk}(t-1)}\right)\mu_k(t)\alpha(t)\mu_k(t-1).\end{aligned} \quad (3)$$

Equation (3) demonstrates that only a calculation of optimal learning parameters, $\eta(t)$ and $\alpha(t)$, can increase the learning dynamics.

Figure 1 represents the analytical evaluation of the learning parameters. We apply the least square error criterion to the output layer and the generalized delta rule to the sigma–pi networks. This new adaptation rule links the system information $B(t)$ (including the pattern information and the approximation error) to the adaptive correction of the learning parameters $D(t)$, and the state vector of the neural network $A(t)$. The result is a new state vector, $A(t+1)$.

The initial values are chosen randomly and the patterns are measured, or given data, respectively. Therefore, an improvement of the learning convergence can only be achieved by an adaptive variation of the learning parameters. This is equivalent to a variation of the network information, $D(t)$. The learning parameters are determined by the approximation error of the last two training steps.

In the first step we apply a sigma–pi network assuming that the network parameters only change if this minimizes the approximation error

$$E(t, A(t+1)) < \gamma(t)E(t-1, A(t)), \quad (4)$$

with $E(t)$ approximation error, $A(t)$ state vector and $\gamma(t) \in]0;1[$ scaling parameter at step t, respectively. Thus equation (4) yields an upper limit of the learning parameters (η_{UL}, α_{UL}).

In the second step we use the knowledge of the boundary value condition [1] and the convergence condition

$$\left\|\widehat{A}(t+1)\right\|_F^2 \le \left\|\widehat{A}(t)\right\|_F^2 \quad (5)$$

$$\text{with} \qquad \widehat{A}(t) = A(t) - A$$
$$\text{and} \qquad A = \lim_{t\to\infty} A(t),$$

where $\|.\|_F$ is a Frobenius norm and A a limiting value of the state vector. After some calculations the equation (5) leads to

$$2 \cdot tr\left\{\widehat{A}(t)\,[\Delta A(t)]^T\right\} + \|\Delta A(t)\|_F^2 \leq 0 \qquad (6)$$
$$\text{with} \qquad \Delta A(t) = f(\eta, \alpha)$$

An interpretation of the equation (6) results in the curve tracking of a polynomial function. There the optimized values of the learning parameters can be calculated using this function in its local/global sub-zero extreme value. The only condition that should be fulfilled is that optimal learning parameters are smaller than their upper limits calculated from equation (4).

2. MODELING OF DIRECTIVITY DIAGRAMS

The above described ACL backpropagation learning was used to model the radiation directivity diagram of loudspeakers. The size of the neural network determines the amount of the resulting model equations and thus the simulation time. Therefore the input data, that are the measured frequency dependent values of levels and phases, are normalized. Activation rules adjusted to the characteristic curves are chosen in order to decrease the size of the neural network which is necessary for yielding an accurate approximation. A coding of the data with first and second order weights increases exactness in spatial representation without making storage or measurement of additional data necessary. Especially when level curves in adjoining frequency bands are similar, the information depth of the networks increases considerable.

The training set was received by measurement with maximum length sequences (MLSSA system, [2]). As examples, we modeled the polar pattern of the loudspeaker Yamaha MS 60 and the radiation pattern of the Fostex 6301 B. Both loudspeakers are widely used for monitoring in studios and in computer based sound editing. Although they are not typically for sound reinforcement systems, they can show the high accuracy of our modeling approach.

2.1. Polar diagram of Yamaha MS 60

Figure 2 shows the polar diagram of the loudspeaker Yamaha MS 60, measured with MLSSA, for different frequencies. We approximated the frequency dependent polar pattern of the Yamaha monitor using a neural network with one hidden layer, consisting of five neurons. The relative error is less than 2 %. This is within the accuracy of measurement. Figure 3 shows the approximated polar diagram for 1 kHz and the relative error versus the polar angle.

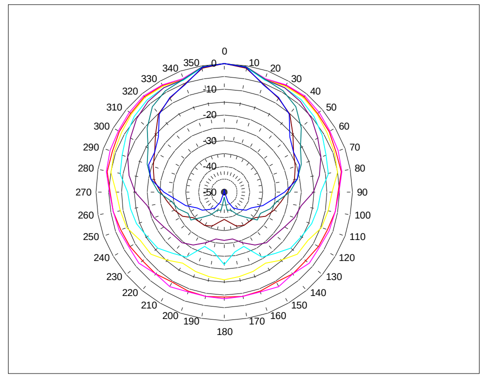

Figure 2: Polar diagram of Yamaha MS 60

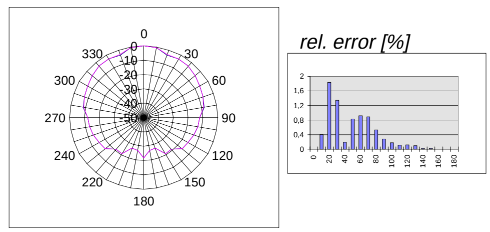

Figure 3: Yamaha MS 60: Approximation of the polar diagram at 1 kHz and relative error in percent versus polar angle

2.2. Radiation pattern of Fostex 6301 B

The spatial directivity pattern of the active loudspeaker Fostex 6301 B was measured in steps of 5 degrees with MLSSA. Figure 4 shows the waterfall diagram, and figure 5 shows the measured spatial data at 1 kHz, prepared for mapping by a neural network. For clearness, we show them in Cartesian coordinates.

The frequency dependent directivity pattern was measured for each spatial angle. For modeling, the data were classified into frequencies and were mapped by an neural network. It consists of one hidden layer with 40 neurons.

Figure 6 shows the angle–dependent relative error of the approximation. It is less than 2.5 %.

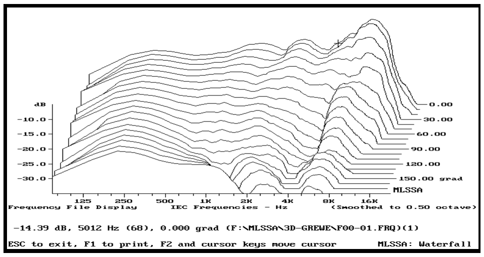

Figure 4: Waterfall diagram of Fostex 6301 B

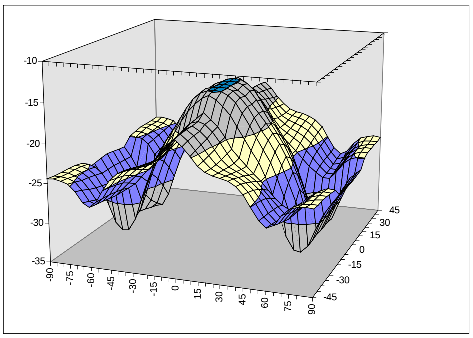

Figure 5: Fostex 6301 B: Measured directivity pattern at 1 kHz in Cartesian coordinates

3. RESULTS

The modeling of loudspeaker directivity pattern for electro-acoustical simulation can widely be done automatically owing to the application of neural networks: First the directivity pattern of the loudspeaker is measured. The measured data are then classified into frequencies and scaled or normalized. They represent the training set of the neural network. The adaptation of the network is continued until a predefined, minimal error is reached.

Finally, the network paradigm can automatically be transformed into equations for use in calculation of sound field distribution.

The proposed improvement of the generalized delta rule causes an acceleration of convergence speed. The learning parameters momentum and learning rate can now be *calculated* and thus optimized for each adaptation step. The presented ACL backpropagation learning was used in sigma-pi networks in the adaptation of loudspeaker directivity patterns. The new algorithm achieved a very high degree of accuracy in smaller number of training periods than other known algorithms – the typical improvement factor is 150.

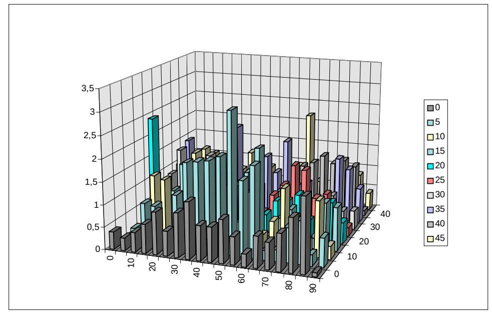

Figure 6: Fostex 6301 B: Relative error in % versus spatial angles, $f = 1$ kHz

We have used the presented method for the development of simulation models of loudspeakers directivity pattern for electro-acoustical simulation. The adapted representation of the measured directivity pattern is yielded accurately and with short development time by using neural networks.

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