DYNAMIC ENUMERATION ALGORITHM USING ARRAY ANTENNAS

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ABSTRACT

This paper develops a robust method to enumerate the incident signals impinging on a uniform but variable size linear array independant of their extent of correlation in a Rayleigh flat fading channel environment. The method also optimizes by minimizing the number of antennas to the number of signals and adapts continously to maintain optimum performance in a mobile environment where users (signals) come and go. The technique is a modification of the matrix decomposition method of Cozzens and Sousa. A new set of stability, stopping and adaptive control criteria is presented. An algorithm is formulated with simulation and field results presented.

1. INTRODUCTION

High resolution parameter estimation algorithms [1] used in modern array processing rely on *apriori* knowledge of the number of incident signals. Even recent work [2] on spatial diversity antennas for distinguishing cochannel signals by exploiting differences in the channel from each user requires the number of active users to be known.

The mobile communication environment is dynamic. The signals that arrive at the base station antenna will therefore consist of a mix of uncorrelated and correlated signals, all subjected to some degree of fading. The ability to detect and correctly enumerate the incident signals independent of their extent of correlation in a fading channel environment is crucial. This paper presents a robust algorithm that achieves this objective.

Information-theoretic techniques of [3, 4] work well in enumerating uncorrelated sources or sources subjected to independent flat fading channels, but fails completely when some or all of the signal sources are correlated.

The matrix decomposition method of Di [5] and the modified rank sequence method of Cozzens and Sousa [6] made it possible for enumeration algorithms to work in a correlated signal environment. These algorithms work for a fixed number of antennas and achieve good enumeration accuracy in an all white Gaussian noise environment.

This present work is based on the information-theoretic and modified rank sequence methods. These methods are then used with a new stopping criterion to develop a robust algorithm that will enumerature all incident signals independent of the extent of their correlation in a fading channel environment. The new algorithm also optimizes the number of antennas to the number of signals and adapts continuously to maintain optimum performance in a mobile environment where users (signals) come and go.

2. PROBLEM FORMULATON

Consider a linear antenna array composed of M sensors. Assume that K narrowband sources, centred around a known frequency, ω_0, impinge on the array from directions-of-arrival $\phi_1, \cdots, \phi_K$.

In matrix notation, the received signal $\boldsymbol{x}(t)$ is an $M \times 1$ complex vector given by

$$\boldsymbol{x}(t) = \boldsymbol{A}(\phi^{(K)})\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{A}(\phi^{(K)})$ is an $M \times K$ matrix of steering vectors such that

$$\boldsymbol{A}(\phi^{(K)}) = [\boldsymbol{a}(\phi_1)\boldsymbol{a}(\phi_2)\boldsymbol{a}(\phi_3)\cdots\boldsymbol{a}(\phi_K)] \tag{2}$$

with $\boldsymbol{a}(\phi_K)$ an $M \times 1$ complex vector characterized by unknown angle ϕ_K associated with the k^{th} signal and

$$\boldsymbol{s}(t) = [\boldsymbol{s_1}(t)\boldsymbol{s_2}(t)\cdots\boldsymbol{s_K}(t)]^T \tag{3}$$

is the signal $K \times 1$ vector with $\boldsymbol{s_k}(t)$ being a complex waveform representing the k^{th} signal and $\boldsymbol{n}(t)$ is the $M \times 1$ complex noise waveform with zero mean and covariance matrix $\sigma_n^2\boldsymbol{I}$ where σ_n^2 is an unknown constant and $\boldsymbol{I}$ is the identity matrix.

If we let the covariance matrix of $\boldsymbol{s}(t)$ be $\boldsymbol{R_s}$ and *positive definite* and $\boldsymbol{A}$ be a *full* column rank matrix then the covariance matrix of $\boldsymbol{x}(t)$ is given by

$$\boldsymbol{R}_x \triangleq E[\boldsymbol{x}(t)\boldsymbol{x}^H(t)] \tag{4}$$

and the rank of $\boldsymbol{AR_sA^H}$ is K. Denoting the eigenvalues of $\boldsymbol{R_x}$ by $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$ it follows, therefore, that the smallest $M - K$of its eigenvalues are all equal to σ_n^2. Thus the number of signals K can be determined from the number of the smallest eigenvalues of R.

3. ENUMERATION OF FULLY CORRELATED SIGNALS

The information-theoretic techniques fail to enumerate correctly when the covariance matrix of $\boldsymbol{s}(t)$ becomes *nonnegative definite* for the coherent signals. They are however still used later in determining the rank of *decorrelated* matrices.

In this section, the generalised matrix decomposition method of [6] will be detailed followed by a new stopping criterion and adaptive selection of antennas. The enumeration algorithm is also described.

3.1. Generalised Matrix Decomposition

From Eq.(4), let

$$\boldsymbol{C}_0 = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H = \boldsymbol{R}_x - \sigma_n^2\boldsymbol{I} \tag{5}$$

The matrix decomposition of $\boldsymbol{C}_0$ is given in [6] as a collection of matrices

$$\boldsymbol{Y}(m,p) = [\varepsilon(\boldsymbol{C}_o)^{(0)}, \varepsilon(\boldsymbol{C}_o)^{(1)}, \ldots, \varepsilon(\boldsymbol{C}_o)^{(m)}] \tag{6}$$

where

$$\varepsilon(\boldsymbol{C}_0)^{(m)} = [\boldsymbol{C}_0^{(m)}, \boldsymbol{J}_p\overline{\boldsymbol{C}_0}^{(M-p-m)}\boldsymbol{J}_M] \tag{7}$$

for $2 \leq p \leq M$ and $0 \leq m \leq M - p$ and

$$\mathbf{C}_0^{(m)} = \begin{pmatrix} c_{1+m,1} & c_{1+m,2} & \cdots & c_{1+m,M} \\ c_{2+m,1} & c_{2+m,2} & \cdots & c_{2+m,M} \\ \vdots & \vdots & \ddots & \vdots \\ c_{p+m,1} & c_{p+m,2} & \cdots & c_{p+m,M} \end{pmatrix} \tag{8}$$

with $\boldsymbol{J}_p$ be the $p \times p$ "exchange matrix"

$$\mathbf{J}_p = \begin{pmatrix} 0 & \ldots & 1 \\ \vdots & 1 & \vdots \\ 1 & \ldots & 0 \end{pmatrix} \tag{9}$$

Useful information can be extracted by studying the ranks of the matrices $\boldsymbol{Y}(m,p)$. There exists a set of rank sequences defined as

$$< \text{rank}\,\boldsymbol{Y}(m,p) : 0 \leq m \leq M - p > \tag{10}$$

Clearly, all such sequences are nondecreasing and bounded above by K.

Di's theorem [5] states that if $\boldsymbol{C}_0$ is an $M \times M$ Hermitian matrix and $\boldsymbol{R}_s$ is a $K \times K$ matrix with no null row (column) vector, then there is a value of m denoted $m_0 < m - p$ such that

$$\text{rank}\,[\boldsymbol{C}_0^{(0)}, \cdots, \boldsymbol{C}_0^{(m_0+j)}] = \text{rank}\,[\boldsymbol{C}_0^{(0)}, \cdots, \boldsymbol{C}_0^{(m_0)}] \tag{11}$$

and equal to $K < p$ where $j = 1, 2, \cdots, M - p - m_0$ then the number of sources is K. Thus, rank $[\boldsymbol{C}_0^{(0)}, \cdots, \boldsymbol{C}_0^{(m)}]$ will increase as k increases until rank $[\boldsymbol{C}_0^{(0)}, \cdots, \boldsymbol{C}_0^{(m)}] = K$ for some m. Also if:

$$\text{rank}\,[\boldsymbol{C}_0^{(0)}, \cdots, \boldsymbol{C}_0^{(M-p-1)}] < \text{rank}\,[\boldsymbol{C}_0^{(0)}, \cdots, \boldsymbol{C}_0^{(M-p)}] \tag{12}$$

and equal to p then the theorem cannot give the exact answer to the number of signals, but only tells us that the number of sigals is not less than p.

If $K < p$, then the rank sequence will strictly increase until it reaches K, and the point at which a rank sequence levels off (two successive terms have the same value) can be used to estimate the number of signals K.

In a real environment, the array covariance matrix which is used in the matrix decomposition and ranking process is not known and one must use the sample covariance matrix given by

$$\hat{\boldsymbol{R}_x} = 1/N \sum_{j=1}^{N} \boldsymbol{x}(t_j)\boldsymbol{x}(t_j)^H \tag{13}$$

where $\boldsymbol{x(t)}$ is sampled at N discrete times, t_j, $j = 1, ..., N$.

It is observed during simulation, using a mix of Rayleigh faded and coherent signals, that the rank sequence can weakly stabilize at more than one point (eg. multiple stabilization points). The previous algorithm [5] which stops after two successive terms having the same value will result in false estimation of the actual number of signals. The dynamic algorithm of this paper is based on calculating the effective ranks in $\boldsymbol{Y}(M - p - 1, p)$ and $\boldsymbol{Y}(M - p, p)$ and selecting K based on the highest frequency of occurrence of the calculated effective ranks.

At initialization, an estimate of the number of uncorrelated sources, $\hat{u}$ is calculated using the MDL algorithm of [4]. Then the effective ranks in $\boldsymbol{Y}(M-p-1, p)$ and $\boldsymbol{Y}(M-p, p)$ are calculated for values of $p = 2, 3, 4$ such that a total of 6 estimated ranks; 3 from $\boldsymbol{Y}(M - p - 1, p)$ and 3 from $\boldsymbol{Y}(M - p, p)$ are available to perform a rank stabilization check. The rank sequence is considered stable only when there exist a unique rank, r', which occurs with the highest frequency and $r' \geq \hat{u}$. Otherwise, p is increased by one and the effective ranks in $\boldsymbol{Y}(M-p-1, p)$ and $\boldsymbol{Y}(M-p, p)$ are recalculated for values of $p = (2, 3, 4, 5, \cdots)$. The process is repeated until stabilization is achieved or until $p = M-1$.

If stabilization is achieved for some $p \leq M-1$, then r' is taken as the estimate of K, i.e. the number of signals. The *next* sample covariance matrix is then taken but this time the effective ranks in $\boldsymbol{Y}(M-p-1,p)$ and $\boldsymbol{Y}(M-p,p)$ are calculated for values of $p = r', r'+1, r'+2$. Recalculations per the method outlined above will be necessary if it does not stabilize.

If stabilization occurs, processing of the *next* sample covariance matrix will start with a value of p based on the *previous* estimate of the rank i.e. $p = r'$.There are 2 exceptions.

The first exception occurs when $r' = \hat{u} = 0$. This implies that the estimated number of signals is zero and this is possible when the signals are at low SNR and experiencing deep fades. Under this condition, p is re-initialized (i.e. $p = 2, 3, 4$).

The second exception occurs when the rank sequence fails to stabilize. When this condition occurs, the rank stabilization check is repeated but this time all estimated ranks with values less than $\hat{u}$ are removed. If stabilization occurs, then this new estimate, r'' becomes the final estimate. Otherwise, the final estimate is taken from the rounded median value of 3 previous estimates. Under this condition, p is re-initialized (i.e. $p = 2, 3, 4$).

If the rank sequence does not stabilize at or before $p = M-1$, then an additional antenna is added,and the process is repeated. To control the build-up of antennas in subsequent trials, the following adaptive control criterion is applied. If the rank sequence stabilizes using 6 estimated ranks, the number of antennas is maintained at M for the *next* trial. If more than 6 estimated ranks are needed to stabilize, then the *next* trial will use one antenna less. If M is small compared to the number of signals, the rank sequence will fail to stabilize and additional antennas will be added one at a time until the rank sequence stabilizes. Likewise if M is large and the number of signals small, the algorithm will remove antennas until an optimum is reached. The algorithm implicitly requires the number of antennas be greater than the number of signals by 2. If the condition is violated, then the algorithm will enumerate $M-2$ signals and treat other signals as noise.

3.2. The enumeration algorithm

The dynamic signal enumeration (DSE) algorithm can now be stated as

1. Form the sample covariance matrix $\hat{\boldsymbol{R}}_x$.

2. Perform a singular value decomposition (SVD) on $\hat{\boldsymbol{R}}_x$ and use the MDL algorithm to estimate the number of uncorrelated sources, $\hat{u}$.

3. Set initial conditions; $M_{initial} = 5$, $2 \leq p \leq 4$ and for $0 \leq m \leq M-p$, and form the matrices $\boldsymbol{Y}(m,p)$ using $\hat{\boldsymbol{R}}_{\boldsymbol{x}}$ as an estimate of $\boldsymbol{C}_0$.

4. Perform a SVD on $\boldsymbol{Y}(M-p-1,p)$ and $\boldsymbol{Y}(M-p,p)$, and then estimate their ranks using the MDL algorithm in an array $\boldsymbol{Y_T}$

5. Perform a rank stabilization check on the elements in $\boldsymbol{Y_T}$. If there exists a unique rank, r' is the final estimate of the number of signals $\hat{K}$ present.

   In a situation where the ranks in $\boldsymbol{Y_T}$ do not stabilize, the process is repeated for $p = p+1$ until stabilization is achieved or until $p = M-1$.

6. Apply the adaptive control criterion to the rank sequence. The criterion determines the number of antennas to be used to obtain the *next* sample covariance matrix.

4. SIMULATION RESULTS

The dynamic performance of the new algorithm is shown in Figure 1. It is simulated using a linear array model with an initial number of antennas $(M_{initial})$ set to 12. They are equally separated at a distance of $\lambda/2$ and receive 3 independant *Rayleigh* fading sources and 3 *coherent* sources arriving at $-45°, -30°, -15°, 0°, 15°$ and $30°$. The number of signals vary at *random* from 3 to 6 users but remain in one particular state over 4 trials. The frequency of the signals are all equal to simulate the arrival of the main and reflected signals and also cochannel signals. The number of snapshots (N) per trial is 100 and 100 trials were conducted.

The top trace in Figure 1 indicates the number of signals, the middle trace show the estimated number of sources and the error between the actual number of sources and the estimate.The bottom trace shows the number of antennas used. The general trend is for the algorithm to ramp down in the first 10 trials to about 7 or 8 antennas. , Performance was best at 97 percent correct estimation at 10dB SNR as shown in Figure 1. At low SNR, the performance is predictably poor and algorithm tends to underestimate.At high SNR, a slight degradation in performance is observed with 91 percent correctly estimated with 9 percent over estimated by 1. There is no under estimation. In general, an over estimation is usually not as serious a problem as an under estimation on the number of users. For example, the Viterbi algorithm of [2] will still work correctly if the number of sources is overestimated albeit making computation more complex but an underestimation will result in system failure.

5. FIELD RESULTS

The algorithm was also verified on a real system at 915 MHz. The system consists of 5 receivers and 2 transmitters. 5 monopoles are equally separated at a distance of $\lambda/2$ and receive 2 *coherent* sources arriving at $-15°$ and $15°$, 10 metres away. The number of signals vary at *random* from 0 to 2 users. The number of snapshots (N) per trial is 100 and 420 trials were conducted.

The upper trace of Figure 2 show the actual transmitted signals. The middle trace is the number of signals enumerated by the algorithm at the receiver and the lower trace is the error. The enumeration accuracy is 99 percent with no underestimation. Performance is better than simulated because the sources do not experience Rayleigh fading in the field experiments.

6. CONCLUSIONS

The enumeration algorithm works well in the presence of correlated signals under fading channel conditions per the conditions outlined in the simulation. The dynamic nature of the algorithm ensures that the number of antennas needed is optimum for the number of signals and will adjust appropriately when additional signals arrive or depart as will happen in a real mobile environment. It will ensure that the amount of computation is kept to a minimum and in a practical system, switching off or putting on standy; RF, IF and digital circuits which are not needed and allowing the reallocation of DSP workload for other applications. The algorithm has also been verified good in limited field experiments.

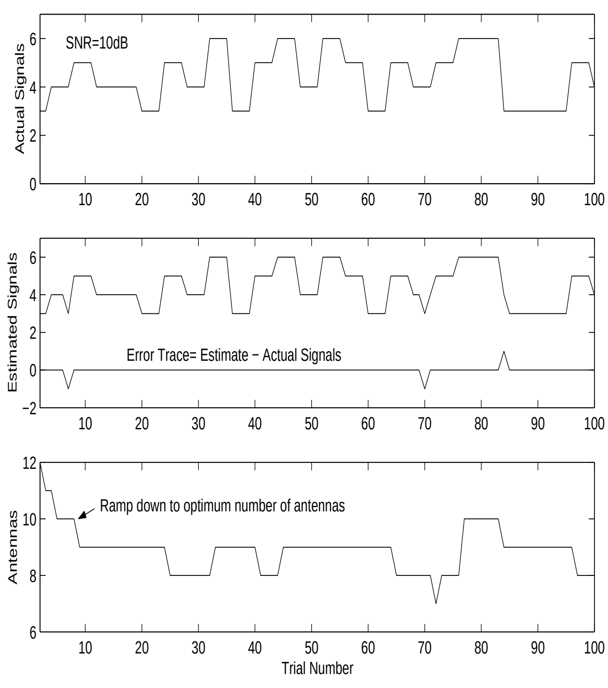

Fig. 1. Simulated performance of algorithm to random number of users at nominal SNR (10dB)

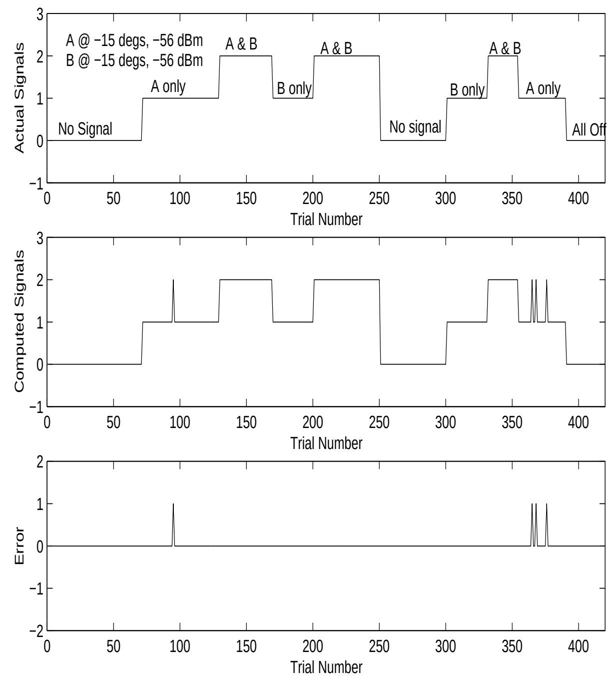

Fig. 2. Performance of algorithm in field test at 915 MHz

7. REFERENCES

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