IMPROVED CDMA MULTIUSER RECEIVERS ROBUST TO TIMING ERRORS

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ABSTRACT

We investigate an improved multiuser detector for asynchronous CDMA uplink channels when there exist timing delay estimation errors. We first formulate a robust decorrelating detector with a capacity limit of 50%. To increase the system capacity, we further propose a robust successive interference cancellation (SIC) implementation. The proposed robust SIC detector adds only a residual error estimation procedure onto the standard SIC detector. Computer simulation results show that its performance and capacity are very close to those of the ideal decorrelating detector which assumes perfect knowledge of time delay.

I. Introduction

CDMA multiuser detectors at the basestation, which utilize known user spreading codes for detection, typically assume perfect time delay information for all users. In practice, these time delays are estimated from the received signal and are prone to estimation errors because of multiple-access interference (MAI) and noise [2]. It has been shown that multiuser detectors are sensitive to delay mismatch, especially in severe near-far environments [3].

There are two kind of approaches to mitigate the effect of timing errors: deterministic and stochastic. The deterministic approach includes the decorrelating detector [4] and the delay-independent decorrelating detector [5] for the quasi-synchronous CDMA (QS-CDMA) channel. The MAI is completely eliminated. However, since these methods double the number of PN codes used, their capacity will not exceed 50% of the spreading factor [5].

The stochastic approach includes the QS-CDMA MMSE detector [4], the improved MMSE (IMMSE) multiuser detector for asynchronous CDMA [11], and two robustfied detectors [6]. The detectors in [4] and [11] achieve robustness by averaging over all possible delay errors, assuming zero-mean Gaussian [11] or uniform delay error distribution [4]. However, they cannot completely eliminate the residual MAI introduced by timing errors and are not near-far resistant.

We consider multiuser detection for the asynchronous CDMA uplink with timing errors using an improved method based on the deterministic approach in [4] and [5]. Section II describes the system model. Section III proposes the robust SIC multiuser detector. Section IV provides simulation results.

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II. System Model

We consider the basestation receiver that has knowledge of the spreading codes of all users. It is assumed that the delays of all users are estimated to within one chip of the true delays. For clarity and brevity, we consider a single-path channel.

Using a similar system model to [1], the equivalent baseband received signal is

$$r(t) = \sum_i \sum_{k=1}^{K} b_k(i) a_k(i) \tilde{s}_k(t - iT - \tau_k) + n(t) \qquad (1)$$

where $a_k(i) \in \mathcal{R}$ is the kth user's received signal amplitude for the ith time interval, $\tau_k \in [0, T)$ is the kth user's propagation delay, T is the bit duration and K is the total number of users.

In (1), the normalized signature waveform of user k is

$$\tilde{s}_k(t) = \sum_{j=0}^{N-1} c_k(j) h(t - jT_c) \qquad (2)$$

where $N = T/T_c$ is the spreading factor, $c_k(j)$ is the j-th PN chip, T_c is the chip duration and $h(t)$ is a rectangular chip pulse with duration $[0, T_c)$.

To focus on the effect of timing errors, we do not consider edge effects by using an isolation bit insertion (IBI) receiver [7], where a blank bit interval is inserted every M bit intervals. Therefore, the observation length is $(M+1)T$ while the number of data bits to be detected for each user is M.

Assume that the channel changes relatively slowly compared to $(M+1)T$, the received signal amplitude can be modeled as a constant, i.e., $a_k(i) = a_k$ for $i = 1, \ldots, M$.

After chip-matched filtering and chip-rate sampling, the received signal is discretized and the $(M+1)T$ observations are organized into a vector

$$\mathbf{r} = \sum_{i=1}^{M+1} \sum_{k=1}^{K} b_k(i) a_k \mathbf{d}_k(i) + \mathbf{n} \qquad (3)$$

where

$$\mathbf{r} = [\mathbf{r}^T(1)\ \mathbf{r}^T(2)\ \ldots\ \mathbf{r}^T(M+1)]^T \in \mathcal{R}^{(M+1)N} \qquad (4)$$

$$\mathbf{n} = [\mathbf{n}^T(1)\ \mathbf{n}^T(2)\ \ldots\ \mathbf{n}^T(M+1)]^T \in \mathcal{R}^{(M+1)N} \quad (5)$$

The noise vector $\mathbf{n}$ is a zero-mean white Gaussian random vector. The mth vector $\mathbf{r}(m)$ in (4) corresponds to the mth observation interval

$$\mathbf{r}(m) = [r(mN+1)\ \ldots\ r(mN+N)]^T \in \mathcal{R}^N \quad (6)$$

The time delay of the kth user is decomposed into an integer p_k and fractional δ_k part as $\tau_k = (p_k + \delta_k)T_c$, where $p_k \in \{0, 1, \ldots, N-1\}$ and $\delta_k \in [0, 1)$. The received signature waveform of the ith bit of the kth user, $\mathbf{d}_k(i) \in \mathcal{R}^{(M+1)N}$, can be expressed as the combination of two adjacent shifted version of user spreading codes [2]

$$\mathbf{d}_k(i) = \delta_k \mathbf{c}_k(p_k+1, i) + (1-\delta_k)\mathbf{c}_k(p_k, i) \quad (7)$$

In (7), $\mathbf{c}_k(p_k, i)$ is defined as $\mathbf{c}_k$ right-shifted by $(i-1)N + p_k$ chips, where $\mathbf{c}_k \in \mathcal{R}^{(M+1)N}$ is the kth user's spreading code vector for the $(M+1)T$ length interval defined as

$$\mathbf{c}_k = [c_k(0)\ c_k(1)\ \ldots\ c_k(N-1)\ \ 0\ 0\ \ldots\ 0]^T \quad (8)$$

The received signal can be expressed in matrix form as

$$\mathbf{r} = \mathbf{DAb} + \mathbf{n} \quad (9)$$

where $\mathbf{b} = [\mathbf{b}^T(1)\ \ \mathbf{b}^T(2)\ \ \ldots\ \ \mathbf{b}^T(M)]^T$, $\mathbf{b}(i) = [b_1(i)\ b_2(i)\ \ldots\ b_K(i)]^T$ is the data bit vector for the ith interval, $\mathbf{A} = \mathbf{I}_M \otimes \mathbf{a}$, where $\mathbf{I}_M$ is an $M \times M$ identity matrix, $\otimes$ denotes the Kronecker product, and $\mathbf{a} = diag(a_1, a_2, \ldots, a_K)$ is diagonal matrix of received signal amplitudes. The code matrix

$$\begin{aligned}\mathbf{D} \;&=\; [\mathbf{d}_1(1)\ \ldots \mathbf{d}_K(1)\ \mathbf{d}_1(2)\ \ldots\ \mathbf{d}_K(2)\ \ldots \\ &\quad \ldots\ \mathbf{d}_1(M)\ \ldots\ \mathbf{d}_K(M)] \in \mathcal{R}^{(M+1)N \times MK}\end{aligned} \quad (10)$$

If the time delays are perfectly known, the decorrelating detector can be constructed by

$$\hat{\mathbf{b}} = \text{sign}\ ([\mathbf{D}^H\mathbf{D}]^{-1}\mathbf{D}^H\mathbf{r}) \quad (11)$$

III. Robust SIC Multiuser Detector

Denote the estimated time delay of the kth user as $\hat{\tau}_k = (\hat{p}_k + \hat{\delta}_k)T_c$. The kth user's signature waveform for the ith interval $\mathbf{d}_k(i)$ in (7) can be expressed as the weighted sum of two signals $\hat{\mathbf{d}}_k(i)$ and $\Delta\mathbf{d}_k(i)$:

$$\begin{aligned}\mathbf{d}_k(i) \;&=\; (\hat{\delta}_k \mathbf{c}_k(p_k+1, i) + (1-\hat{\delta}_k)\mathbf{c}_k(p_k, i)) \\ &\quad +(\delta_k - \hat{\delta}_k)(\mathbf{c}_k(p_k+1, i) - \mathbf{c}_k(p_k, i)) \\ &\stackrel{\text{def}}{=} \hat{\mathbf{d}}_k(i) + (\delta_k - \hat{\delta}_k)\Delta\mathbf{d}_k(i)\end{aligned} \quad (12)$$

From (12), each user is decomposed into two virtual users, one with estimated code vector $\hat{\mathbf{d}}_k(i)$, and the other with error code vector $\Delta\mathbf{d}_k(i)$. This is different from the approach of [4] and [5], where two virtual users have chip-synchronous adjacent shifted versions of the same spreading code as in (7).

If it is assumed that the true delay and the estimated delay have the same integer part, i.e, $p_k = \hat{p}_k$ for $1 \leq k \leq K$, then the expression in (12) is exact [6].

Using (12), the received signal vector (9) can be expressed in the alternative form

$$\mathbf{r} = \mathbf{D}'\mathbf{A}'\mathbf{b}' + \mathbf{n} \quad (13)$$

where $\mathbf{b}' = [\mathbf{b}^T\ \ \mathbf{b}^T]^T \in \mathcal{R}^{2MK}$, $\mathbf{A}' = \mathbf{I}_{2M} \otimes \mathbf{a}'$, $\mathbf{a}' = diag(\ a_1, a_2,\ \ldots,\ a_K, (\delta_1 - \hat{\delta}_1)a_1, (\delta_2 - \hat{\delta}_2)a_2,\ \ldots,\ (\delta_K - \hat{\delta}_K)a_K\ )$, and code matrix

$$\begin{aligned}\mathbf{D}' \;&=\; [\mathbf{D}\ \Delta\mathbf{d}_1(1)\ \ldots\ \Delta\mathbf{d}_K(1)\ \Delta\mathbf{d}_1(2)\ \ldots\ \Delta\mathbf{d}_K(2)\ \ldots \\ &\quad \Delta\mathbf{d}_1(M)\ \ldots\ \Delta\mathbf{d}_K(M)] \in \mathcal{R}^{(M+1)N \times 2MK}\end{aligned} \quad (14)$$

We can now construct a robust decorrelating detector

$$\hat{\mathbf{b}}' = \text{sign}\ ([\mathbf{D}'^H\mathbf{D}']^{-1}\mathbf{D}'^H\mathbf{r}) \quad (15)$$

If the estimated delay is close to the true delay, most of the signal energy is contained in the estimated virtual user. There will be a very small SNR degradation if we use only this part of the signal energy for bit detection:

$$\hat{b}_k(i) = \hat{b}'_k(i)\ , \quad \text{k = 1, \ldots, K, and i =1, \ldots, M} \quad (16)$$

Since (15) is a decorrelating detector, it does not depend on amplitude information and is near-far resistant even with timing errors. However, since each user is decomposed into two virtual users, the total number of users that can be detected is upper bounded by $N/2$, where N is the spreading factor [5].

One possible way to improve capacity and performance is to compress the dimension of the error vectors and use a multistage version of the above robust decorrelating detector. At each stage, the M error vectors of each user, $\Delta\mathbf{d}_k(1) \ldots \Delta\mathbf{d}_k(M)$, are combined into a long error vector $\mathbf{e}_k$ based on the tentative data bit decisions, $\hat{b}_k(i)$

$$\mathbf{e}_k = \sum_{i=1}^{M} \Delta\mathbf{d}_k(i)\hat{b}_k(i) \quad (17)$$

We construct a new code matrix $\mathbf{D}''$, but with smaller dimension than that of (14)

$$\mathbf{D}'' = [\mathbf{D}\ \mathbf{e}_1\ \ldots\ \mathbf{e}_K] \in \mathcal{R}^{(M+1)N \times (M+1)K} \quad (18)$$

The tentative data bit decisions are obtained for the next stage via decorrelating detector

$$\hat{\mathbf{b}}'' = \text{sign}\ ([\mathbf{D}''^H\mathbf{D}'']^{-1}\mathbf{D}''^H\mathbf{r}) \quad (19)$$

The final decision is made after several iterations of (17)-(19).

The number of users that can be supported is now $NM/(M+1)$. For moderate block lengths, such as $M = 9$, the capacity is 90% of the spreading factor. However, the inversion of an $(M+1)K \times (M+1)K$ matrix in (19) is still computationally complex, and the tentative data bit update through decorrelating is not efficient.

The linear successive interference cancellation (SIC) receiver is a computationally attractive iterative implementation of the decorrelating detector with proven convergence properties [8]. If we implement the above multistage decorrelator iteratively, then we may lower computational complexity. Therefore, we propose the following iterations:

For $j = 0, 1, \ldots$ do:

For $k = 1, 2, \ldots, K$ do steps (1) and (2):

(1) Denote the long error vector of the lth user at the jth iteration as $\mathbf{e}_l^j = \sum_{i=1}^{M} \Delta \mathbf{d}_l(i) \hat{b}_l^j(i)$, and the error vector amplitude estimate as $\widehat{\Delta a}_l^j$. The reconstructed estimated signal of the lth user at the jth iteration is $\hat{\mathbf{r}}_l^j = \sum_{i=1}^{M} \hat{b}_l^j(i) \hat{a}_l^j(i) \hat{\mathbf{d}}_l(i)$. The received signal of the kth user for the *(j+1)*st iteration is obtained by subtracting other users' reconstructed signals and the residual signals from the received signal:

$$\mathbf{r}_k^{j+1} = \mathbf{r} - \sum_{l=1}^{k-1} \hat{\mathbf{r}}_l^{j+1} - \sum_{l=k+1}^{K} \hat{\mathbf{r}}_l^j - \sum_{l=1}^{K} \widehat{\Delta a}_l^j \mathbf{e}_l^j \quad (20)$$

The estimate of the residual signal of the kth user due to timing error is:

$$\Delta \mathbf{r}_k^{j+1} = \mathbf{r} - \sum_{l=1}^{K} \hat{\mathbf{r}}_l^{j+1} - \sum_{l=1}^{k-1} \widehat{\Delta a}_l^{j+1} \mathbf{e}_l^{j+1} - \sum_{l=k+1}^{K} \widehat{\Delta a}_l^j \mathbf{e}_l^j \quad (21)$$

(2) Update the user signal amplitude, user data bits and the amplitude of the error vector:

$$\hat{a}_k^{j+1}(i) = \text{abs}\left((\hat{\mathbf{d}}_k(i))^T \, \mathbf{r}_k^{j+1} \right) \quad (22)$$

$$\hat{b}_k^{j+1}(i) = \text{sign}\left((\hat{\mathbf{d}}_k(i))^T \, \mathbf{r}_k^{j+1} \right) \quad (23)$$

$$\widehat{\Delta a}_k^{j+1} = \frac{1}{M} (\mathbf{e}_k^{j+1})^T (\Delta \mathbf{r}_k^{j+1}) \quad (24)$$

As a simple method to terminate the iterations, we end the calculation as soon as $\hat{a}_k^j(i)$, for all $k = 1$ to K and $i = 1$ to M, changes by less than 1%.

The above algorithm can also be derived by the space alternating generalized EM (SAGE) algorithm to maximize the log-likelihood function with the unknown parameters to be the user signal amplitudes, user data bits and the amplitude of the error vectors [10]. The log-likelihood function is monotonically increasing after each SAGE iteration, so it is guaranteed that the robust SIC will converge at least to a local maximum.

Our robust SIC is novel due to the error vector estimation procedure in (20), (21) and (24). When some bits of the tentative data bit decisions are incorrect, the estimated amplitude of the error vector may be smaller than the actual value, which is equivalent to soft cancellation with a factor less than 100%. This robust SIC implicitly incorporates soft interference cancellation into its iterations, so it is numerically stable and it will likely converge to the global maximum. In the case when it does not converge to a global maximum, good performance will still be expected: strong users will more likely have an accurate residual error signal estimate and cancellation.

IV. Numerical and Simulation Results

Throughout the simulations, the delay estimation errors are independent zero-mean Gaussian variables with equal standard deviation $\sigma_\tau = 0.1$ for all users and normalized to the chip interval as in [3]. Gold code sequences of length 31 and a block size of $M = 9$ are used. The signal-to-noise ratio (SNR) is defined with respect to user 1.

In Figures 1-4, the *Decorrelator(True Delay)* curves refer to the decorrelating detector with true time delays, Eq. (11). *Decorrelator(Est. Delay)* refers to the decorrelating detector with estimated time delays, *Robust SIC* refers to the robust SIC detector of Eqs. (12), (20)-(24), *Robust Decorrelator* refers to the robust decorrelating detector with $2K$ virtual users of Eq. (15).

The Asymptotic multiuser efficiency (AME) [1] simulations in Fig. 1 and 2 are calculated for user 1. In Fig. 1, the number of users is $K = 5$ and the near-far ratio is increased from 0 to 30 dB. As shown, the AME of the proposed robust SIC detector stays constant as the near-far ratio increases, and is very close to the AME value of the ideal decorrelator with true delays.

Fig. 2 compares the AME performance as the number of users is increased from 3 to 31, while the near-far ratio is fixed at 20 dB. The robust SIC can support $NM/(M+1) = 31 \times 9/(9+1) \simeq 28$ users. The decorrelator with estimated delays can only support 5 or 6 users while the robust decorrelator can support 16 users, about half the spreading factor, N/2.

For the BER simulations in Figures 3-4, user 2 is the strongest user, and the user of interest, user 1, is the weakest user. The near-far ratio is 20 dB and all other users have power ratio uniformly distributed between 20 dB and 0 dB, compared to user 1.

In Fig. 3, the results show that with 5 users the BER of the robust SIC detector is lower than that of the robust decorrelator and is close to the BER of the ideal decorrelator with known time delays.

While in Fig. 3, the performance improvement of the robust SIC detector over the robust decorrelator is not large for $K = 5$ users, the improvement is obvious in Fig. 4 when the number of users is increased to $K = 10$.

V. Conclusion

We have proposed a new SIC detector that is robust to timing errors. It adds modest complexity to the standard SIC detector. Its capacity and performance are close to the ideal decorrelating detector. This robust SIC detector is also the iterative maximization of the log-likelihood function using the SAGE algorithm, so its convergence is guaranteed. Currently, we are extending the robust detection method to the general case of band-limited chip-pulse shapes.

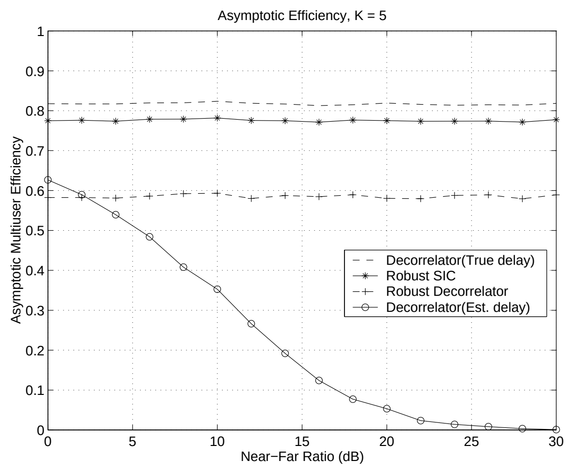

Fig. 1. Asymptotic multiuser efficiency (AME) as a function of near-far ratio for $K = 5$ users.

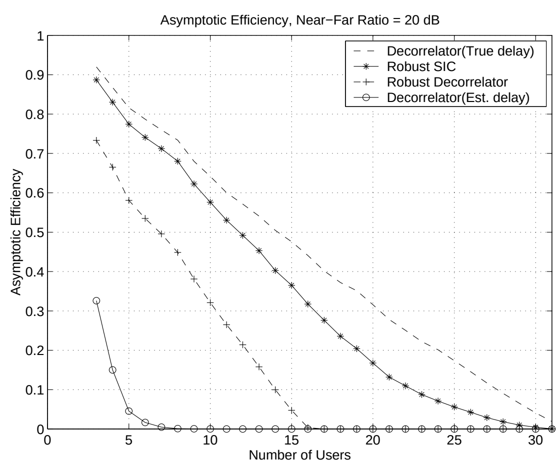

Fig. 2. Asymptotic multiuser efficiency (AME) as a function of number of users for Near-far Ratio $P_k/P_1 = 20dB$.

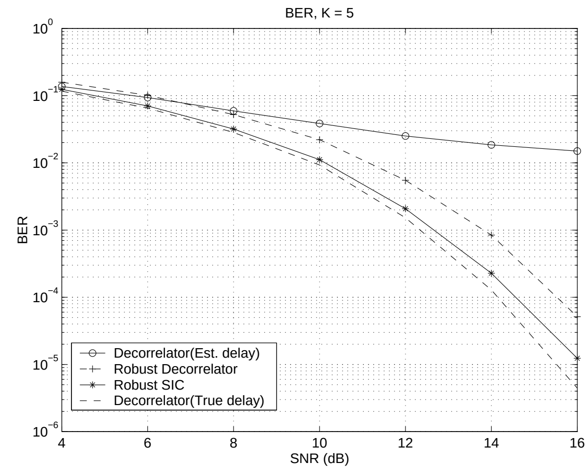

Fig. 3. Bit Error rate (BER) of user 1 for proposed robust SIC detector with $K = 5$ users.

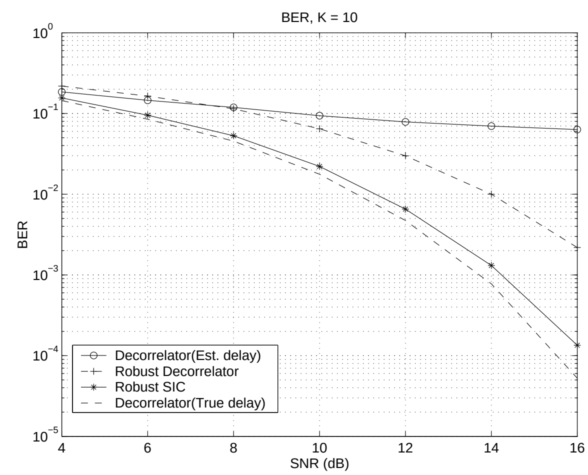

Fig. 4. Bit Error rate (BER) of user 1 for proposed robust SIC detector with $K = 10$ users.

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