DECOUPLED ESTIMATION OF DOA AND COHERENCE LOSS FOR MULTIPLE SOURCES IN UNCERTAIN PROPAGATION ENVIRONMENTS

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ABSTRACT

In this paper, the problem of Direction of Arrival (DOA) estimation of multiple sources is addressed considering possible coherence loss along the impinging wavefronts. The loss results from wave propagation through a fluctuating medium and leads to a decreasing signal correlation from sensor to sensor. Two new algorithms are proposed that are significantly less computationally complex than the well-known Covariance Matching (CM) approach. Furthermore a polynomial approximation of the coherence loss parameters is introduced, which permits a decoupling of the DOA estimation from the estimation of all other parameters. The proposed algorithms and theoretical results are verified by numerical examples.

1. INTRODUCTION

Conventional DOA estimation methods exploit the assumption that signals received by a sensor array are fully correlated [1] from sensor to sensor. They rely on fully coherent wavefronts and point sources. In many situation this assumption is not true, e.g. in the presence of local scatterers around the source or long range propagation through a random medium. The first situation is typical for wireless communication in urban environments, see [2] and the references therein, whereas the second situation occurs in underwater acoustics [3, 4]. Although the underlying physical models are different the resulting parameter estimation methods are based on similar covariance matrices.

One of the first studies taking into account coherence loss along wavefronts has been conducted in [3]. There, the resolution capability of high-resolution DOA estimation methods has been improved, but the unrealistic assumption of known coherence loss has been exploited. An early CM approach to estimate DOA and coherence loss simultaneously has been proposed in [4], but there, the coherence loss was assumed to be equal for all wavefronts. This restriction has been dropped in [5], where the computationally expensive CM approach has been used as well. Recent publications have been concentrated on the one-source problem. For this special case fast algorithms have been developed. In [2] a decoupled estimation of both DOA and coherence loss has been introduced successfully. This paper has been followed by further studies of Maximum Likelihood (ML) [6] and Least Square (LS) methods [7].

In this paper, the original CM approach is reformulated as a mathematically equivalent weighted LS problem by exploiting the Toeplitz structure of the covariance matrix. Then, two algorithms are proposed for the multiple source case. The first algorithm estimates both DOA and coherence loss simultaneously and the second one delivers a decoupled estimation. However, this approach is based on a polynomial approximation of the coherence loss. Both algorithm are significantly less computationally complex than the CM approach.

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2. SIGNAL MODEL

Consider a uniform linear array (ULA) of n sensors receiving q wavefronts of narrowband stationary zero-mean far-field sources, which are mutually stochastically independent. Then the array output can be modeled as [3, 5, 6]

$$\boldsymbol{x}(i) = \sum_{k=1}^{q} \boldsymbol{g}_k(i) \odot \boldsymbol{a}(\omega_k) + \boldsymbol{n}(i), \quad i = 1, \ldots, N \qquad (1)$$

where $\boldsymbol{g}_k(i)$ is the $n \times 1$ vector of the kth fluctuated wavefront and $\boldsymbol{a}(\omega_k)$ is the corresponding steering vector. The latter is defined as $\boldsymbol{a}(\omega) = [1,\, e^{-j\omega},\, \ldots,\, e^{-j\omega(n-1)}]^T$, with $\omega = 2\pi\Delta\sin\theta$, the sensor spacing Δ in wavelengths, and the DOA θ. Furthermore, $\boldsymbol{n}(i)$ is the $n \times 1$ vector of i.i.d. sensor noise, N is the number of independent snapshots, and $\odot$ is the elementwise matrix product. $(\cdot)^T$ and $(\cdot)^H$ denote transposition and Hermitian transposition, respectively. The conventional model [1] of the array output is a special case of (1), where all entries of $\boldsymbol{g}_k(i)$ are equal to the originally emitted source signal $s_k(i)$. All random processes are assumed to be zero-mean circularly symmetric Gaussian processes. The covariance matrix of the complex zero-mean Gaussian distribution $\mathcal{CN}(\mathbf{0}, \boldsymbol{R})$ can be written as [5]

$$\begin{aligned}\boldsymbol{R} &= \mathrm{E}\{\boldsymbol{x}(i)\boldsymbol{x}^H(i)\} \\ &= \sum_{k=1}^{q} \sigma_k^2 \boldsymbol{B}_k \odot [\boldsymbol{a}(\omega_k)\boldsymbol{a}^H(\omega_k)] + \sigma_N^2 \boldsymbol{I},\end{aligned} \tag{2}$$

where the introduced symbols are the source variance σ_k^2 of the kth source, the so-called coherence loss matrix $\boldsymbol{B}_k = \mathrm{E}\{\boldsymbol{g}_k(i)\boldsymbol{g}_k^H(i)\}/\sigma_k^2$, which is normalized to formally separate medium and source parameters, the variance of the sensor noise σ_N^2 and the identity matrix $\boldsymbol{I}$. Usually, the coherence loss matrix is parameterized by a real valued symmetric Toeplitz matrix [3, 5, 2]

$$[\boldsymbol{B}_k]_{lm} = \rho_k^{|l-m|^r}, \tag{3}$$

where ρ_k is the fluctuation strength and r varies application dependent between 1 and 2. The appropriate choice of the latter is not topic of this paper and therefore we set r equal to 1.

3. COVARIANCE MATCHING

Let $\boldsymbol{\xi} = [\theta_1, \ldots, \theta_q, \rho_1, \ldots, \rho_q, \sigma_1^2, \ldots, \sigma_q^2, \sigma_N^2]^T$ be the vector of all unknown parameters. Then, a CM estimation of $\boldsymbol{\xi}$ is given by [5]

$$\hat{\boldsymbol{\xi}} = \arg\min_{\boldsymbol{\xi}} \|\hat{\boldsymbol{R}} - \boldsymbol{R}(\boldsymbol{\xi})\|^2, \tag{4}$$

where $\|\cdot\|$ denotes the Frobenius norm and $\hat{\boldsymbol{R}}$ is the sample covariance matrix

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{x}(i)\boldsymbol{x}^H(i). \tag{5}$$

Now let us exploit the Toeplitz structure of the covariance matrix and rewrite (4) in an analytically equivalent form as

$$\hat{\boldsymbol{\xi}} = \arg\min_{\boldsymbol{\xi}} \|\hat{\boldsymbol{r}} - \boldsymbol{r}(\boldsymbol{\xi})\|_{\boldsymbol{W}}^2, \tag{6}$$

where

$$\hat{\boldsymbol{r}} = (\hat{r}_0, \ldots, \hat{r}_{n-1})^T, \quad \hat{r}_l = \frac{1}{n-l}\sum_{p=1}^{n-l} \hat{\boldsymbol{R}}(p+l, p),$$

$$\boldsymbol{r} = (\boldsymbol{R}(1,1), \ldots, \boldsymbol{R}(n,1))^T,$$

$$\boldsymbol{W} = \mathrm{diag}\{n, 2(n-1), \ldots, 2\}.$$

Here $\boldsymbol{R}(s,t)$ means the matrix entry of row s and column t. In the following sections (6) is used to obtain fast algorithms.

3.1. Exact Least-Squares Estimator

To separate the linear parameters summarized by $\boldsymbol{\gamma} = [\sigma_1^2, \ldots, \sigma_q^2, \sigma_N^2]^T = [\boldsymbol{\sigma}^T, \sigma_N^2]^T$ from the nonlinear parameters summarized by $\boldsymbol{h}_l = [\rho_1^l e^{-j\omega_1 l}, \ldots, \rho_q^l e^{-j\omega_q l}, \delta_l]^T$, where δ_l is the Kronecker delta, (6) can be rewritten as

$$\hat{\boldsymbol{\xi}} = \arg\min_{\boldsymbol{\xi}} \|\hat{\boldsymbol{r}} - \boldsymbol{H}(\boldsymbol{\omega}, \boldsymbol{\rho})\boldsymbol{\gamma}\|_{\boldsymbol{W}}^2, \tag{7}$$

where $\boldsymbol{H}(\boldsymbol{\omega}, \boldsymbol{\rho}) = [\boldsymbol{h}_0, \ldots, \boldsymbol{h}_{n-1}]^T$. For any fixed $\boldsymbol{H}$, the minimum of (7) is achieved if

$$\hat{\boldsymbol{\gamma}} = \left(\mathrm{Re}\left\{\boldsymbol{H}^H \boldsymbol{W} \boldsymbol{H}\right\}\right)^{-1} \mathrm{Re}\left\{\boldsymbol{H}^H \boldsymbol{W} \hat{\boldsymbol{r}}\right\}. \tag{8}$$

Inserting the value $\hat{\boldsymbol{\gamma}}$ into (7), the estimates of $\hat{\boldsymbol{\omega}}$ and $\hat{\boldsymbol{\rho}}$ are readily obtain as

$$\hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\rho}} = \arg\min_{\boldsymbol{\omega}, \boldsymbol{\rho}} \boldsymbol{y}^T \boldsymbol{Y}^{-1} \boldsymbol{y}, \tag{9}$$

$$\hat{\boldsymbol{\sigma}} = \boldsymbol{Y}^{-1}\boldsymbol{y}\Big|_{\boldsymbol{\omega}=\hat{\boldsymbol{\omega}}, \boldsymbol{\rho}=\hat{\boldsymbol{\rho}}}, \tag{10}$$

$$\hat{\sigma}_N^2 = \hat{r}_0 - \sum_{k=1}^{q} \hat{\sigma}_k^2, \tag{11}$$

where

$$\boldsymbol{y} = \sum_{l=1}^{n-1} (n-l)\mathrm{Re}\{\hat{r}_l \boldsymbol{c}_l\} \odot \boldsymbol{x}_l,$$

$$\boldsymbol{Y} = \sum_{l=1}^{n-1} (n-l)\mathrm{Re}\{\boldsymbol{c}_l \boldsymbol{c}_l^H\} \odot \boldsymbol{x}_l \boldsymbol{x}_l^T,$$

and finally $\boldsymbol{c}_l = [e^{-j\omega_1 l}, \ldots, e^{-j\omega_q l}]^T$, and $\boldsymbol{x}_l = [\rho_1^l, \ldots, \rho_q^l]^T$. Note, $\boldsymbol{Y}$ and $\boldsymbol{y}$ are a $q \times q$ matrix and a $q \times 1$ vector, respectively, i.e. that the computationally costly inversion of $\boldsymbol{Y}$ does not depend on the number of sensors. The number of floating point operations of the new algorithm is about 50 times less than of the original algorithm [5] providing all the same estimates. This value was measured for 2 sources and 15 sensors and increases for more sources or sensors.

3.2. Approximate Least-Squares Estimator

An additional decoupling of DOA and coherence loss estimation can be achieved if the latter together with the source variances are approximated by a polynomial, thus

$$\sigma_k^2 \rho_k^l \approx \sum_{\nu=0}^{L_k-1} \beta_k(\nu) l^{\alpha\nu}, \tag{12}$$

where the $\beta_k(\nu)$ are real valued polynomial coefficients. $\alpha = 1$ leads to general polynomials and $\alpha = 2$ to even ones, which should be chosen application dependent based on (3). L_k determines the individual number of coefficients of the respective source. Therefore, *a priori* knowledge about the character or strength of the coherence loss can be incorporated source dependent, e.g. by choosing $L_k = 1$ for nearly undisturbed sources.

With $\boldsymbol{\beta}_k = [\beta_k(0), \ldots, \beta_k(L_k-1)]^T$ and $\boldsymbol{u}_k(l) = [1, l^\alpha, \ldots, l^{\alpha(L_k-1)}]^T$ the polynomial can be formulated in vector notation. Then, the lth component of $\boldsymbol{r}$ is given by

$$r_l = \sum_{k=1}^{q} e^{-j\omega_k l} \boldsymbol{u}_k(l)^T \boldsymbol{\beta}_k + \sigma_n^2 \delta_l. \tag{13}$$

Note, the conventional model is obtained if we choose $L_k = 1$ for all k. Let us now define $\boldsymbol{\gamma} = [\boldsymbol{\beta}_1^T, \ldots, \boldsymbol{\beta}_q^T, \sigma_N^2]^T = [\boldsymbol{\zeta}^T, \sigma_N^2]^T$ containing all linear parameters and additionally $\boldsymbol{h}_l = [e^{-j\omega_1 l}\boldsymbol{u}_1(l)^T, \ldots, e^{-j\omega_q l}\boldsymbol{u}_q(l)^T, \delta_l]^T$. Finally, by denoting $\boldsymbol{H}(\boldsymbol{\omega}) = [\boldsymbol{h}_0, \ldots, \boldsymbol{h}_{n-1}]^T$, which only depends on the DOA, we get

$$\hat{\boldsymbol{\xi}} = \arg\min_{\boldsymbol{\xi}} \|\hat{\boldsymbol{r}} - \boldsymbol{H}(\boldsymbol{\omega})\boldsymbol{\gamma}\|_W^2. \tag{14}$$

A calculation similar to the one in the last subsection leads to the second algorithm, which can be summarized as

$$\hat{\boldsymbol{\omega}} = \arg\min_{\boldsymbol{\omega}} \boldsymbol{y}^T \boldsymbol{Y}^{-1} \boldsymbol{y} \tag{15}$$

$$\hat{\boldsymbol{\zeta}} = \boldsymbol{Y}^{-1}\boldsymbol{y}\Big|_{\boldsymbol{\omega}=\hat{\boldsymbol{\omega}}} \tag{16}$$

$$\hat{\sigma}_N^2 = \hat{r}_0 - \sum_{k=1}^{q} \hat{\beta}_k(0) \tag{17}$$

where

$$\boldsymbol{y} = \sum_{l=1}^{n-1}(n-l)\mathrm{Re}\{\hat{r}_l \boldsymbol{c}_l\}, \quad \boldsymbol{Y} = \sum_{l=1}^{n-1}(n-l)\mathrm{Re}\{\boldsymbol{c}_l \boldsymbol{c}_l^H\}$$

and $\boldsymbol{c}_l = [e^{-j\omega_1 l}\boldsymbol{u}_1(l)^T, \ldots, e^{-j\omega_q l}\boldsymbol{u}_q(l)^T]^T$.

Note, $\boldsymbol{Y}$ and $\boldsymbol{y}$ are a $L \times L$ matrix and a $L \times 1$ vector, respectively, where $L = \sum_{k=1}^q L_k$. The estimates of the source variances $\boldsymbol{\sigma}$ and coherence losses $\boldsymbol{\rho}$ have to be calculated from the estimated polynomial coefficients $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_q$. This is possible by additional LS fits:

$$\hat{\sigma}_k^2 = \hat{\beta}_k(0) \tag{18}$$

$$\hat{\rho}_k = \arg\min_{\rho_k} \left| \sum_{l=1}^{n-1} \boldsymbol{u}_k^T(l)\hat{\boldsymbol{\beta}}_k - \hat{\sigma}_k^2 \rho_k^l \right|^2 \tag{19}$$

for $k = 1, \ldots, q$, i.e. further q successive 1-dimensional searches are necessary to estimate all unknown parameters. A closed form expression of $\hat{\rho}_k$ is given by

$$\ln\hat{\rho}_k = \frac{6}{n(n-1)(2n-1)} \left(\sum_{l=1}^{n-1} l \ln \boldsymbol{u}_k^T(l)\hat{\boldsymbol{\beta}}_k \right) - \frac{3\ln\hat{\sigma}_k^2}{2n-1},$$

if $\boldsymbol{u}_k^T(l)\hat{\boldsymbol{\beta}}_k > 0$ for all l. The polynomial approximation is expected to introduce a degradation of estimation performance compared to the exact LS estimator. On the other hand, the estimation of $\boldsymbol{\omega}$ is quite robust since the approximative model for $\boldsymbol{B}$ does not depend on a specific coherence loss model.

4. STATISTICAL ANALYSIS

The covariance matrix (2) has Toeplitz structure. Therefore, a necessary condition of identifiability is that the number of unknown parameters is less than $2n-1$. A prove of global identifiability has not been conducted yet.

Generally, for any identifiable Toeplitz problem as formulated in (6) the covariance matrix of the asymptotic normal distribution is given by

$$\begin{aligned}\lim_{N\to\infty} N\mathrm{E}(\hat{\boldsymbol{\xi}} - \boldsymbol{\xi}_0)(\hat{\boldsymbol{\xi}} - \boldsymbol{\xi}_0)^T = \\ \frac{1}{2}\mathrm{Re}\{\boldsymbol{DWD}^H\}^{-1}\mathrm{Re}\{\boldsymbol{D}^*\boldsymbol{WS}(\boldsymbol{\Gamma SWD}^T + \tilde{\boldsymbol{\Gamma}}\boldsymbol{SWD}^H)\} \\ \times \mathrm{Re}\{\boldsymbol{DWD}^H\}^{-1},\end{aligned} \tag{20}$$

where

$$\boldsymbol{D} = \frac{\partial \boldsymbol{r}^T}{\partial \boldsymbol{\xi}}\Big|_{\boldsymbol{\xi}=\boldsymbol{\xi}_0},$$

$$\boldsymbol{S} = \mathrm{diag}\{1/n, \ldots, 1\},$$

$$\boldsymbol{\Gamma}(k,l) = \sum_{p=1}^{n-k}\sum_{q=1}^{n-l} \boldsymbol{R}(p+k, q+l)\boldsymbol{R}^*(p,q),$$

$$\tilde{\boldsymbol{\Gamma}}(k,l) = \sum_{p=1}^{n-k}\sum_{q=1}^{n-l} \boldsymbol{R}(p+k, q)\boldsymbol{R}^*(p, q+l),$$

where $*$ denotes complex conjugation. The derivation of (20) is based on [7]. Numerical results are depicted in the figures of section 5. Furthermore, the Cramér-Rao bound has been calculated and is shown in the figures as well, while stating the exact expression is omitted due to lack of space.

5. NUMERICAL EXAMPLES

The estimation performances of the proposed algorithms are now demonstrated by means of 200 Monte-Carlo simulations. They are compared with two CM algorithm. One of them does not take into account coherence loss, in the figures indicated by (wrong) for wrong model. The comparison is focused on the estimation of the DOA because the other parameters are mainly considered as nuisance parameters.

As the first example a ULA was modeled consisting of $n = 20$ sensors, impinging wavefronts with DOAs $\theta_1 = 0°$ and $\theta_2 = 8°$, and coherence loss parameters $\rho_1 = 0.95$ and $\rho_2 = 0.85$. The Signal-to-Noise Ratio (SNR) was set to 10

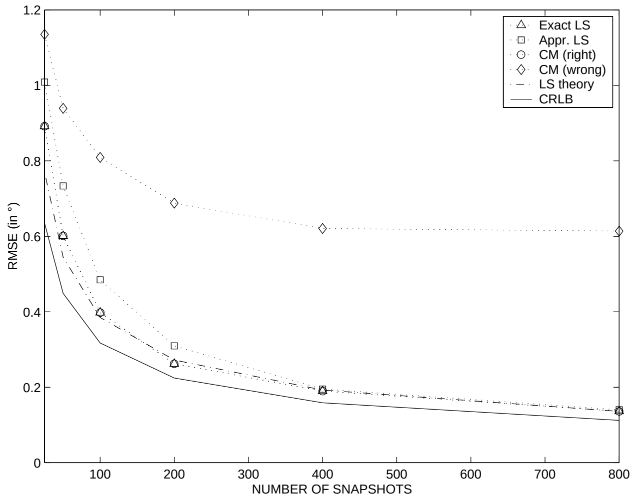

Fig. 1. RMSE versus the number of snapshots

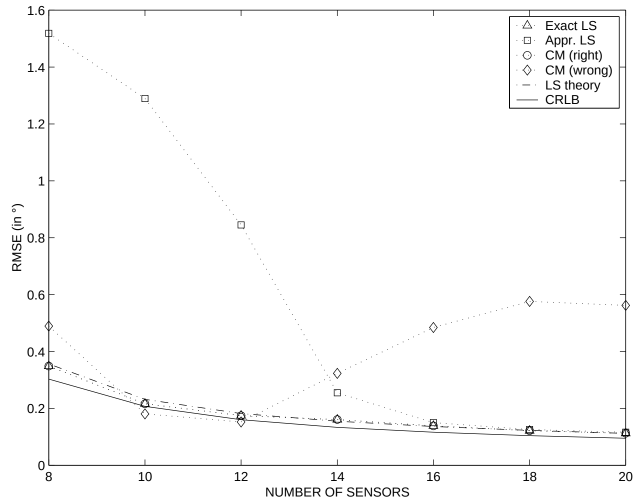

Fig. 2. RMSE versus the number of sensors

dB and the number of snapshots varied between 25 and 800. The number of polynomial coefficients was $L_k = 3$ for every single source and general polynomials were used. The experiments were conducted in the following way: A global and local optimization of the CM was performed ignoring coherence loss. Then, the estimated DOA were taken as initial values for the other three algorithms and the coherence loss parameters were set to 1, i.e. no coherence loss was assumed initially. In Fig. 1 the results are displayed. The estimation performances of the exact LS and the conventional CM considering coherence loss are exactly the same, even for every single snapshot. This two algorithms and the approximate LS outperform the CM exploiting the wrong model.

As the second example a ULA was modeled with varying number of sensors between 8 and 20. The DOA and the SNR were not changed, whereas both coherence loss parameters were set to 0.9 and the number of snapshots was 1000. The simulation results are displayed in Fig. 2. There, a drawback of the approximate LS becomes visible. It suffers from low number of degrees of freedom when the array consists only of a small number of sensors. The estimation performances of the proposed algorithms are similar for larger arrays. The CM exploiting the wrong model behaves unpredictable.

6. CONCLUSION

Two new algorithms have been proposed. It has been shown that the estimation of the DOA can be decoupled from the estimation of all other parameters. Additionally, the computationally complexity of the new algorithms is low and depends on the number of sensors only linearly. Therefore, the areas of application are large sensor arrays, where coherence loss is critical even for weakly fluctuated wavefronts.

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