NORMALISED CONSTANT MODULUS ALGORITHM WITH SELECTIVE PARTIAL UPDATES

Kutluyıl Doğançay[*]

School of Electrical and Information Engineering
University of South Australia
Mawson Lakes, SA 5095, Australia
E-mail: Kutluyil.Dogancay@unisa.edu.au

Oğuz Tanrıkulu

Tellabs Media Processing Group
175 Middlesex Turnpike
Bedford, MA 01730, USA
E-mail: Oguz.Tanrikulu@tellabs.com

ABSTRACT

A reduced complexity realisation for the normalised constant modulus algorithm (NCMA) and its soft criterion satisfaction (SCS) version is proposed based on selective partial updating. The computational complexity of NCMA and SCS is reduced by updating a block of equaliser parameters at every iteration rather than the entire equaliser. This results in a smaller number of multiplications for updating the equaliser parameters. A simple block selection criterion is derived from the solution of a constrained minimisation problem that underpins the development of NCMA. In fractionally-spaced equalisation, the proposed selective partial updating is shown to be capable of maintaining comparable convergence speed to its full-update counterpart. This implies a significant reduction in implementation cost without necessarily penalising the convergence speed.

1. INTRODUCTION

Normalised constant modulus algorithm (NCMA) is an adaptive blind equalisation algorithm that can be derived from the solution of a constrained optimisation problem [1]. The computational complexity of NCMA is proportional to the number of equaliser parameters. If the number of equaliser parameters is large, as is often the case in practical channel equalisation applications, the computational complexity can become prohibitively large. Indeed, one of the problems with the use of blind equalisers in modern communication systems is the large computational complexity.

This paper proposes a reduced complexity realisation for NCMA and its soft criterion satisfaction (SCS) variant [2] based on selective partial updating [3]. The computational complexity of NCMA and SCS is reduced by updating a block of equaliser parameters at every iteration rather than the entire equaliser, thereby decreasing the number of multiplications required for updating the equaliser parameters. A simple block selection criterion is also derived, requiring the channel outputs to be ranked according to their Euclidean norm. The only additional overhead is due to the ranking of regressor vector elements. This can be done using the heapsort [4] or the sortline algorithm [5] with minimal additional complexity, which is often far less than the complexity reduction achieved by selective partial updating. The block selection criterion does not result in undue decrease in convergence speed. This is thanks to the use of a well-defined optimisation problem as the basis for the selection criterion.

[*]This work was partially supported by a DITEE Research Grant of the University of South Australia.

2. BLIND EQUALISATION USING CONSTANT MODULUS CRITERION

In communication systems, transmitted symbols $u(k)$ undergo distortion due to intersymbol interference (ISI). The task of an equaliser is to remove ISI from the received signal. The baud-rate (T-spaced) equalisation problem consists of estimating the equaliser parameter vector $\boldsymbol{\theta}(k) = [\theta_0(k), \theta_1(k), \cdots, \theta_{N-1}(k)]^T$ such that, for a given regressor (equaliser input) vector $\boldsymbol{x}(k) = [x(k), x(k-1), \cdots, x(k-N+1)]^T$, the equaliser output signal $y(k) = \boldsymbol{\theta}^T(k)\boldsymbol{x}(k)$ obeys the relation

$$y(k) = u(k - \Delta) \quad \forall k \tag{1}$$

where $\Delta \geq 0$ is a constant equalisation delay. In most blind equalisation problems, the equaliser is allowed to be insensitive to phase rotations to facilitate separation of equalisation from carrier recovery [6]. This is achieved by employing the following constant modulus criterion in lieu of (1)

$$|y(k)| = R \quad \forall k \tag{2}$$

where R is a constellation-dependent dispersion factor. For constant modulus constellations such as BPSK, M-PSK, R is simply equal to the magnitude of transmitted symbols [7]. For other constellations, R can be obtained from higher-order moments of transmitted symbols [6].

3. THE FULL-UPDATE NCMA AND SCS-1

The full update NCMA can be derived from the solution of a constrained optimisation problem much in the same as in the normalised least-mean-square (NLMS) algorithm [8]. For blind equalisation employing the constant modulus criterion (2) the constrained optimisation problem takes the form [1]:

$$\min_{\boldsymbol{\theta}(k+1)} \|\boldsymbol{\theta}(k+1) - \boldsymbol{\theta}(k)\|_2^2 \tag{3a}$$

$$\text{subject to} \quad |\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)| = R. \tag{3b}$$

The solution of the above optimisation problem gives the full-update NCMA:

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) + \frac{\mu}{\|\boldsymbol{x}(k)\|_2^2}\big(\mathrm{sgn}(y(k))R - y(k)\big)\boldsymbol{x}(k) \tag{4}$$

where μ is a stepsize.

A soft criterion satisfaction (SCS) version of NCMA can be obtained by replacing the hard constraint in (3b) with a "soft" constraint [2]. The resulting SCS-1 algorithm is given by

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) + \frac{\mu}{\|\boldsymbol{x}(k)\|_2^2}\left(1 - \frac{|y(k)|}{R}\right)y(k)\boldsymbol{x}(k).$$

4. NCMA AND SCS-1 WITH SELECTIVE PARTIAL UPDATES

Partition the regressor vector and the equaliser parameter vector into M blocks of length $L = N/M$:

$$\boldsymbol{x}(k) = \begin{bmatrix} \boldsymbol{x}_1(k) \\ \boldsymbol{x}_2(k) \\ \vdots \\ \boldsymbol{x}_M(k) \end{bmatrix} \qquad \boldsymbol{\theta}(k) = \begin{bmatrix} \boldsymbol{\theta}_1(k) \\ \boldsymbol{\theta}_2(k) \\ \vdots \\ \boldsymbol{\theta}_M(k) \end{bmatrix}$$

For the selective-partial-update version of NCMA, the constrained minimisation problem (3) can be reformulated as

$$\min_{1 \le i \le M} \min_{\boldsymbol{\theta}_i(k+1)} \|\boldsymbol{\theta}_i(k+1) - \boldsymbol{\theta}_i(k)\|_2^2 \tag{5a}$$

$$\text{subject to} \quad |\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)| = R. \tag{5b}$$

We will first consider the minimisation problem for a given block. If i is fixed, then (5) becomes a constrained minimisation problem over $\boldsymbol{\theta}_i(k+1)$:

$$\min_{\boldsymbol{\theta}_i(k+1)} \|\boldsymbol{\theta}_i(k+1) - \boldsymbol{\theta}_i(k)\|_2^2 \tag{6a}$$

$$\text{subject to} \quad |\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)| = R \tag{6b}$$

which can be solved in a similar way to NCMA by using the method of Lagrange multipliers. The cost function to be minimised is given by

$$J_i(k) = \|\boldsymbol{\theta}_i(k+1) - \boldsymbol{\theta}_i(k)\|_2^2 + \lambda\big(R - |\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)|\big)$$

where λ is a Lagrange multiplier. Setting $\partial J_i(k)/\partial \boldsymbol{\theta}_i(k+1) = \mathbf{0}$ and $\partial J_i(k)/\partial \lambda = 0$, we get

$$\boldsymbol{\theta}_i(k+1) - \boldsymbol{\theta}_i(k) - \frac{\lambda}{2}\operatorname{sgn}\big(\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)\big)\boldsymbol{x}_i(k) = \mathbf{0} \tag{7a}$$

$$R - |\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)| = 0. \tag{7b}$$

Substituting (7a) into (7b) yields

$$\frac{\lambda}{2} = \frac{R - \operatorname{sgn}\big(\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)\big)y(k)}{\|\boldsymbol{x}_i(k)\|_2^2}.$$

Plugging this into (7a) and introducing a positive stepsize μ, we get

$$\boldsymbol{\theta}_i(k+1) = \boldsymbol{\theta}_i(k) + \frac{\mu}{\|\boldsymbol{x}_i(k)\|_2^2}\big(R\operatorname{sgn}(y(k)) - y(k)\big)\boldsymbol{x}_i(k) \tag{8}$$

where we have replaced $\operatorname{sgn}\big(\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)\big)$ with $\operatorname{sgn}(y(k))$, assuming that the update term is sufficiently small such that it does not flip the sign of the resulting equaliser output for the regressor vector at time k. Equation (8) solves the fixed-block-update constrained minimisation problem in (6).

According to (5), the selection of the block to be updated is performed by finding the block with the smallest squared-Euclidean-norm update, i.e.,

$$\begin{aligned} i &= \arg\min_{1 \le j \le M} \|\boldsymbol{\theta}_j(k+1) - \boldsymbol{\theta}_j(k)\|_2^2 \\ &= \arg\min_{1 \le j \le M} \left\| \frac{\big(R\operatorname{sgn}(y(k)) - y(k)\big)\boldsymbol{x}_j(k)}{\|\boldsymbol{x}_j(k)\|_2^2} \right\|_2^2 \\ &= \arg\max_{1 \le j \le M} \|\boldsymbol{x}_j(k)\|_2^2. \end{aligned}$$

After determining the block that satisfies the above criterion, (8) is used to update the selected block, resulting in the recursion

$$\boldsymbol{\theta}_i(k+1) = \boldsymbol{\theta}_i(k) + \frac{\mu}{\|\boldsymbol{x}_i(k)\|_2^2}\big(R\operatorname{sgn}(y(k)) - y(k)\big)\boldsymbol{x}_i(k), \quad \text{where} \quad i = \arg\max_{1 \le j \le M} \|\boldsymbol{x}_j(k)\|_2^2. \tag{9}$$

Equation (9) assumes that only one block is updated at every iteration. A natural extension of (9) would be to consider adaptation of B blocks out of M per iteration. In this vein, we have the following constrained optimisation problem

$$\min_{\mathcal{I}_B} \min_{\substack{\boldsymbol{w}_i(k+1) \\ i \in \mathcal{I}_B}} \sum_{i \in \mathcal{I}_B} \|\boldsymbol{w}_i(k+1) - \boldsymbol{w}_i(k)\|_2^2 \tag{10a}$$

$$\text{subject to} \quad |\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)| = R \tag{10b}$$

where $\mathcal{I}_B = \{i_1, i_2, \ldots, i_B\}$ is a B-subset (subset with B members) of $\{1, 2, \ldots, M\}$.

If $\mathcal{I}_B$ is given and fixed, (10) can be solved by minimising the cost function:

$$\begin{aligned} J_{\mathcal{I}_B}(k) = {} & \|\boldsymbol{\theta}_{\mathcal{I}_B}(k+1) - \boldsymbol{\theta}_{\mathcal{I}_B}(k)\|_2^2 \\ & + \lambda\big(R - |\boldsymbol{\theta}^T(k+1)\boldsymbol{x}(k)|\big) \end{aligned}$$

where the $LB \times 1$ vector $\boldsymbol{\theta}_{\mathcal{I}_B}(k)$ is defined by

$$\boldsymbol{\theta}_{\mathcal{I}_B}(k) = \begin{bmatrix} \boldsymbol{\theta}_{i_1}^T(k) & \boldsymbol{\theta}_{i_2}^T(k) & \cdots & \boldsymbol{\theta}_{i_B}^T(k) \end{bmatrix}^T.$$

Minimisation of $J_{\mathcal{I}_B}(k)$ with respect to $\boldsymbol{\theta}(k+1)$ and λ results in the recursion

$$\boldsymbol{\theta}_{\mathcal{I}_B}(k+1) = \boldsymbol{\theta}_{\mathcal{I}_B}(k) + \frac{\mu\big(R\operatorname{sgn}(y(k)) - y(k)\big)\boldsymbol{x}_{\mathcal{I}_B}(k)}{\|\boldsymbol{x}_{\mathcal{I}_B}(k)\|_2^2} \tag{11}$$

where $\boldsymbol{x}_{\mathcal{I}_B}(k)$ is defined as

$$\boldsymbol{x}_{\mathcal{I}_B}(k) = \begin{bmatrix} \boldsymbol{x}_{i_1}^T(k) & \boldsymbol{x}_{i_2}^T(k) & \cdots & \boldsymbol{x}_{i_B}^T(k) \end{bmatrix}^T.$$

There are $\frac{M!}{B!(M-B)!}$ unique B-subsets of $\{1, 2, \ldots, M\}$. Let us denote the collection of all B-subsets by $\mathcal{S}$. To determine which subset to use, we need to find B parameter blocks with a minimum squared-Euclidean-norm update:

$$\begin{aligned} \mathcal{I}_B &= \arg\min_{\mathcal{J}_B \in \mathcal{S}} \|\boldsymbol{\theta}_{\mathcal{J}_B}(k+1) - \boldsymbol{\theta}_{\mathcal{J}_B}(k)\|_2^2 \\ &= \arg\min_{\mathcal{J}_B \in \mathcal{S}} \left\| \frac{\big(R\operatorname{sgn}(y(k)) - y(k)\big)\boldsymbol{x}_{\mathcal{J}_B}(k)}{\|\boldsymbol{x}_{\mathcal{J}_B}(k)\|_2^2} \right\|_2^2 \\ &= \arg\max_{\mathcal{J}_B \in \mathcal{S}} \sum_{j \in \mathcal{J}_B} \|\boldsymbol{x}_j(k)\|_2^2. \end{aligned}$$

Thus, if the regressor vector blocks are ranked according to their squared Euclidean norms, $\mathcal{I}_B$ must contain the B largest blocks:

$$\|\boldsymbol{x}_{i_1}(k)\|_2^2 \geq \|\boldsymbol{x}_{i_2}(k)\|_2^2 \geq \cdots \geq \|\boldsymbol{x}_{i_B}(k)\|_2^2 \geq \|\boldsymbol{x}_i(k)\|_2^2, \quad \forall i \in \{1, 2, \ldots, M\} \setminus \mathcal{I}_B.$$

The *selective-partial-update NCMA (SPU-NCMA)*, which is the most general form of NCMA with selective partial updates, is given by

$$\boxed{\begin{array}{l} \boldsymbol{\theta}_{\mathcal{I}_B}(k+1) = \boldsymbol{\theta}_{\mathcal{I}_B}(k) + \dfrac{\mu\big(R\,\mathrm{sgn}(y(k)) - y(k)\big)\boldsymbol{x}_{\mathcal{I}_B}(k)}{\|\boldsymbol{x}_{\mathcal{I}_B}(k)\|_2^2} \\ \text{where } \mathcal{I}_B = \{i : \|\boldsymbol{x}_i(k)\|_2^2 \text{ is one of the } B \text{ largest among} \\ \|\boldsymbol{x}_j(k)\|_2^2, j \in \{1, 2, \ldots, M\}\}. \end{array}} \tag{12}$$

Note that setting $M = N$ and $B = M$ corresponds to the full-update NCMA (4).

Following the same line of development as above, the *selective-partial-update SCS-1 (SPU-SCS-1) algorithm* can be obtained as

$$\boxed{\begin{array}{l} \boldsymbol{\theta}_{\mathcal{I}_B}(k+1) = \boldsymbol{\theta}_{\mathcal{I}_B}(k) + \dfrac{\mu\Big(1 - \frac{|y(k)|}{R}\Big)y(k)\boldsymbol{x}_{\mathcal{I}_B}(k)}{\|\boldsymbol{x}_{\mathcal{I}_B}(k)\|_2^2} \\ \text{where } \mathcal{I}_B = \{i : \|\boldsymbol{x}_i(k)\|_2^2 \text{ is one of the } B \text{ largest among} \\ \|\boldsymbol{x}_j(k)\|_2^2, j \in \{1, 2, \ldots, M\}\}. \end{array}} \tag{13}$$

5. FRACTIONALLY-SPACED IMPLEMENTATION

In $T/2$-spaced equalisation, which is the most common form of fractionally spaced equalisation, the communication channel is modelled as two subchannels with outputs $x_1(k)$ and $x_2(k)$. The subchannel outputs are applied to subequalisers with $N/2 \times 1$ parameter vectors $\boldsymbol{\theta}_1(k)$ and $\boldsymbol{\theta}_2(k)$. Rewriting the regressor vector as $\boldsymbol{x}^T(k) = [\boldsymbol{x}_1^T(k), \boldsymbol{x}_2^T(k)]$ and the equaliser parameter vector as $\boldsymbol{\theta}^T(k) = [\boldsymbol{\theta}_1^T(k), \boldsymbol{\theta}_2^T(k)]$, which are both $1 \times N$ vectors, the algorithms described so far can be used with no modification as fractionally-spaced equalisers.

6. SIMULATION STUDIES

This section includes computer simulations for the proposed algorithms. For purposes of performance comparison, we use the open-eye measure (OEM), which, for BPSK signals, is defined by

$$\mathrm{OEM}(k) = \frac{\|\boldsymbol{c}(k)\|_1 - \|\boldsymbol{c}(k)\|_\infty}{\|\boldsymbol{c}(k)\|_\infty}$$

where $\boldsymbol{c}(k)$ is the combined channel and equaliser impulse response. If $\mathrm{OEM}(k) < 1$, the eye is open. Otherwise, the eye is closed.

6.1. T-Spaced NCMA and SCS-1

The channel is assumed to have the transfer function

$$H(z) = \frac{1}{1 - 0.6z^{-1} + 0.3z^{-2} + 0.2z^{-3} + 0.01z^{-4}}.$$

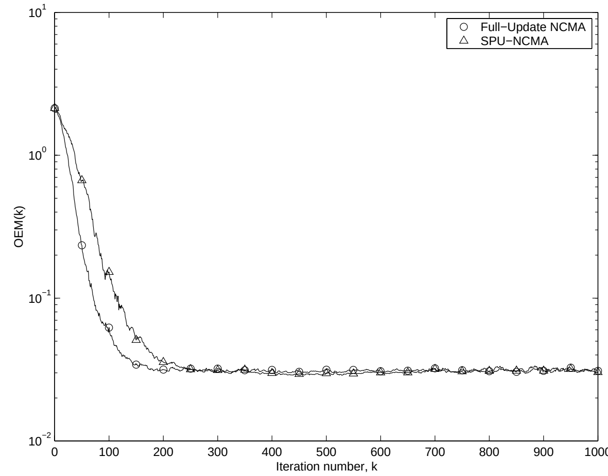

Figure 1: OEM plots for T-spaced NCMA and SPU-NCMA.

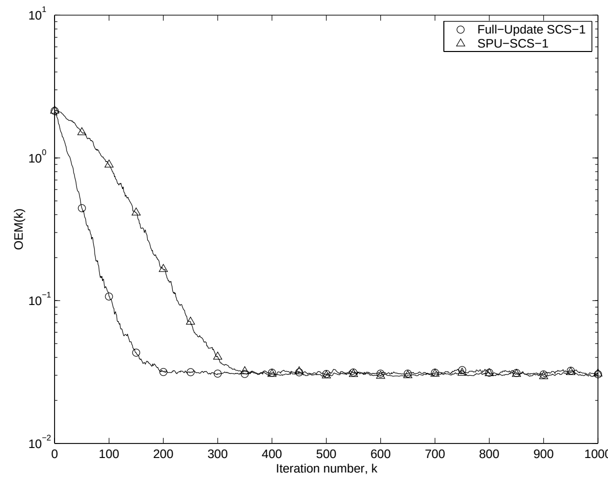

Figure 2: OEM plots for T-spaced SCS-1 and SPU-SCS-1.

The T-spaced implementations of NCMA and SCS-1 and their selective-partial-update versions have been simulated on this channel for $N = M = 4$ and $B = 1$. Note that perfect equalisation can be achieved only if $N \geq 5$. All algorithms were initialised to $[1, 0, 0, 0]^T$. The algorithm stepsizes were chosen to get similar OEM values at steady-state ($\mu = 0.2$ for NCMA and SCS-1, and $\mu = 0.125$ for SPU-NCMA and SPU-SCS-1). The OEM plots for NCMA and SPU-NCMA, averaged over ten realisations, are shown in Fig. 1. The OEM plots for SCS-1 and SPU-SCS-1 are depicted in Fig. 2. Evidently, selective-partial-update algorithms converge more slowly than their full-update counterpart. However, in the case of SPU-NCMA and SPU-SCS-1, only one quarter of the equaliser parameters are updated at every iteration ($B/M = 1/4$).

6.2. $T/2$-Spaced NCMA and SCS-1

The $T/2$-spaced channel impulse response is given by

$$\boldsymbol{h} = [-0.2, -0.3, 0.4, 0.1, -0.35, -0.15, -0.005, -0.002].$$

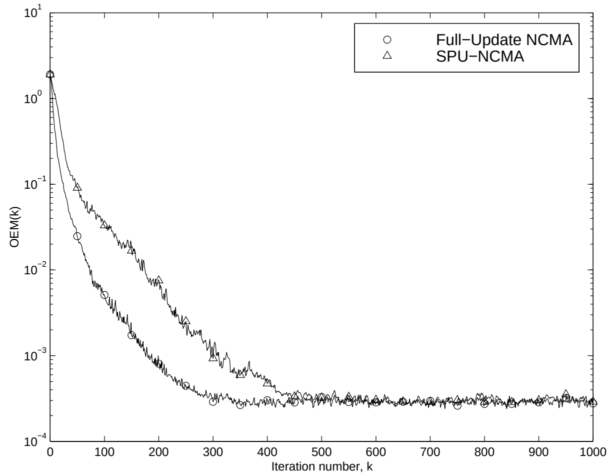

Figure 3: OEM plots for $T/2$-spaced NCMA and SPU-NCMA.

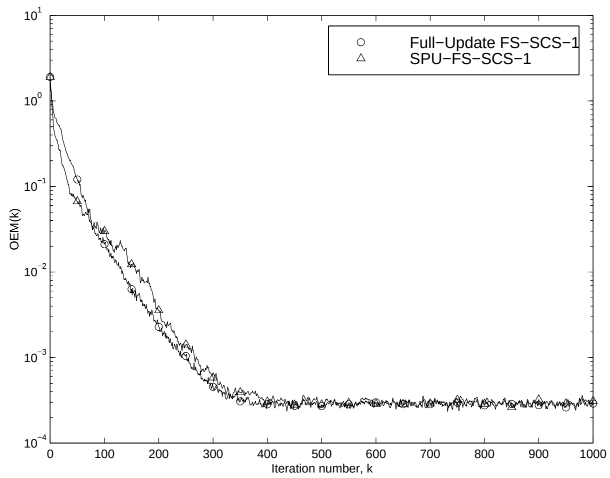

Figure 4: OEM plots for $T/2$-spaced SCS-1 and SPU-SCS-1.

Fractionally spaced implementations of the algorithms have been simulated for $N = 4$, $M = N$ and $B = 1$. The chosen equaliser length N is an underestimate of the channel length so as to impose a finite lower bound on OEM. The algorithms were initialised to $[-0.1, 0, 5, -3]^T$ and the stepsizes were selected to produce approximately the same OEM on convergence ($\mu = 0.5$ for FS-NCMA and FS-SCS-1, and $\mu = 0.25$ for SPU-FS-NCMA and SPU-FS-SCS-1). The OEM values, averaged over twenty realisations, are shown in Figs. 3 and 4. We note that FS-SCS-1 and SPU-FS-SCS-1 appear to have almost identical convergence speeds. Fig. 5 depicts the OEM curves for these algorithms with their stepsizes set to maximum stable values. In this case, SPU-FS-SCS-1 attains a lower OEM value (better ISI mitigation) than FS-SCS-1 by not escaping to other minima with different Δ and larger MSE.

7. CONCLUSION

We have developed selective-partial-update NCMA and SCS algorithms by drawing on the principle of minimum disturbance that

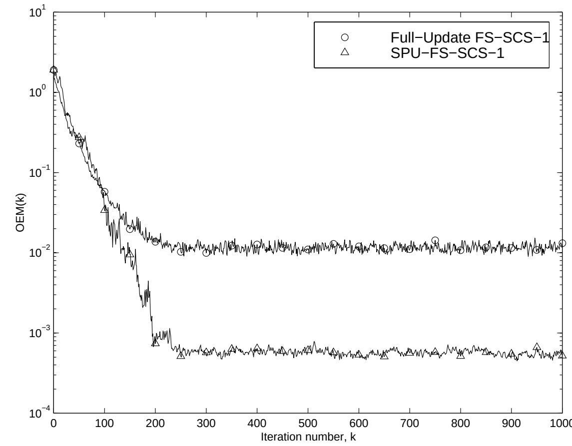

Figure 5: Comparison of OEM plots for fastest convergence.

underpins the derivation of the NLMS algorithm. The simulation results show that complexity reduction by selective-partial-updating does not necessarily slow down the convergence speed for fractionally-spaced implementations. This means that the full-update algorithm performance can be achieved at reduced computational complexity. Selective partial updating can also be applied to the affine projection realisation of NCMA and SCS by introducing multiple constraints. This extension follows the same line of development as in [3], and is not included here for space reasons.

8. REFERENCES

[1] K. Hilal and P. Duhamel, "A convergence study of the constant modulus algorithm leading to a normalized-CMA and block-normalized-CMA," in *Proc. EUSIPCO-92*, Brussels, Belgium, August 1992, pp. 135–138.

[2] O. Tanrıkulu, B. Baykal, A. G. Constantinides, and J. A. Chambers, "Soft constraint satisfaction (SCS) blind channel equalization algorithm," *Int. Journal of Adapt. Contr. and Sig. Proc.*, vol. 12, no. 2, pp. 117–134, March 1998.

[3] K. Doğançay and O. Tanrıkulu, "Selective-partial-update NLMS and affine projection algorithms for acoustic echo cancellation," in *Proc. of IEEE Int. Conf. on Acoustics, Speech, and Signal Processing, ICASSP 2000*, Istanbul, Turkey, June 2000, vol. 1, pp. 448–451.

[4] D. E. Knuth, *Sorting and Searching*, vol. 3 of *The Art of Computer Programming*, Addison-Wesley, Reading, MA, 1973.

[5] I. Pitas, "Fast algorithms for running ordering and max/min calculation," *IEEE Trans. on Circuits and Systems*, vol. 36, no. 6, pp. 795–804, June 1989.

[6] D. N. Godard, "Self-recovering equalization and carrier tracking in two-dimensional data communication systems," *IEEE Trans. Commun.*, vol. 28, pp. 1867–1875, November 1980.

[7] J. R. Treichler and B. G. Agee, "A new approach to multipath correction of constant modulus signals," *IEEE Trans. on Acoustics, Speech, Signal Processing*, vol. ASSP-31, pp. 459–472, April 1983.

[8] G. C. Goodwin and K. S. Sin, *Adaptive Filtering, Prediction, and Control*, Prentice Hall, New Jersey, 1984.