ANALYSIS OF ECHOES IN SINGLE-IMAGE RANDOM-DOT-STEREOGRAMS

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ABSTRACT

Three-dimensional depth information of a surface can be encoded in a two-dimensional image called single-image random-dot-stereograms or, more widely known, autostereograms. It is achieved by using the correlations of pixels in the horizontal direction. Using the correspondences between pixels in human brains or computer algorithms, surfaces can be reconstructed from autostereograms. However, in some cases, the reconstructed surfaces are not unique because of "echoes". In the presence of echoes, reconstruction of the original surface from an autostereogram cannot be guaranteed since no cue of the original surface is available in autostereograms. In this paper, the causes of echoes are investigated and conditions for echo-free reconstructions are derived. Based on these conditions, an improved autostereogram generation algorithm is proposed to guarantee echo-free autostereograms. Besides, the surface reconstruction algorithm is modified such that the originally encoded surfaces can always be reconstructed from echo-free autostereograms.

1. INTRODUCTION

In 1960, Julesz [2] studied binocular depth perceptions based on stereo information embedded in randomly generated images. These images appear completely random when viewed monocularly, but if viewed binocularly, depth information is perceptible. These images used by Julesz are known as *random-dot-stereograms*. Tyler and Clark [3] invented a new type of stereograms which combine random-dot-stereogram pairs into one image called *single-image random-dot-stereograms* or, more widely known, *autostereograms*. Autostereograms (or stereograms) are images containing depth information of surfaces. The depth information is encoded using the correlations of pixels in the horizontal direction.

A human being reconstructs a surface from a stereogram by using the correspondences of pixels in his or her brain. To establish correspondences, features in the left and the right eye images of the stereogram are matched to one another. The locations of the matched features are used to calculate the disparity and hence the depth information. However, the surface reconstructed from an stereogram is not necessarily unique since there can be more than one matches within a distance on a stereogram. In this situation, surface reconstruction from stereograms becomes a very difficult problem. Nevertheless, this problem can be overcome by choosing the "building block" of a stereogram such that it is uncorrelated in the horizontal direction. But in some cases, due to the nature of the surface encoded, we still cannot guarantee the uniqueness of the reconstructed surface. This is called "echo". Echo is a problem of many stereogram generation algorithms [4]. Echoes may not be noticeable when stereograms are viewed by human eyes. However, they can be a problem in some applications in which reconstructions of the original surfaces are needed. Echoes are described in [5] together with an echo reduction method called *hidden surface removal*. However, this technique can only remove part of the echoes.

The objective of this paper is to present the causes of echoes, and conditions under which echoes can be eliminated in stereograms. Using these conditions, an echo-free stereogram generation algorithm, as well as a surface reconstruction algorithm, are obtained. These algorithms guarantee that the original surfaces can always be reconstructed from echo-free stereograms.

2. THE PRINCIPLE OF AUTOSTEREOGRAMS

The principle underlying stereograms can be explained using geometry of stereo visions. Fig. 1 shows the top view when the surface of an object is viewed through an image plane. The variation of depth of the surface is represented by the surface profile $S_c(x, y)$, where the subscript c stands for "continuous" function. The image plane can be considered as a transparent plane with zero thickness placing between the eyes and the surface. As we will see, the stereograms form on this plane. Light rays CAL and CBR coming from the same source C through the image plane enter the eyes L and R, respectively. But this source can be reproduced by two identical (same color and intensity) light sources A and B separated by a distance on the image plane. This distance is called "image stereo separation (IS-separation)" [5] $\sigma(x, y)$ which will be discussed in the next section. If the eyes focus behind image plane at a suitable distance, an illusion of the surface is perceptible even if the surface does not exist. According to the above principle, a stereogram can be constructed by placing pixels A and B apart with a distance equal to the IS-separation for every pixel on the stereogram.

Depth information can be retrieved from stereograms using the correspondence of pixels. However, the depth information retrieved is not necessarily unique because of the presence of echoes. For instance, as shown in Fig 1, depth information of the surface at point C can be retrieved using the correspondence of pixels A and B. In the presence of echoes, not only point C, but point W (which does not exist in the original surface) is also perceptible since A, B and N are three identical pixels. In this case, we cannot tell whether C or W is at the original surface since no cue of the original surface is available.

3. IS-SEPARATIONS, STEREOGRAM GENERATIONS AND RECONSTRUCTIONS

Stereograms are generated by computers in most applications. Therefore, we will consider mainly discrete stereograms in the following.

A discrete stereogram $R(n_1, n_2)$ is a two-dimensional sequence defined on the points (n_1, n_2) for all $n_1 \in N_1$ and $n_2 \in N_2$, where $N_1 = \{1, 2, \ldots, L_1\}$ and $N_2 = \{1, 2, \ldots, L_2\}$ are sets of horizontal and vertical co-ordinates, respectively. In discrete stereograms, "discrete" surface profiles are encoded. A discrete surface profile $S(n_1, n_2)$ is obtained from a continuous surface profile $S_c(x, y)$ which is uniformly sampled and quantized to integers:

$$S(n_1, n_2) = Q(S_c(n_1, n_2)),$$

where the quantization process is denoted by $Q(\cdot)$. We also assume that $S(n_1, n_2)$ has the same size as $R(n_1, n_2)$.

For stereograms generated by computer algorithms, distance between any two points must be an integer. It is desirable if the values of IS-separations $\sigma(n_1, n_2)$ are integers for all values of $S(n_1, n_2)$ since truncation errors make analysis of echoes very difficult. It was suggested that IS-separation $\sigma(n_1, n_2)$ can be computed by using the geometry of stereo visions [5]. However, $\sigma(n_1, n_2)$ computed under this scheme are not necessarily integers for all values of $S(n_1, n_2)$. For implementations on computers, we suggest that IS-separations $\sigma(n_1, n_2)$ is computed using the following expression

$$\sigma(n_1, n_2) = M - aS(n_1, n_2), \qquad (1)$$

where M and a are integers. The values of M and a can be chosen such that $0 < \sigma(n_1, n_2) \leq E$ is satisfied, where E is the distance between the eyes L and R. Notice that $\sigma(n_1, n_2)$ computed from Eq. (1) are integers for all values of $S(n_1, n_2)$, and hence truncation problem is avoided. However, Eq. (1) will create visual distortions in depth, which may be acceptable if they are not sensitive to human eyes.

By definition, stereograms $R(n_1, n_2)$ are determined by the following formula

$$R(n_1, n_2) = \begin{cases} P(n_1, n_2), & 1 \leq n_1 \leq M \\ R(n_1 - \sigma(n_1, n_2), n_2), & M < n_1 \leq L_1, \end{cases} \qquad (2)$$

for all $n_1 \in N_1$ and $n_2 \in N_2$. The left most area ($1 \leq n_1 \leq M$) of a stereogram are "filled" with a L_2-by-M sequence $P(n_1, n_2)$. This sequence is called the "pre-defined pattern" which can be considered as the building block of stereograms. To simplify feature matching processes of right eye and left eye images, we assume that $P(n_1, n_2)$ is "horizontally uncorrelated", namely, $P(i, n_2) \neq P(j, n_2)$ for all integers $1 \leq i, j \leq M$ and $i \neq j$. For the values of other pixels ($M < n_1 \leq L_1$), they are obtained by copying from the pixels on the left according to the IS-separation $\sigma(n_1, n_2)$. These recursive "copying steps" proceeds along the horizontal direction until $n_1 = L_1$.

To reconstruct a surface $\widetilde{S}(n_1, n_2)$ from a stereogram, correspondences of pixels in the horizontal direction are established. By algorithmic definition, correspondence is established between two pixels (n_1, n_2) and $(n_1 - i, n_2)$ if

$$R(n_1, n_2) = R(n_1 - i, n_2)$$

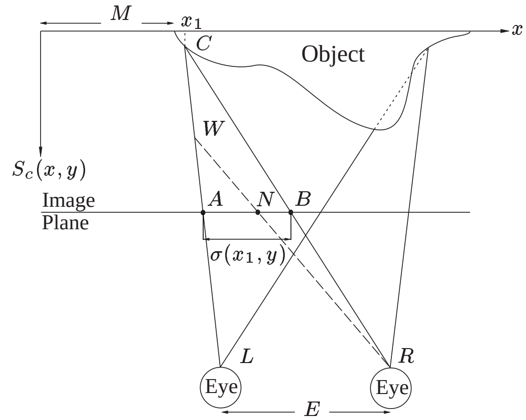

Fig. 1: Viewing a surface of an object through an image plane.

for all integers $1 \leq i \leq M$. The locations i of the correspondences are used to compute $\widetilde{S}(n_1, n_2)$:

$$\widetilde{S}(n_1, n_2) = \begin{cases} 0, & 1 \leq n_1 \leq M \\ \frac{M-i}{a}, & R(n_1, n_2) = R(n_1 - i, n_2), M < n_1 \leq L_1 \\ \text{Undefined}, & \text{otherwise} \end{cases} \qquad (3)$$

for all integers $1 \leq i \leq M$.

Recall that the left most area ($1 \leq n_1 \leq M$) of a stereogram is the pre-defined pattern $P(n_1, n_2)$ which is horizontally uncorrelated. By Eq. (3), $\widetilde{S}(n_1, n_2)$ is undefined and we have no way to recover depth information encoded in this area. To avoid lost of depth information in the reconstruction process, the original surface profile should be designed such that no depth information is contained in this area by simply setting them to zeros, i.e., $S(n_1, n_2) = 0$ for all $1 \leq n_1 \leq M$ (as shown in Fig. 1). The actual surface starts at $n_1 = M + 1$.

The reconstructed surface $\widetilde{S}(n_1, n_2)$ from Eq. (3) is not necessarily unique in the presence of echoes. The original surface is said to be "completely reconstructed" from the stereogram if $\widetilde{S}(n_1, n_2) = S(n_1, n_2)$ for all values of n_1 and n_2.

4. ECHOES AND CONDITIONS FOR ECHO SUPPRESSION

From Eq. (3), it can be derived that a surface $\widetilde{S}(n_1, n_2)$ reconstructed from a stereogram $R(n_1, n_2)$ is unique if and only if the maximum separation among any three consecutive pixels having the same value along the horizontal direction is greater than M. Based on this fact, we observe that there are two causes of echoes: "overlappings" of copying steps, and the small length of pre-defined pattern $P(n_1, n_2)$. Stereograms are generated in row-wise fashion such that the generation processes are independent of the vertical axis. To simplify discussions, only one-dimensional cases are considered in the following. These results can be applied directly to two-dimensional cases.

4.1. Condition Associated with "Overlappings" of Copying Steps

Let $p, q \in N_1$ be the horizontal co-ordinates of the stereogram $R(n_1)$, in which $p < q$. The values of $R(p)$ and $R(q)$ were copied from $R(p-\sigma(p))$ and $R(q-\sigma(q))$, respectively. These copying steps are depicted in Fig. 2 using arrows which are pointing to the directions of copying. As seen in Fig. 2(a), the depth level at p is lower than the depth level at q. Therefore, by Eq. (1), we have

$$q-\sigma(q) > p-\sigma(p) \tag{4}$$

which implies that the pixel copied by $R(q)$ is on the right of that copied by $R(p)$. However, the scenario is very different if $S(p)$ is greater than $S(q)$. In this case, a pixel copied by $R(q)$ is not necessarily on the right of that copied by $R(p)$. In the worst case, the copying step "overlaps" which means that $R(p)$ and $R(q)$ are copying the same pixel (as shown in Fig. 2(b)). Consequently, the values of $R(p)$, $R(q)$ and $R(q-\sigma(q))$ are the same. The maximum separation among these three equally valued pixels is $\sigma(q)$, which is definitely smaller than M. It is concluded that echoes appear on the resulting stereogram if the copying step overlaps. Clearly, the copying steps do not overlap if Eq. (4) is satisfied. Using Eq. (1), it becomes

$$q-p > a[S(p)-S(q)]. \tag{5}$$

Thus, we require that $R(q)$ and $R(p)$ should be separated apart by a distance greater than $a[S(p)-S(q)]$ pixels.

Using the above arguments and Eq. (5), the stereogram generation algorithm is modified to avoid echoes associated with overlappings of copying steps. It is required that any pixel being copied in the present copying step should be on the right of the pixel which has been copied in the most recent copying step. Define a "transition-point" $n_t \in N_1$ to be the horizontal co-ordinates of a surface profile $S(n_1)$ at which the depth level changes, i.e., $S(n_t) \neq S(n_t+1)$. Furthermore, a "down transition-point" n_t^d is a transition-point at which the depth level decreases, i.e., $S(n_t^d) > S(n_t^d+1)$. To avoid echoes in stereograms, it was suggested in [1] that the values of $a[S(n_t^d)-S(n_t^d+1)]$ pixels after a down transition-point n_t^d are assigned with arbitrary values (not copied from the left as in normal copying steps). These values should be horizontally uncorrelated to the pre-defined pattern $P(n_1)$. The copying process resumes when $n_1 = n_t^d + a[S(n_t^d) - S(n_t^d+1)]+1$. Therefore, to avoid the problem of echoes (caused by overlapping of copying steps) in stereogram generation processes, Eq. (2) is modified to Eq. (6) as shown at the bottom of this page.

We will show an example to demonstrate the above arguments. Stereogram $R_1(n_1)$, as shown in Fig. 3, was generated using Eq. (2) with $M=7$, $a=1$, and $P(n_1)=(1,2,\ldots,7)$. Since the problem of overlappings of copying steps is not considered in this stereogram generation process, the reconstructed surface $\widetilde{S}_1(n_1)$ from $R_1(n_1)$ is not unique. In contrast, a unique surface $\widetilde{S}_2(n_1)$ is reconstructed from another stereogram $R_2(n_1)$ which was generated from the same surface profile $S(n_1)$ using Eq. (6). Therefore, it can be shown that echoes caused by overlappings of copying steps can be effectively avoided using Eq. (6).

In the above example, however, $\widetilde{S}_2(13)$ and $\widetilde{S}_2(14)$ are not defined. It is because the copying steps are not taken at these

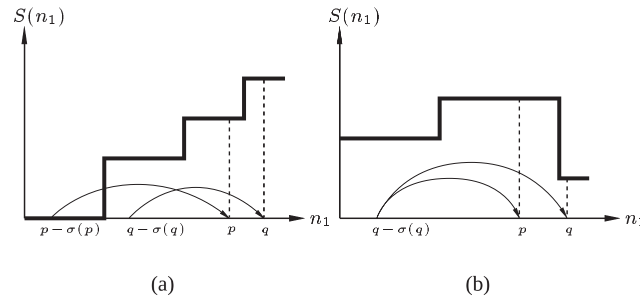

Fig. 2: Illustration of (a) non-overlapping, (b) overlapping copying steps.

points since a down-transition point is located at $n_1 = 12$. In this situation, although the reconstructed surface is unique, complete reconstruction of the original surface is still not guaranteed. To solve this problem, restriction on the "shape" of surface profiles is imposed such that the depth levels at $S(n_t^d+1),\ldots,S\big(n_t^d + a[S(n_t^d)-S(n_t^d+1)]+1\big)$ should be constant, i.e,

$$S(n_1) = S\big(n_t^d + a[S(n_t^d)-S(n_t^d+1)]+1\big), \tag{7}$$

for all $n_1 = n_t^d+1,\ldots,n_t^d+a[S(n_t^d)-S(n_t^d+1)]$. This ensure that the lost depth information can always be recovered from $\widetilde{S}\big(n_t^d+a[S(n_t^d)-S(n_t^d+1)]+1\big)$. In other words, for undefined $\widetilde{S}(n_1)$, its value can be always obtained from the nearest defined value of $\widetilde{S}(n_1)$ on the right (as shown in Fig. 3 using dotted line) provided that Eq. (7) is satisfied. In this way, complete reconstructions of the original surfaces are guaranteed.

4.2. Condition Associated with the Length of Pre-Defined Pattern

In this sub-section, we assume that copying steps do not overlap for all pixels $R(n_1)$. We will analyze the problem of echoes associated with the length M of the pre-defined pattern $P(n_1)$. Again, we let $p, q \in N_1$ be the horizontal co-ordinates of one-dimensional stereograms $R(n_1)$ in which $p < q$. Further, we assume that the value of $R(q)$ is copied from $R(p)$ such that $p = q-\sigma(q)$. Since the value of $R(p)$ is copied from $R(p-\sigma(p))$ and copying steps do not overlap for all values of n_1, pixels $R(q)$, $R(p)$ and $R(p-\sigma(p))$ are three consecutive pixels having the same value such that $q > p > p-\sigma(p)$. The maximum separation among these three pixels is the distance between $R(q)$ and $R(p-\sigma(p))$, i.e., $q-p+\sigma(p)$. To prevent echoes, we require that this distance should be greater than M such that

$$q-p+\sigma(p) > M.$$

Substituting $p = q-\sigma(q)$, we have

$$R(n_1,n_2) = \begin{cases} P(n_1,n_2), & 1 \le n_1 \le M \\ \text{Arbitrary(horizontally uncorrelated to } P(n_1,n_2)\text{)}, & n_t^d < n_1 \le n_t^d + a[S(n_t^d,n_2) - S(n_t^d+1,n_2)] \\ R(n_1-\sigma(n_1,n_2),n_2), & \text{otherwise.} \end{cases} \tag{6}$$

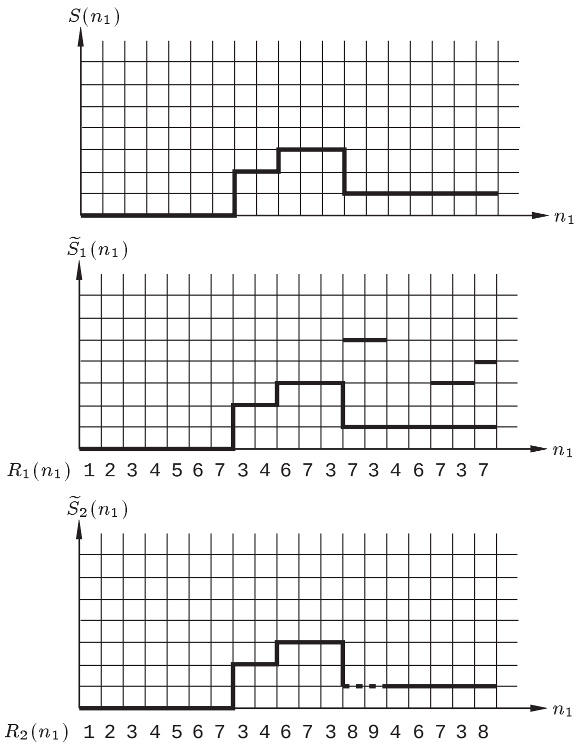

Fig. 3: Avoiding echoes caused by overlapping of copying steps.

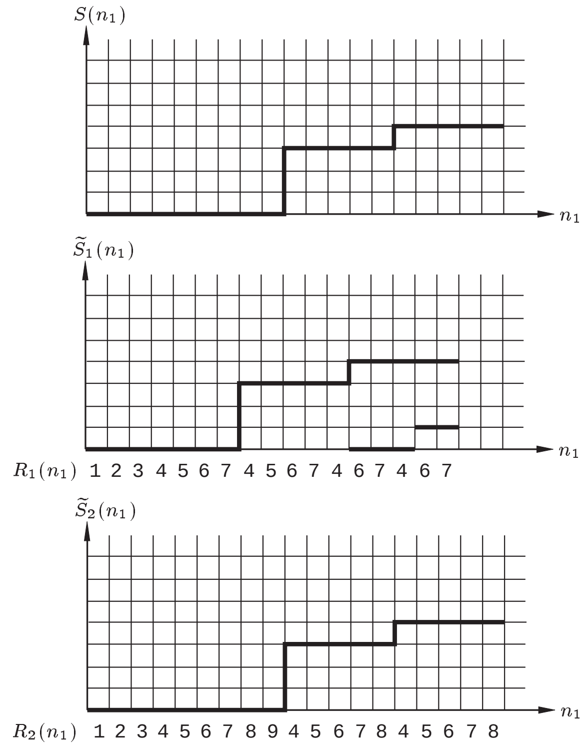

Fig. 4: Avoiding echoes caused by the small length of pre-defined pattern.

$$\sigma(q) + \sigma(q) > M,$$

and by Eq. (1), the inequality becomes

$$M > a[S(q) + S(p)]. \qquad (8)$$

For algorithmic convenience, Eq. (8) is generalized to

$$M > 2aS_{max}, \qquad (9)$$

where S_{max} is the greatest value that the surface profile $S(n_1)$ attains. Therefore, we conclude that echoes can be avoided for all pixels $R(n_1)$ if the length M of the pre-defined pattern $P(n_1)$ is greater than $2aS_{max}$ provided that copying steps do not overlap.

Here, we will demonstrate the condition given above. A stereogram $R_1(n_1)$ was generated from a surface profile $S(n_1)$ with $M = 7$, $a = 1$ and $P(n_1) = (1, 2, \ldots, 7)$ as shown in Fig. 4. The reconstructed surface $\widetilde{S}_1(n_1)$ is not unique since Eq. (8) is not satisfied at $n_1 = 13, 14, \ldots, 17$. Another stereogram $R_2(n_1)$ was generated from the same surface profile with $M = 9$, $a = 1$ and $P(n_1) = (1, 2, \ldots, 9)$. In this case, the length M of $P(n_1)$ is lengthened by 2 such that Eq. (8) and hence Eq. (9) are satisfied. Therefore, the reconstructed surface $\widetilde{S}_2(n_1)$ is unique and hence echo is avoided. Besides, $\widetilde{S}_2(n_1)$ equals to $S(n_1)$ for all n_1, therefore the original surface encoded in the stereogram is completely reconstructed.

5. CONCLUSION

This paper studies echoes in autostereograms. The principle of autostereograms have been stated followed by a discussion of echoes. There are two causes of echoes: "overlappings" of copying steps; and the small length of pre-defined pattern. Conditions for echo eliminations have been derived. In addition, the autostereogram generation and the reconstruction algorithms have been modified to satisfy these conditions such that echoes are avoided over an autostereogram. By using examples, we have shown that the originally encoded surfaces can always be reconstructed from echo-free autostereograms.

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