EFFECTS OF SECONDARY PATH MODELING ERRORS ON THE MODIFIED FX-LMS ALGORITHM FOR ACTIVE NOISE CONTROL

Paulo A. C. Lopes, Moisés S. Piedade

Universidade Técnica de Lisboa, IST/INESC
Rua Alves Redol n.9, 1000-029 Lisboa, Portugal
Paulo.C.Lopes@inesc.pt

ABSTRACT

The Modified FX-LMS (MFX-LMS) algorithm has been the subject of some research. It effectively removes the delay from the adaptation mechanism allowing faster convergence rates than the FX-LMS. It also allows the use of more sophisticated algorithms such as the RLS or the Kalman. Although this is true for perfect secondary path modeling, there is no extensive study of the influence of secondary path modeling errors in this algorithm. This paper tries to do just that. It shows that it's maximum allowed step-size is in fact almost independent of the delay in the secondary path, even in the presence of modeling errors. It also shows that for very large delays the algorithm is stable for phase errors of $\pm 60^o$ and for an estimated amplitude of more than half of the correct one.

1. INTRODUCTION

The most successful algorithm for Active Noise Control is the Filtered-X least-mean-square algorithm (FX-LMS, fig. 1) but it suffers from slow convergence due to high sensitivity to: eigenvalue spread and delay in the error path [5]. In fact it's maximum allowed step-size has been shown to be approximately equal to the inverse of the power of the filtered reference signal times the filter length plus the secondary path delay, $1/(P_x(L+\Delta))$ [3]. The Modified Filtered X-LMS (MFX-LMS) algorithm [2][9] (fig. 2) does not have this problem, namely if $\hat{S} = S$ then it corresponds to an exact implementation of the LMS algorithm. Also, the configuration on fig. 2 allows the use of other algorithms, such as the Kalman or the RLS [4][6][7] to improve on the tracking abilities of the LMS. The effect of secondary-path modeling errors on the FX-LMS has already been studied by several researchers [8][10][11][1]. The results from [10] show that the maximum allowed step size is proportional to cosine of the phase error, but the combined effect of delay and secondary path modeling errors haven't been considered. On the other hand, the effects of modeling errors on the Modified FX-LMS algorithm haven't had much attention. This paper tried to study just that, and compare the results with the FX-LMS algorithm. The paper will process as follows: first an analysis of the stability of the MFX-LMS for very small step sizes will be made, with results similar to the FX-LMS. Then stability bounds for the step size, in function of phase errors and amplitude errors, for the case of very large delays will be obtained. Finally the same analysis will be made for the FX-LMS, and the results will be compared.

This research was supported by PRAXIS XXI under contract number BD / 13791 / 97

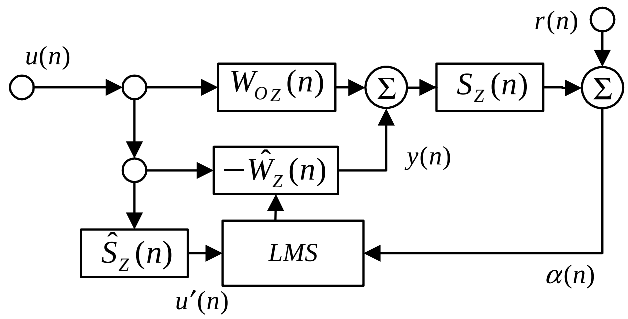

Fig. 1. The Filtered-X LMS algorithms for ANC.

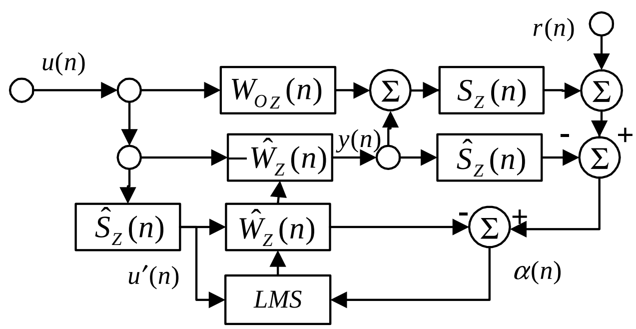

Fig. 2. The Modified Filtered-X LMS algorithm for ANC.

2. ANALYSIS OF THE MODIFIED FILTERED-X LMS

A good way to simplify the analysis, and following the path of other researchers [8][10], is to make the calculations in the frequency domain. However, if we want to compare both algorithms, then a pure frequency domain analysis isn't enough. In fact one of the major differences between them is that Modified Filtered-X LMS algorithm isn't sensitive to secondary path delay (at least when there is no modeling errors of the secondary path) while the Filtered-X LMS is. So, in order to take this issue into account a mixed time-domain frequency-domain analysis will be made, where the secondary path filters are described by an amplitude and phase at a given frequency, plus an integer delay. So $\hat{S}_z$ and S_z will be described by respectively: $\hat{S}$, $\theta_{\hat{S}}$, $\hat{d}$ and S, θ_S, d.

In this case, any of the algorithms used in fig. 1 or fig. 2 can be written as:

$$\hat{w}(n) \;=\; \hat{w}(n-1) + \mu u(n-\hat{d})\hat{S}_z^* \alpha^*(n) \qquad (1)$$

This gives for the MFX-LMS algorithm:

$$\begin{aligned}\alpha(n) \;=\;& u(n-d) w_o^* {S_z}^* - u(n-d)\hat{w}^*(n-1-d){S_z}^* + \\ & u(n-\hat{d})\hat{w}^*(n-1-\hat{d})\hat{S}_z^* + \\ & -u(n-\hat{d})\hat{S}_z^*\hat{w}^*(n-1) + r(n)\end{aligned}$$

The discussion will be limited to convergence of the mean. So, replacing $\alpha(n)$ in eq. 1, taking expected values and letting $R_{\hat{d}\hat{d}} = E[u(n-\hat{d})u(n-\hat{d})]$ and $R_{\hat{d}d} = E[u(n-\hat{d})u(n-d)]$, it is obtained:

$$\begin{aligned}E[\hat{w}(n)] \;=\;& E[\hat{w}(n-1)] + \\ & \mu R_{\hat{d}d}\hat{S}_z^* S_z w_o + \\ & -\mu R_{\hat{d}d}\hat{S}_z^* S_z E[\hat{w}(n-1-d)] + \\ & \mu R_{\hat{d}\hat{d}}\hat{S}_z^*\hat{S}_z E[\hat{w}(n-1-\hat{d})] + \\ & -\mu R_{\hat{d}\hat{d}}\hat{S}_z^*\hat{S}_z E[\hat{w}(n-1)]\end{aligned} \qquad (2)$$

Taking the Z-Transform, the poles of the system are given by:

$$z - 1 + \left(R_{\hat{d}\hat{d}}\hat{S}^2 - \frac{R_{\hat{d}\hat{d}}\hat{S}^2}{z^{\hat{d}}} + \frac{R_{\hat{d}d}\, S\, \hat{S}}{e^{i\,(\hat{\theta}_S - \theta_S)}\, z^d} \right) \mu = 0 \qquad (3)$$

This equation allows us to write μ as a function of z, $\mu = \Gamma(z)$. This is an interesting property. It means that for any given pole there is only one step size that can result in a system with that pole. The inverse is obviously not true, since for any given step size the algorithm has obviously several modes of convergence, or poles. Zero step size is an exception, the only pole is $z = 1$ for this case. The same is true for very small step sizes. In this case the function can be inverted, and using the rule for derivative of the inverse a linear approximation for very small step-sizes can be made.

$$z \approx 1 - \mu\, e^{-i(\theta_S - \theta_{\hat{S}})} R_{\hat{d}d} S\hat{S} \qquad (4)$$

This equation tells us that, for positive but very small step-sizes the algorithm is stable if and only if: $\hat{d} = d$ (or else $R_{\hat{d}d}$ may be negative) and the phase error is less than 90^o. These results are very intuitively satisfying, and in agreement with previous results for the FX-LMS algorithm.

Assuming these conditions are met, it still remains to determine what are the maximum allowed step-sizes that guarantee stability. In order to do this a different approach has to be taken. The limiting values are when the poles of the system lie on the unit circle, that is $z = e^{-\theta_z i}$. The poles are the values of z for which $\mu = \Gamma(z)$, as given by eq. 3, is a real number. So, to get the desired result one can first calculate the values of θ_z for which μ is real, and then calculate $\mu = |\Gamma(e^{-\theta_z i})|$. In order to obtain analytical results some simplification where made, which give sufficient conditions for stability, but that are only necessary for very large delays.

The absolute value of μ is given by:

$$\frac{\sqrt{2}\,\sqrt{1-\cos(\theta_z)}}{R_{\hat{d}\hat{d}}\,\hat{S}\,\left|\hat{S} + e^{i\,\theta_z\, d}\left(-\hat{S} + \frac{S}{e^{i\,(\hat{\theta}_S - \theta_S)}}\right)\right|} \qquad (5)$$

This function is plotted in fig. 3. A lower bound for this is:

$$\frac{\sqrt{2-2\,\cos(\theta_z)}}{R_{\hat{d}\hat{d}}\,\hat{S}\,\left(\hat{S} + \left|\hat{S} - S\, e^{i\,(\hat{\theta}_S - \theta_S)}\right|\right)} \qquad (6)$$

To determine the values of θ_z of the poles of the system,

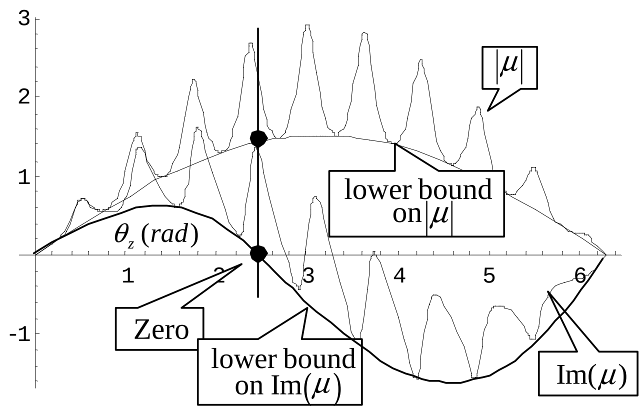

Fig. 3. Plot of $|\mu|$ and $\mathrm{Im}(\mu)$ vs θ_z for the MFX-LMS algorithm and phase error of 18^o. The amplitude follows a $\sqrt{1-\cos(\theta_z)}$ trend.

one must make the imaginary part of μ equal to zero, as in eq. 7. In fig. 3 it it can be seen that the smallest values for θ_z is the one which results in a lower limit for the step-size.

$$2\left(\hat{S}\,\cos(\frac{\theta_z}{2}) + S\,\cos(\hat{\theta}_S - \theta_S - \frac{\theta_z}{2} - d\,\theta_z) - \hat{S}\,\cos(\frac{\theta_z + 2\,d\,\theta_z}{2})\right)\,\sin(\frac{\theta_z}{2}) = 0 \qquad (7)$$

Making $\Delta\theta_S = \theta_S - \hat{\theta}_S$, and, if there are no amplitude errors, then this expression simplifies to:

$$\sin(\theta_z) - 4\sin(\frac{\Delta\theta_S}{2})\ \sin(\frac{\theta_z}{2})\ \sin(\frac{\Delta\theta_S - \theta_z - 2\theta_z d}{2}) \qquad (8)$$

which has as a lower bound:

$$\sin(\theta_z) - 4|\sin(\frac{\Delta\theta_S}{2})|\ \sin(\frac{\theta_z}{2}) \qquad (9)$$

The roots of this equation correspond to lower bounds for the values of θ_z of the poles of the system. So replacing these values in the lower bound for $|\mu|$, and since this grows with θ_z, finally, a bound for the value of step-size which assures stability is obtained:

$$\frac{2\sqrt{2\cos(\Delta\theta_S) - 1}}{R_{\hat{d}\hat{d}}\, S^2\left(1 + \sqrt{2 - 2\cos(\Delta\theta_S)}\right)} \qquad (10)$$

following a similar approach, but now ignoring phase errors, the following expression is obtained:

$$\frac{2\sqrt{\frac{S\,(2\,\hat{S} - S)}{\hat{S}^2}}}{R_{\hat{d}\hat{d}}\,\hat{S}\,\left(\hat{S} + |\hat{S} - S|\right)} \qquad (11)$$

This equations are plotted in fig. 4. It can be seen that it is

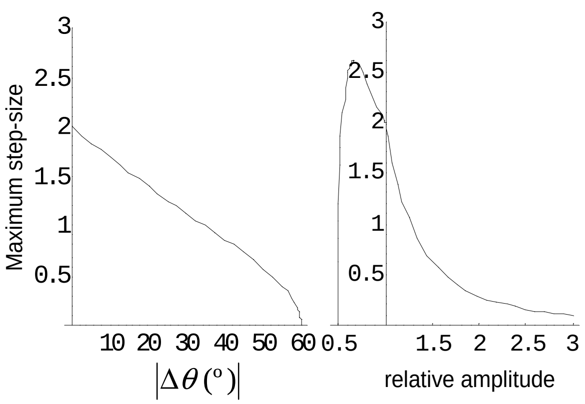

Fig. 4. Value of μ which assures stability versus phase and amplitude modeling errors for the MFX-LMS

possible to stabilize the algorithm as long as the phase errors are less than 60^o and the estimated amplitude is more than half of the real one. It should also be noted that although these are only bound on the step-size for finite d, they should be very close to the actual limits for very large delay.

In fig. 5 is presented the result of computer simulations. The chart was obtained with a time domain implementation of the FX-LMS algorithm, using a synchronous sinusoid as the reference signal, and a delay of twenty samples in the secondary path. As expected the convergence is much slower for the case of 50^o phase error. The values for the errors in the secondary that maintained the stability of the algorithm where almost exactly the ones predicted.

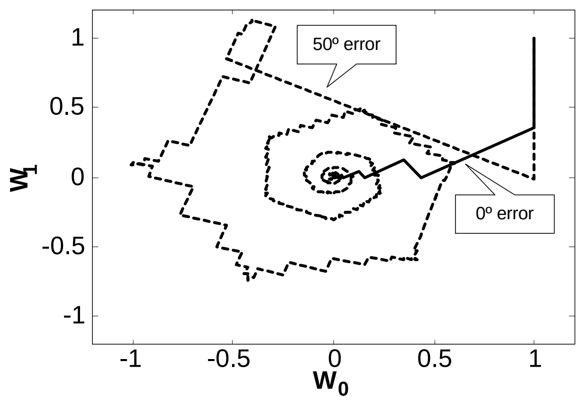

Fig. 5. Experimental results for the convergence of the MFX-LMS algorithm for secondary modeling phase-errors of 50^o and 0^o.

3. ANALYSIS OF THE FILTERED-X LMS

The Filtered-X LMS algorithm has been extensively analyzed, so this will only be a brief exposition of the results obtained using a similar approach as before. For this case

$$\begin{aligned} E[\hat{w}(n)] &= E[\hat{w}(n-1)] + \\ &\quad \mu R_{\hat{d}d}\hat{S}_z^* S_z w_o + \\ &\quad -\mu R_{\hat{d}d}\hat{S}_z^* S_z E[\hat{w}(n-1-d)] \end{aligned} \qquad (12)$$

Taking the Z-Transform to obtain the equation for the poles, and then doing, as before, a linear approximation for very small step-sizes the exact same equation as in 4 is obtained, confirming that both algorithms perform the same way for small step-sizes. The values of the maximum allowed step-sizes are different however. Making $z = e^{-\theta_z i}$, as before, the amplitude of μ is given by:

$$\frac{\sqrt{2}\sqrt{1 - \cos(\theta_z)}}{R_{\hat{d}d} S\hat{S}} \qquad (13)$$

and the imaginary part is:

$$\frac{2\cos(\hat{\theta}_S - \theta_S - \frac{\theta_z}{2} - \theta_z\, d)\ \sin(\frac{\theta_z}{2})}{R_{\hat{d}\hat{d}}\, S\,\hat{S}} \qquad (14)$$

Calculating the zeros and replacing in $|\mu|$ gives for the maximum allowed step size:

$$\min\left(\frac{\sqrt{2 - 2\cos(\frac{\pi \pm 2\Delta\theta_S}{1 + 2\,d})}}{R_{\hat{d}\hat{d}}\, S\,\hat{S}}\right) \qquad (15)$$

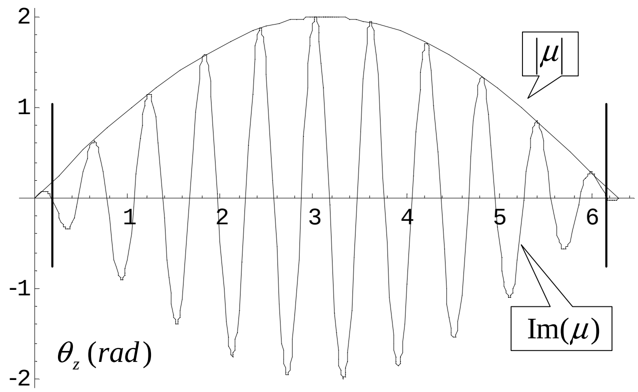

Fig. 6. Plot of $|\mu|$ and $\mathrm{Im}(\mu)$ vs θ_z for the FX-LMS algorithm and a phase error of 18^o. The amplitude follows a $\sqrt{1-\cos(\theta_z)}$ trend.

This equation is plotted in fig. 7 for several values for the secondary path delay. For $d = 0$ the shape is of a cosine, as shown in [10]. For higher delays, the maximum value for the step size is decreased, and the function assumes a more linear behavior.

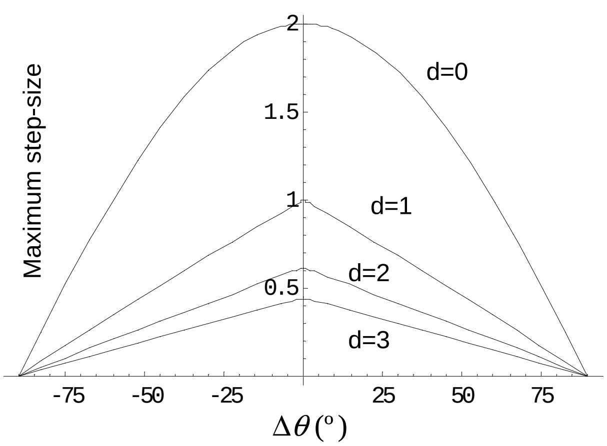

Fig. 7. Value of μ which assures stability versus phase and amplitude modeling errors for the FX-LMS algorithm.

4. CONCLUSION

For very small step-sizes, both the FX-LMS and the Modified FX-LMS algorithm are stable as long as the secondary path model has a phase error smaller than 90^o. However, the maximum allowed step-size of the Modified FX-LMS algorithm is almost independent of the secondary path delay, even in the presence of modeling errors, which is not the case for the FX-LMS algorithm. Quantitative bounds for the step-size which insure stability in the presence of modeling errors and large delays were derived for both the algorithms. Namely, the MFX-LMS algorithm is stable for phase errors of $\pm 60^o$ and for an estimated amplitude more than half of the correct one, even for arbitrarily large values of the secondary path delay.

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