SPACE-TIME DIVERSITY SYSTEMS BASED ON UNITARY CONSTELLATION-ROTATING PRECODERS*

Yan Xin, Zhengdao Wang, Georgios B. Giannakis

Dept. of ECE, University of Minnesota, 200 Union Str. SE, Minneapolis, MN 55455, USA
E-mails: {yxin,zhengdao,georgios}@ece.umn.edu

ABSTRACT

We present a unified approach to constructing linear space-time (ST) block codes based on unitary constellation-rotating (ST-CR) precoders. We show that with an arbitrary number of M-transmit and N-receive antennas, ST-CR precoders achieve 1 symbol/sec rate and enjoy maximum diversity gain MN over both quasi-static and fast fading channels. We also compare real with complex rotations to delineate the tradeoff between performance and complexity. Based on a simplified decoder, we study diversity and coding gains as well as information-theoretic aspects of the proposed ST-CR scheme. Compared with ST orthogonally designed (ST-OD) codes, ST-CR precoding provides larger coding gain and maximum mutual information. Though ST-OD codes afford simpler decoding, the tradeoff between performance and rate versus complexity favors the ST-CR codes when M, N or the spectral efficiency of the system increase.

1. INTRODUCTION

Considered as a major effective technique in combating fading effects, transmit diversity schemes have been widely studied and applied in practice (see [1, 12] and references therein). In particular, transmit diversity based on space-time coding across M antennas leads to significant improvement in performance when appropriate signal processing is employed at the receiver.

ST trellis codes enjoy maximum diversity and large coding gains but decoding complexity grows exponentially in M and in the transmission rate [13]. The latter can be prohibitive when using large size constellations. On the other hand, ST-OD block codes can afford a low complexity linear decoding scheme [1, 12]. But for complex constellations and $M > 2$, ST-OD codes can only achieve rate of $1/2$ symbol/sec except for the sporadic codes with rate $3/4$ symbol/sec that are known so far only for $M = 3, 4$. Moreover, the complexity of encoding and decoding grows considerably for $M > 8$.

A new class of ST block codes based on constellation-rotating precoders was introduced recently in [6, 14]. It turns out that for any M and N, there exist ST-CR precoders that can achieve maximum diversity gain MN and transmission rate of 1 symbol/sec [14]. The complex rotations of [14] offer the potential to improve coding gain relative to the real rotations considered in [6]. Unitary CR precoders are available in closed form with $M = 2$ transmit antennas and for BPSK constellations [6, 14]. However, finding maximum diversity CR precoders with large coding gains for $M > 2$ requires computer search which can be feasible only for small size constellations [14]. Based on the algebraic codes of [3], ST-CR codes were independently presented in [4] only for real precoders and for certain specific choices of M. Using the sphere decoding (SD) algorithm of [5], reduced complexity (relative to exhaustive search) and near-optimum (in the maximum likelihood (ML) sense) performance were purported in [4] based on simulations.

In this paper, we present a unified approach to constructing ST-CR codes. We compare real with complex CR precoders in terms of performance and complexity. We also apply SD to reduce the ML decoding complexity. In addition to diversity and coding gains, the maximum mutual information criterion of [7] is employed to evaluate the performance of ST-CR precoders and compare them with the ST-OD codes of [1, 12].

2. ST-CR PRECODING

We consider a wireless link with M transmit and N receive antennas over Rayleigh fading channels. The data stream s_i from the constellation set $\mathcal{C}$ is first parsed into T_0-dimensional signal vectors $\mathbf{s}$, and then linearly precoded by a $T_0 \times T_0$ rotation matrix $\mathbf{\Theta}$. The precoded block $\mathbf{\Theta s}$ is fed to a ST encoder. The ST encoder maps $\mathbf{\Theta s}$ to an $M \times T_0$ signal matrix $\bar{\mathbf{S}}$. The (m, i)th entry $\bar{s}_{mi} := u_{mi}\boldsymbol{\theta}_i^T\mathbf{s}$, is transmitted through the mth antenna at the ith time interval, where u_{mi} is a weighting coefficient, $\boldsymbol{\theta}_i^T$ denotes the ith row of $\mathbf{\Theta}$, and T_0 is chosen to be equal to M. Defining $[\mathbf{U}]_{mi} := u_{mi}$ and $\mathbf{D}_s := \text{diag}\{\boldsymbol{\theta}_1^T\mathbf{s}, \ldots, \boldsymbol{\theta}_M^T\mathbf{s}\}$, we can write the $M \times M$ transmitted signal matrix $\bar{\mathbf{S}}$ as

$$\bar{\mathbf{S}} = \mathbf{U}\mathbf{D}_s. \tag{1}$$

The signal x_{ni} received by antenna n at the ith time interval after receive-filtering and symbol rate sampling is given by:

$$x_{ni} = \sum_{m=1}^{M} h_{nm}u_{mi}\boldsymbol{\theta}_i^T\mathbf{s} + w_{ni}, \tag{2}$$

where h_{nm} are fading coefficients (between the mth transmit and nth receive antenna), which are assumed to be: i.i.d. zero mean complex Gaussian random variables with variance 0.5 per dimension; constant during the T_0 symbol interval (*quasi-static fading*); and available to the receiver. Moreover, w_{ni} are assumed to be independent samples of a zero mean complex Gaussian random variable with variance $\sigma^2/2$ per dimension. Let $\mathbf{X}$ be the $N \times M$ received signal matrix with (n, i)th entry $[\mathbf{X}]_{ni} = x_{ni}$; $\mathbf{H}_o$ the $N \times M$ channel matrix with $[\mathbf{H}_o]_{nm} = h_{nm}$; and $\bar{\mathbf{W}}$ the $N \times M$ noise matrix with $[\bar{\mathbf{W}}]_{ni} = w_{ni}$. Applying these notational conventions, under the quasi-static fading assumption, we can write (2) in matrix form as follows:

$$\mathbf{X} = \mathbf{H}_o\bar{\mathbf{S}} + \bar{\mathbf{W}} = \mathbf{H}_o\mathbf{U}\mathbf{D}_s + \bar{\mathbf{W}}. \tag{3}$$

Matrix $\mathbf{U}$ was chosen to be identity in [6, 14]. We restrict $\mathbf{U}$ to be a unitary matrix in this paper. Since $\mathbf{H}_o$ is a complex Gaussian

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matrix with i.i.d. entries having zero mean and equal variance, the distribution of $\mathbf{H}_o\mathbf{U}$ is the same as the distribution of $\mathbf{H}_o$. Thus, the average probability error rate is not affected by the choices of $\mathbf{U}$. However, $\mathbf{U}$ could be used to alleviate high power amplifier non-linearity problems by selecting its rows to have constant modulus entries (note that with $\mathbf{D}_s$ only, (3) corresponds to a TDMA-like transmission with each antenna pausing for $M-1$ out of M time slots).

3. DESIGN CRITERIA

Under quasi-static fading, our approach is to minimize the pairwise error probability (PEP) in the ML detection of the codeword $\mathbf{s}$ (detected erroneously as $\tilde{\mathbf{s}}$) according to the following design criteria (see also [13, 14]):

- *Diversity criterion:* Maximize the minimum rank l of $\mathbf{D}_s - \mathbf{D}_{\tilde{s}}$ over all distinct pairs of $\{\mathbf{s}, \tilde{\mathbf{s}}\}$ with $\mathbf{D}_{\tilde{s}} := \text{diag}\{\boldsymbol{\theta}_1^T\tilde{\mathbf{s}}, \ldots, \boldsymbol{\theta}_M^T\tilde{\mathbf{s}}\}$ (l is called the diversity gain). The maximum diversity gain in our setting will turn out to be MN.
- *Coding criterion:* Maximize the minimum of the l-th root of the sum of determinants of all $l \times l$ principal cofactors of $(\mathbf{D}_s - \mathbf{D}_{\tilde{s}})(\mathbf{D}_s - \mathbf{D}_{\tilde{s}})^{\mathcal{H}}$ taken over all the distinct pairs $\{\mathbf{s}, \tilde{\mathbf{s}}\}$.

Consider $\boldsymbol{\Theta}$ to be an $M \times M$ rotation matrix with entries from $\mathbb{R}$ (real) or $\mathbb{C}$ (complex) satisfying $\boldsymbol{\Theta}^{\mathcal{H}}\boldsymbol{\Theta} = \mathbf{I}_M$, where $\mathcal{H}$ denotes conjugate transpose of a matrix. Given a constellation $\mathcal{C}$ that entries of $\mathbf{s}$ are drawn from, we define the set $\mathcal{Q} := \{\boldsymbol{\Theta}\mathbf{s}, \mathbf{s} \in \mathcal{C}^M\}$ to be a rotated version of $\mathcal{C}^M$ by $\boldsymbol{\Theta}$. Special rotations were designed in [9] so that: given an M-dimensional constellation $\mathcal{Q}$, if a vector $\boldsymbol{\Theta}\mathbf{s} \in \mathcal{Q}$ has its components $\boldsymbol{\theta}_1^T\mathbf{s}, \ldots, \boldsymbol{\theta}_M^T\mathbf{s}$ all different from those of any other vector in $\mathcal{Q}$, then according to the diversity criterion, the minimum rank of $\mathbf{D}_s - \mathbf{D}_{\tilde{s}}$ is equal to M. Hence, by selecting special CR precoders $\boldsymbol{\Theta}$, we can achieve maximum transmit diversity M.

It was proved in [14] that for any finite constellation there exists at least one linear unitary precoder achieving the maximum transmit diversity M. Based on this fact, the matrix $\boldsymbol{\Theta}$ that achieves maximum diversity and coding gains is found by maximizing the minimum product distance defined as follows:

$$D_{M,\min} := \min_{\mathbf{s} \neq \tilde{\mathbf{s}}} \prod_{m=1}^{M} |\boldsymbol{\theta}_m^T(\mathbf{s} - \tilde{\mathbf{s}})|. \tag{4}$$

As we only consider $\boldsymbol{\Theta}$ to be a rotation matrix, the resulting optimization problem can be formulated as follows [c.f. (4)]:

$$\boxed{\boldsymbol{\Theta}_{\text{opt}} = \arg\max_{\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathcal{H}}=\mathbf{I}} \min_{\mathbf{s} \neq \tilde{\mathbf{s}}} \prod_{m=1}^{M} |\boldsymbol{\theta}_m^T(\mathbf{s} - \tilde{\mathbf{s}})|.} \tag{5}$$

The design of the rotation $\boldsymbol{\Theta}$ was carried out in [6, 14] by using computer search on parsimonious $\boldsymbol{\Theta}$ parameterizations for small values of M and small size constellations. But as M and/or the size of constellations grows large, computer search is not feasible. In [3, 9], matrix $\boldsymbol{\Theta}$ was constructed by applying algebraic number theory tools for some specific values of M. Finding the optimal $\boldsymbol{\Theta}_{\text{opt}}$ in (5) for any given M remains an interesting problem.

4. ST-CR DECODING

The received signal (3) can be written as

$$\mathbf{X} = \mathbf{H}_o\mathbf{U}\mathbf{D}_s + \bar{\mathbf{W}} := \bar{\mathbf{H}}\mathbf{D}_s + \bar{\mathbf{W}}$$

Using the $\text{vec}(\cdot)$ operator to put the columns of $\mathbf{X}^T$ one after the other, we can rewrite Eq. (6) as

$$\begin{aligned} \bar{\mathbf{x}} := \text{vec}(\mathbf{X}^T) &= \begin{bmatrix} \text{diag}(\bar{\mathbf{h}}_1^T) \\ \vdots \\ \text{diag}(\bar{\mathbf{h}}_m^T) \end{bmatrix} \boldsymbol{\Theta}\mathbf{s} + \text{vec}(\bar{\mathbf{W}}^T) \\ &:= \mathbf{H}\boldsymbol{\Theta}\mathbf{s} + \bar{\mathbf{w}}, \end{aligned} \tag{7}$$

where $\bar{\mathbf{h}}_j^T$ denotes the j-th row of $\bar{\mathbf{H}}$ corresponding to the j-th receiver, and $\mathbf{H}$ is an $MN \times M$ block diagonal matrix. The received vector in (7) is equivalent to a received block from M uncoded transmit-antennas to MN receive-antennas with a channel matrix $\mathbf{H}$ that is almost always full rank. One can apply either successive interference cancellation (SIC) of [8, 10], or the sphere decoder (SD) [5] which has polynomial complexity in M regardless of the constellation size. Thanks to the special structure of $\mathbf{H}$ in (7), applying a maximum ratio combiner at the receiver yields:

$$\begin{aligned} \mathbf{x} &= \mathbf{H}^{\mathcal{H}}\bar{\mathbf{x}} \\ &= \text{diag}\Big(\sum_{n=1}^{N} |\bar{h}_{n1}|^2, \ldots, \sum_{n=1}^{N} |\bar{h}_{nM}|^2\Big)\boldsymbol{\Theta}\mathbf{s} + \boldsymbol{\eta} \\ &:= \mathbf{D}(\bar{\mathbf{h}})\boldsymbol{\Theta}\mathbf{s} + \boldsymbol{\eta}, \end{aligned}$$

where $\boldsymbol{\eta} := \mathbf{H}^{\mathcal{H}}\bar{\mathbf{w}}$ is an $M \times 1$ Gaussian noise of uncorrelated entries of possiblly different variances. The latter can be pre-whitened to obtain

$$\mathbf{y} = \mathbf{D}^{-\frac{1}{2}}(\bar{\mathbf{h}})\mathbf{x} = \mathbf{D}^{\frac{1}{2}}(\bar{\mathbf{h}})\boldsymbol{\Theta}\mathbf{s} + \mathbf{w}, \tag{8}$$

with $\mathbf{w}$ being an $M \times 1$ AWGN vector. When $\boldsymbol{\Theta}$ is real and $\mathbf{s}$ is complex, we use the SD to decode separately real and imaginary parts of $\mathbf{y}$ [5]. When both $\boldsymbol{\Theta}$ and $\mathbf{s}$ is complex, one should write an M-dimensional complex vector $\mathbf{y}$ as a $2M$-dimensional real vector. The decoding complexity will increase because we need to apply the SD for a $2M$-dimensional vector.

5. ST-CR PRECODER PROPERTIES

In this section, we assess the performance of ST-CR precoding and compare it with competing alternatives.

5.1. Performance Evaluation

We first state the optimality features of our ST-CR precoders:

Proposition 1 *Given a CR precoder $\boldsymbol{\Theta}$ with $D_{M,\min} > 0$, the ST-CR code given by (1) achieves full transmit diversity gain; its coding gain is given by*

$$CG_M \quad := \quad (D_{M,\min})^{2/M}. \tag{9}$$

The proof is straightforward since $D_{M,\min} > 0$ implies that the minimum rank of $\mathbf{D}_s - \mathbf{D}_{\tilde{s}}$ is M over all distinct pairs $\{\mathbf{s}, \tilde{\mathbf{s}}\}$.

In addition to Proposition 1, ST-CR has the following attractive properties:

1. ST-CR precoders are delay-optimal; i.e., the size T_0 of the ST encoded blocks is equal to M, and transmission achieves a rate of 1 symbol/sec. For example, when $M = 5$ and complex constellations are used, ST-OD codes require $T_0 \geq 16$, while for ST-CR codes, $T_0 = 5$. In general, ST-CR has smaller encoding delays than ST-OD.

2. Since by construction, over $T_0 = M$ periods, the M transmitted symbols are all different, the ST-CR code (1) is also suitable for fast fading (i.e., it is "smart and greedy" as codes suitable for fast fading were termed in [13]).

Remark: When $\mathbf{\Theta}$ is complex, $D_{M,\min}$ is given by (4) and hence the coding gain (9) increases because we have more dimensions to optimize over the rotation parameters. Another parameter that should be taken into account is the *product kissing number* κ_M which is the number of pairs $\{\mathbf{s}, \tilde{\mathbf{s}}\}$ in the constellation with distance equal to $D_{M,\min}$. Our experience is that one should use complex rotations when M increases, so that the increase in complexity (from M to $2M$ dimensions) is worth the performance enhancement. This point will be validated by Example 2 in Section 6.

5.2. Mutual Information Comparisons

In this subsection, we will prove that ST-CR precoders can achieve higher mutual information than ST-OD. First, we state without proof the following lemma.

Lemma 1. *Consider two random variables $X, Y > 0$ (a.s.) with $X = U\bar{\gamma}^m + f(\bar{\gamma})$ and $Y = V\bar{\gamma}^n + g(\bar{\gamma})$, where $f(\bar{\gamma}), g(\bar{\gamma})$ are two polynomials degree $< m$ and $< n$ respectively and both have with nonnegative random coefficients. Moreover, U, V, W are random variables > 0 (a.s.), $\bar{\gamma} > 0$, $m - n > 0$, and $E(V + \sum_{i=0}^{n-1} g_i)$, and $|E(\log U)|$ are bounded, where $g_i (i = 0, \ldots, n-1)$ denotes the ith coefficient of $g(\bar{\gamma})$. It then holds that $E(\log X) > E(\log Y)$ for sufficiently large values of $\bar{\gamma}$.*

From (7), the channel capacity is given by [7, 11],

$$C = \max_{tr\mathbf{R}_s = M} E[\log\det(\mathbf{I}_N + \frac{\bar{\gamma}}{M}\mathbf{H}\mathbf{R}_\mathbf{s}\mathbf{H}^{\mathcal{H}})], \qquad (10)$$

where $\bar{\gamma}$ is the average SNR per receive antenna and $\mathbf{R}_\mathbf{s}$ is the covariance matrix of $\mathbf{s}$.

By using the approach of [11], we obtain from (10) the maximum mutual information C_{CR} of our ST-CR precoders:

$$C_{\mathsf{CR}} = E[\log(\prod_{m=1}^{M}(1 + \bar{\gamma}\sum_{n=1}^{N}|h_{nm}|^2))].$$

For ST-OD codes with $M \in [5, 8]$ and complex constellations, only rate $1/2$ symbol/sec ST block codes are known so far [12]. The maximum mutual information achieved with ST-OD codes is given for these cases as:

$$C_{\mathsf{OD}} = \tfrac{1}{2}E[\log(1 + \tfrac{\bar{\gamma}}{M}\sum_{m=1}^{M}\sum_{n=1}^{N}|h_{nm}|^2)].$$

According to Lemma 1, it can be seen that $C_{\mathsf{CR}} > C_{\mathsf{OD}}$ for all these rate $1/2$ symbol/sec codes.

In particular, for $M = 3$ and rate $3/4$ symbol/sec ST-OD codes, the maximum mutual information is given by:

$$C_{\mathsf{OD}} = \frac{3}{4}E[\log(1 + \frac{\bar{\gamma}}{3}\sum_{m=1}^{3}\sum_{n=1}^{N}|h_{nm}|^2)]. \qquad (11)$$

We can draw the same conclusion that $C_{\mathsf{CR}} > C_{\mathsf{OD}}$ for large enough $\bar{\gamma}$ by applying Lemma 1. The upper bound of the average PEP is closely related to the channel capacity when the channel input is Gaussian [7]. Since $\mathbf{u} := \mathbf{\Theta}\mathbf{s}$ is more Gaussian than $\mathbf{s}$ in our case, symbol error rate (SER) performance of Figure 2 and the maximum mutual information curves of Figure 3 validate this upper bound approximation which becomes more accurate as the size of $\mathbf{\Theta}$ increases.

M	2	3	4
CG_M, κ_M (real)	$0.895, 48$	$0.547, 432$	$0.316, 2560$
CG_M, κ_M (complex)	$1, 48$	$0.620, 280$	$0.5, 3136$
Gain [dB]	0.484	0.544	1.99

Table 1: Coding gains of ST-CR codes for $M = 2, 3, 4$.

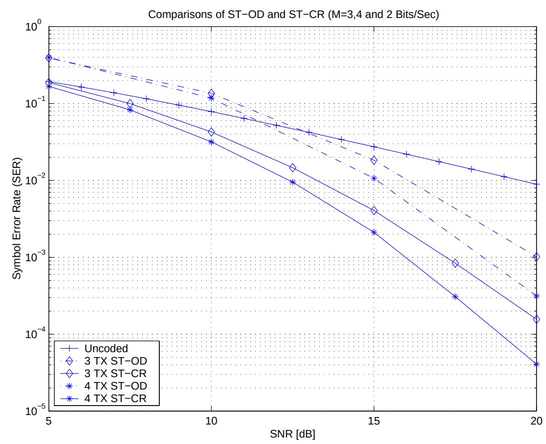

Figure 1: One receive antenna ($N = 1$)

6. SIMULATED PERFORMANCE

When simulating ST-CR codes, we normalized the average energy per symbol so that $\bar{\mathcal{E}}_s = 1$. The channel matrix $\mathbf{H}_o$ is modeled as in (3) and remains unchanged over the ST code length, before changing randomly; the AWGN has variance $\sigma^2 = 1/(2\text{SNR})$ per real dimension. Comparisons of ST-CR with the ST-OD block codes of [12] (denoted by $\mathtt{OD}_M$) are carried out for $M = 3, 4$, at the same spectral efficiency. We used only real rotations with the best values of the minimum product distance found in [3] for $M = 2, 3, 4$. For complex rotations we used the ones in [9, 14], except for $M = 3$, where we used the one obtained by computer search since it has a better minimum product distance than that found in [14] (see Table 1).

Example 1: Table 1 lists the coding gains of ST-CR codes (1) over normalized constellations for $M = 2, 3, 4$, with real rotations taken from [3], and complex rotations taken from [9, 14]. It also gives the product kissing number κ_M computed over $(4\text{-QAM})^M$.

Example 2: Figure 1 shows the performance of the ST-CR codes with complex $\mathbf{\Theta}$ for $M = 3, 4$ with 4-QAM modulation, and $N = 1$ receive antenna. It also shows the performance of the codes $\mathtt{OD}_3$ and $\mathtt{OD}_4$ with 16-QAM modulation. At 2 bits/sec, and at the same diversity gain ($M = 3$) ST-CR precoders exhibit a coding gain of more than 2 dB over $\mathtt{OD}_3$; for $M = 4$, the ST-CR precoders show a gain of more than 2 dB over $\mathtt{OD}_4$.

Example 3: Figure 2 shows the performance of ST-CR and $\mathtt{OD}_M$ codes for $M = 3, 4$ at spectral efficiency of 4 bits/sec with $N = 2$. We notice that the improvement over the ST-OD codes is enhanced when the spectral efficiency increases, or, when the number of receivers increases, which confirms our analytical results obtained on the maximum mutual information computed for both codes. For example, at spectral efficiency of 4 bits/sec and $N = 2$ receive antennas, ST-CR real codes gain more than 7 dB over $\mathtt{OD}_M$ for $M = 3, 4$. We notice also that the ST-CR complex codes have a slight performance gain over real ones, that is also confirmed by Table 1. Also, it can be seen that the performance gain over real

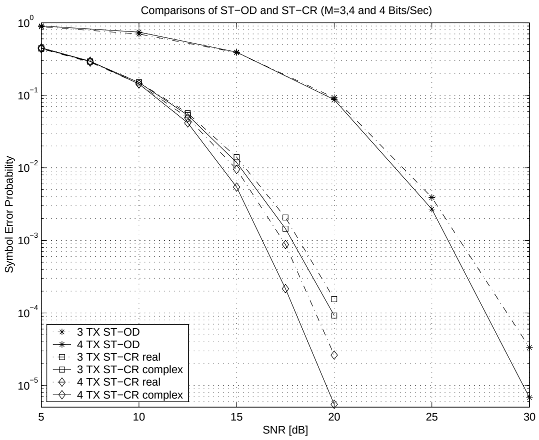

Figure 2: Two receive antennas ($N = 2$)

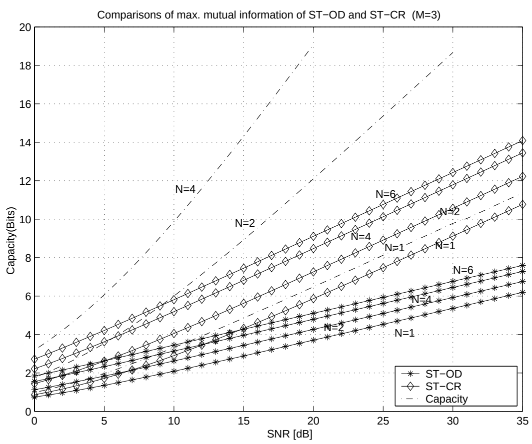

Figure 3: Max. mutual information of ST-CR and ST-OD

rotations increases as M increases.

Example 4: Figure 3 depicts the maximum mutual information for both ST-CR and ST-OD when $M = 3$ and $N = 1, 2, 4, 6$. The capacity loss of ST-OD is significant at high SNR compared with ST-CR. When N increases, the capacity loss becomes larger at high SNR. Also, when $N = 2$, at spectral efficiency of 4 bits/sec, ST-CR has about 9 dB gain over ST-OD at SNR$=$ 20 dB. This gain matches quite well with the performance gain of 7 dB in SER performance in Figure 2. Also we notice that the maximum mutual information achieved by ST-CR codes is still far from the channel capacity especially when N is large at high SNR.

7. CONCLUSIONS

We presented in this paper a unified approach for exploiting the transmit diversity in a multi-antenna environment using real and complex rotating precoders. We have shown that one can transmit at 1 symbol/sec with maximum transmit diversity and large coding gain by applying unitary constellation-rotating precoders that can be decoded with moderate complexity at the receiver. We have verified that complex rotations increase coding gain and complexity when SD is applied. It is worth using complex rotations when M and/or N are large. Finally, we have established that ST-CR codes can achieve better performance and larger maximum mutual information compared with ST-OD codes. We are currently investigating low-complexity decoders including the SIC-ILS algorithm of [10] that offers an attractive alternative particularly when the channel matrix is unknown.

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