BLIND CHANNEL IDENTIFICATION AND PROJECTION RECEIVER DETERMINATION FOR MULTICODE AND MULTIRATE SITUATIONS IN DS-CDMA SYSTEMS

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ABSTRACT

We consider multicode and multirate transmission scenarios in a DS-CDMA system operating in an asynchronous fashion in a multipath environment. Oversampling w.r.t. the chip rate is applied to the cyclostationary received signal and multisensor reception is considered, leading to a linear multichannel model. Channels for different users are considered to be finite-impulse response (FIR) and of possibly different lengths, depending upon their processing gains. We consider an individualized linear MMSE-ZF or projection receiver for a given user, exploiting its spreading sequence and timing information. In the multicode case, a certain user is considered to use several spreading codes in order to transmit at a higher rate. Considering different code sequences to be issuing from different *virtual* users, the propagation channel impulse responses of all these users are the same. However, the total channel impulse response which includes spreading sequences is different for all users. On the other hand, in the multirate case, a periodically varying set of periodic spreading codes spread successive symbols of a certain user. Symbols spread by different codes can therefore by considered to be issuing from different *virtual* users. The problem therefore boils down to classical multiuser detection with time-invariant interference canceling filters for each virtual user. A blind channel estimate is also obtainable through Capon's method (first used by Tsatsanis) as a by-product of the MMSE-ZF receiver algorithm.

1. INTRODUCTION

Multicode transmission is an alternative to multirate transmission for achieving high data rates in in third generation wireless networks based upon DS-CDMA. The former technique employs several channelization codes from a given set of spreading sequences of the same length to transmit data. Hence the input symbol stream at a high rate is demultiplexed and is spread by different spreading sequences to achieve greater capacity through parallel transmission. Multirate transmission, on the other hand, achieves faster data rates by sacrificing the processing gain. Each symbol is thus spread by a shorter code. The choice between the two modes of transmission is essentially open and depends on several parameters like the dynamic range of the receiver power amplifier, channel delay spread, number of actual users in the system, and the receiver algorithm. One advantage of multicode communications is that in large delay spreads, the intersymbol interference (ISI) is still small since the delay spread, even though significant in some cases, is much shorter than the symbol duration.

The purpose of this paper is to give improved insight into the multicode and multirate problems from a signal processing standpoint while presenting the blind MMSE-ZF [1] or projection receiver [2], taking into account the very particular structure of the problem in terms of the synchrony of the *virtual* multicode subusers and the fact that the propagation channel seen by all is the same. It is further shown that Capon's method can be employed to blindly estimate the channel impulse response for the user of interest.

The receiver presented in this paper is a per user (decentralized) receiver in the sense that its estimation requires the knowledge of the desired user's spreading sequence and timing information along with the second order statistics of the received signal.

2. CONVERTING MULTIRATE TO MULTICODE

Fig. 1 shows the multirate scenario and depicts the discretized signal model for a $u_k = 2$ times faster user in the system ($P_k = P/2$), with P the basic processing gain of the system. $\boldsymbol{h}_k$ represents the discrete time chip-rate channel for the kth user, and $\mathcal{T}(h_k)$ is the corresponding channel convolution matrix. The overall system therefore depicts spreading of successive symbols $a_k(n)$ by spreading sequences $\boldsymbol{c}_k$, which are later passed through $\boldsymbol{h}_k$, the discrete-time chip rate channel. It can be seen that for a user transmitting u_k times faster than the slowest rate, the block diagonal spreading matrices $\mathcal{T}(\boldsymbol{c}_k)$ have periodically varying (with period u_k) vector elements on the diagonal. Due to the ***i.i.d.*** nature of the input data sequence, $a_k(n)$, this user can be viewed as u_k cyclostationary users with modified spreading sequences shown in the figure with zeros padded either at the top or at the bottom.

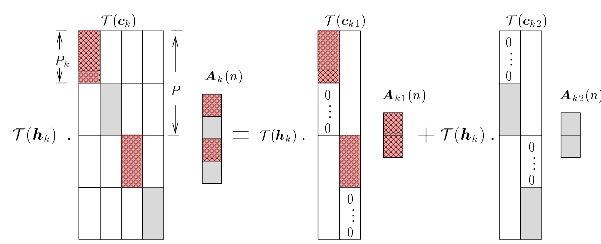

Fig. 1. Representation of a high-rate user as two slow-rate users, $P_k = P/2$.

This representation of faster users in a multirate system motivates the design of periodically time variant filters or yet better,

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independent parallel receivers for the successive symbols of a certain faster user. The advantage of the split-up approach is that the estimation of these filters is independent of each other, exactly as if they were independent users. If the system has $K^{'}$ concurrent users transmitting at different rates, then let us denote by K, where, $K = \sum_{k=1}^{K^{'}} u_k$, the modified number of the basic rate users in the system. In the following, we shall consider this slowest rate representation of a multirate system with K effective users, and a processing gain, P, and concentrate on the detection of data symbols for the kth user, which might be a sub-user of a higher rate.

It is clear, therefore, that the case of multirate communications can be cast into a multicode framework by considering the u_k sub-users of a high rate user k to be multicode users with modified spreading sequences (appended by zeros to keep a common processing gain, P). We shall henceforth consider a multirate description of the model and emphasize on its properties.

3. GENERAL MULTIUSER DATA MODEL

Fig. 2 shows the baseband signal model. The K users are assumed to transmit linearly modulated signals over a linear multipath channel with additive Gaussian noise. K is the number of virtual users. Any multirate user, $k^{'}$, using $u_k^{'}$ spreading sequences will simply contribute $u_k^{'}$ virtual users to K. It is assumed that the receiver employs M sensors to receive the mixture of signals from all users. The receiver front-end is an anti-aliasing low-pass filter. The continuous-time signal received at the mth sensor can be written in baseband notation as

$$y_m(t) = \sum_{k=1}^{K} \sum_{n} a_k(n) g_{k,m}(t - nT) + v_m(t), \qquad (1)$$

where the $a_k(n)$ are the transmitted symbols from user k, T is the common symbol period, $g_{k,m}(t)$ is the overall channel impulse response (including the spreading sequence, and the transmit and receive filters) for the kth user's signal at the mth sensor, and $\{v_m(t)\}$ is the complex circularly symmetric AWGN with power spectral density $\mathcal{N}_0$. Assuming the $\{a_k(n)\}$ and $\{v_m(t)\}$ to be jointly wide-sense stationary, the process $\{y_m(t)\}$ is wide-sense cyclostationary with period T. The overall channel impulse response $g_{k,m}(t)$, is the convolution of the spreading code c_k and $h_{k,m}(t)$, itself the convolution of the chip pulse shape, the receiver filter, and the actual channel representing the multipath environment. This can be expressed as

$$g_{k,m}(t) = \sum_{p=0}^{P-1} c_k(p) h_{k,m}(t - pT_c), \qquad (2)$$

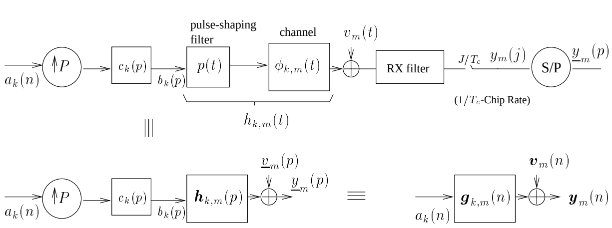


Fig. 2. Signal model in continuous and discrete time for the mth sensor, showing only the contribution from one user.

where T_c is the chip duration. The symbol and chip periods are related through the processing gain/spreading factor P: $T = PT_c$. S/P in fig. 2 denotes serial-to-parallel conversion (vectorization) with downsampling of a factor J. Sampling the received signal at J (oversampling factor) times the chip rate, we obtain the wide-sense stationary $PJ \times 1$ vector signal $\boldsymbol{y}_m(n)$ at the symbol rate. It is to be noted that the oversampling aspect (with respect to the symbol rate) is inherent to DS-CDMA systems by their very nature, due to the large (extra) bandwidth and the need to acquire chip-level resolution. This aspect directly translates into temporal diversity and explains the interference cancelation capability of these systems.

We consider the channel delay spread between the kth user and all of the M sensors to be of length $l_k T_c$. Let $n_k \in \{0, 1, \cdots P - 1\}$ be the chip-delay index for the kth user: $\boldsymbol{h}_{k,m}(n_k)$ is the first non-zero $J \times 1$ chip-rate sample of $\boldsymbol{h}_{k,m}(p)$. Let us denote by N_k, the FIR duration of $g_{k,m}(t)$ in symbol periods. It is a function of l_k, n_k, and P. We nominate the user 1 as the user of interest and assume that $n_1 = 0$ (synchronization to user 1). The symbol sequences for other users are relabeled (delayed or advanced), so that their relative delay with respect to user 1 falls in $[0, T)$.

Let $N = \sum_{k=1}^{K} N_k$. The vectorized oversampled signals at M sensors lead to a discrete-time $PMJ \times 1$ vector signal at the symbol rate that can be expressed as

$$\begin{aligned} \boldsymbol{y}(n) &= \sum_{k=1}^{K} \sum_{i=0}^{N_k - 1} \boldsymbol{g}_k(i) a_k(n - i) + \boldsymbol{v}(n) \\ &= \sum_{k=1}^{K} \boldsymbol{G}_{k,N_k} A_{k,N_k}(n) + \boldsymbol{v}(n) = \boldsymbol{G}_N \boldsymbol{A}_N(n) + \boldsymbol{v}(n), \end{aligned} \qquad (3)$$

$$\boldsymbol{y}(n) = \begin{bmatrix} \boldsymbol{y}_1(n) \\ \vdots \\ \boldsymbol{y}_P(n) \end{bmatrix}, \boldsymbol{y}_p(n) = \begin{bmatrix} \boldsymbol{y}_p^1(n) \\ \vdots \\ \boldsymbol{y}_p^M(n) \end{bmatrix}, \boldsymbol{y}_p^m(n) = \begin{bmatrix} y_{p,1}^m(n) \\ \vdots \\ y_{p,J}^m(n) \end{bmatrix}$$

$\boldsymbol{G}_{k,N_k} = [\boldsymbol{g}_k(N_k - 1) \ldots \boldsymbol{g}_k(0)]$, $\boldsymbol{G}_N = [\boldsymbol{G}_{1,N_1} \ldots \boldsymbol{G}_{K,N_K}]$, $A_{k,N_k}(n) = [a_k(n - N_k + 1) \ldots a_k(n)]^T$, $\boldsymbol{A}_N(n) = \left[A_{1,N_1}^T(n) \ldots A_{K,N_K}^T(n)\right]$, and the superscript ^{T} denotes transpose. For the user of interest (user 1), $\boldsymbol{g}_1(i) = (\boldsymbol{C}_1(i) \otimes I_{MJ})\, \boldsymbol{h}_1$, where, $\boldsymbol{h}_1$ is the $l_1 MJ \times 1$ propagation channel vector given by

$$\boldsymbol{h}_1 = \begin{bmatrix} \boldsymbol{h}_{1,1} \\ \vdots \\ \boldsymbol{h}_{1,l_1} \end{bmatrix}, \boldsymbol{h}_{1,l} = \begin{bmatrix} \boldsymbol{h}_{1,l}^1 \\ \vdots \\ \boldsymbol{h}_{1,l}^M \end{bmatrix}, \boldsymbol{h}_{1,l}^m = \begin{bmatrix} h_{1,l}^m(1) \\ \vdots \\ h_{1,l}^m(J) \end{bmatrix},$$

$\otimes$ denotes the Kronecker product, and the Toeplitz matrices $\boldsymbol{C}_1(i)$ are banded and the band consists of the spreading code $(c_0 \cdots c_{P-1})^T$ shifted successively to the right and down by one position. For the interfering users, we have a similar setup except that owing to asynchrony, the band in $\boldsymbol{C}_1$ is shifted down n_k chip periods and is no longer coincident with the top left edge of the box. We denote by $\boldsymbol{C}_1$, the concatenation of the code matrices given above for user 1: $\boldsymbol{C}_1 = [\boldsymbol{C}_1^T(0) \cdots \boldsymbol{C}_1^T(N_1 - 1)]^T$.

It is clear that the signal model above addresses a multiuser setup suitable for joint interference cancelation provided the timing information and spreading codes of all sources are available. As we shall see in the following, it is possible to decompose the problem into single user ones, thus making the implementation suitable for decentralized applications such as at mobile terminals or as a suboptimal processing or initialization stage at the base station. To this end, let us stack L successive $\boldsymbol{y}(n)$ vectors in a super vector

$$\boldsymbol{Y}_L(n) = \mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{A}_{N+K(L-1)}(n) + \boldsymbol{V}_L(n), \qquad (4)$$

where, $\mathcal{T}_L(\boldsymbol{G}_N) = [\mathcal{T}_L(\boldsymbol{G}_{1,N_1}) \cdots \mathcal{T}_L(\boldsymbol{G}_{K,N_K})]$, and $\mathcal{T}_L(\boldsymbol{x})$ is a banded block Toeplitz matrix with L block rows and $\left[\boldsymbol{x} \quad \boldsymbol{0}_{p\times(L-1)}\right]$ as first block row (p is the number of rows in $\boldsymbol{x}$), and $\boldsymbol{A}_{N+K(L-1)}(n)$ is the concatenation of user data vectors ordered as $[A^T_{1,N_1+L-1}(n), A^T_{2,N_2+L-1}(n) \cdots A^T_{K,N_K+L-1}(n)]^T$. We shall refer to $\mathcal{T}_L(\boldsymbol{G}_{k,N_k})$ as the channel convolution matrix for the kth user.

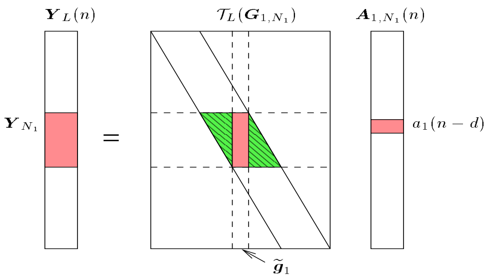


Fig. 3. ISI for the desired user.

Consider the noiseless received signal shown in fig. 3 for the contribution of user 1, from which the following observations can be made. Due to the limited delay spread, the effect of a particular symbol, $a_1(n-d)$, influences N_1 symbol periods, rendering the channel a moving average (MA) process of order $N_1 - 1$. We are interested in estimating the symbol $a_1(n-d)$ from the received data vector $\boldsymbol{Y}_L(n)$. One can notice that $a_1(n-d)$ appears in the portion $\boldsymbol{Y}_{N_1}$ of $\boldsymbol{Y}_L(n)$. The shaded triangles constitute the ISI, i.e., the effect of neighboring symbols on $\boldsymbol{Y}_{N_1}$. The contributions from the other (interfering) users to the received data vector have a similar structure. Note that to handle ISI and MAI, it may be advantageous to consider the longer received data vector $\boldsymbol{Y}_L(n)$.

4. THE MULTICODE SCENARIO

In fig. 3, $\tilde{\boldsymbol{g}}_1 = \boldsymbol{T}_1^H \boldsymbol{h}_1$, with $\boldsymbol{T}_1 = \left[\boldsymbol{0} \quad \boldsymbol{C}_1^H \quad \boldsymbol{0}\right] \otimes \boldsymbol{I}_{MJ}$ is the overall channel impulse response vector for the desired symbol, $a_1(n-d)$ of user 1. Let us consider user 2 to be a multicode co-user of user 1 (a certain user is employing two codes for higher rate transmission). The symbol of interest for this user is $a_2(n-d)$ and the channel impulse response for this user is

$$\tilde{\boldsymbol{g}}_2 = \boldsymbol{T}_2^H \boldsymbol{h}_1, \quad \text{with} \quad \boldsymbol{T}^\perp = \left[\boldsymbol{0} \quad \boldsymbol{C}_2^H \quad \boldsymbol{0}\right] \otimes \boldsymbol{I}_{MJ}.$$

Note that since the two users' signals issue from the same point in space, the propagation channel is the same (given by $\boldsymbol{h}_1$) for both of them. The matrix $\boldsymbol{C}_2$ is the same as $\boldsymbol{C}_1$ except that a different code $\boldsymbol{c}_2$ builds up the band.

In the above setup, the symbols of interest at the nth instant are both symbols $a_1(n-d)$ and $a_2(n-d)$, and the matrices $\boldsymbol{T}_1$ and $\boldsymbol{T}^\perp$ are code correlators matched to all multipath components of these users. The goal of the present discussion is to obtain decorrelating[1] receivers $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ to obtain, in a blind manner, the estimates of the desired symbols of the multicode user as $\hat{a}_1(n-d) = \boldsymbol{f}_1\boldsymbol{Y}_L(n)$ and $\hat{a}_2(n-d) = \boldsymbol{f}_2\boldsymbol{Y}_L(n)$.

[1] also termed as MMSE-ZF receiver and projection receiver

5. THE DECENTRALIZED MMSE-ZF RECEIVER

It was shown in [1], that the MMSE-ZF receiver could be obtained by a proper implementation of the unbiased minimum output energy [2] (MOE) criterion. We shall refer to [1] for details while mentioning that the MMSE-ZF receiver can be implemented in the generalized sidelobe canceler (GSC) fashion as in the following.

Let us denote by

$$\boldsymbol{T}_1 = \left[\boldsymbol{0} \quad \boldsymbol{C}_1^H \quad \boldsymbol{0}\right] \otimes \boldsymbol{I}_{MJ}; \; \boldsymbol{T}^\perp = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C}_1^\perp & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \otimes \boldsymbol{I}_{MJ}, \qquad (5)$$

the partial signature of the desired user and its orthogonal complement employed, respectively, in the upper and lower branches of the GSC, as shown in fig. 4. $\boldsymbol{C}_1^{\perp H}$ is the orthogonal complement of $\boldsymbol{C}_1$, the tall code matrix given in section 3 ($\boldsymbol{C}_1^\perp \boldsymbol{C}_1 = \boldsymbol{0}$). Then, $\boldsymbol{C}_1^H \boldsymbol{Y}_{N_1} = \boldsymbol{T}_1 \boldsymbol{Y}_L$ and the matrix $\boldsymbol{T}^\perp$ acts as a blocking transformation for all components of the signal of interest. Note that $P_{T_1^H} + P_{T_2^H} = \boldsymbol{I}$, where, P_X is the projection operator (projection on the column space of $\boldsymbol{X}$). Then the LCMV problem can be written as

$$\min_{f:fT_1^H=(h_1^H h_1)^{-1}h_1^H} \boldsymbol{f}\boldsymbol{R}^d_{YY}\boldsymbol{f}^H = \min_{\substack{f \,:\, fT_1^H h_1 = 1 \\ fT_1^H h_1^\perp = 0}} \boldsymbol{f}\boldsymbol{R}^d_{YY}\boldsymbol{f}^H, \qquad (6)$$

where, $\left[\boldsymbol{h}_1 \;\; \boldsymbol{h}_1^\perp\right]$ is a square non-singular matrix, and $\boldsymbol{h}_1^H \boldsymbol{h}_1^\perp = \boldsymbol{0}$. Note that in the LCMV problem (GSC formulation) there is a number of constraints to be satisfied. However, imposing the second set of constraints, namely $\boldsymbol{f}\boldsymbol{T}_1^H \boldsymbol{h}_1^\perp = 0$ has no consequence because the criterion automatically leads to their satisfaction once, $\text{span}\{\boldsymbol{R}^d_{YY}\} \cap \text{span}\{\boldsymbol{T}_1^H\} = \text{span}\{\boldsymbol{T}_1^H \boldsymbol{h}_1\}$, i.e., when the intersection of the signal subspace and the subspace spanned by the columns of $\boldsymbol{T}_1^H$ is one dimensional.

The matrix $\boldsymbol{T}_1$ is nothing but a bank of correlators matched to the l_1 delayed multipath components of user 1's code sequence. Note that the main branch in fig. 4 by itself gives an unbiased response for the desired symbol, $a_1(n-d)$, and corresponds to the (normalized) coherent RAKE receiver. For the rest, we have an estimation problem, which can be solved in the least squares sense, for some matrix $\boldsymbol{Q}$. This interpretation of the GSC corresponds to the pre-combining (or pathwise) interference (ISI and MAI) canceling approach.

The vector of estimation errors is given by

$$\boldsymbol{Z}(n) = \left[\boldsymbol{T}_1 - \boldsymbol{Q}\boldsymbol{T}^\perp\right] \boldsymbol{Y}_L(n). \qquad (7)$$

Since the goal is to minimize the estimation error variances, or in other words, estimate the interference term in the upper branch as closely as possible from $\boldsymbol{T}^\perp \boldsymbol{Y}_L(n)$, the interference cancelation problem settles down to minimization of the trace of the estimation error covariance matrix $\boldsymbol{R}_{ZZ}$ for a matrix filter $\boldsymbol{Q}$, which results in

$$\boldsymbol{Q} = \left(\boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}^{\perp H}\right)\left(\boldsymbol{T}^\perp \boldsymbol{R}^d \boldsymbol{T}^{\perp H}\right)^{-1}, \qquad (8)$$

and where, $\boldsymbol{R}^d$ is the noiseless (denoised) data covariance matrix, $\boldsymbol{R}_{YY}$, with the subscript removed for convenience. The output

[2] a derivative of the minimum variance distortionless response (MVDR) method, and a particular instance of the linearly constrained minimum-variance (LCMV) criterion

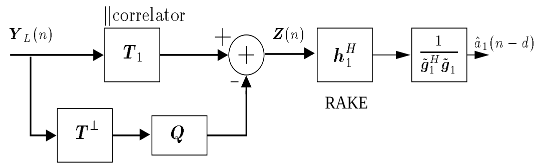


Fig. 4. GSC implementation of the MMSE-ZF receiver.

$\boldsymbol{Z}(n)$ can directly be processed by a multichannel matched filter to get the symbol estimate, $\hat{a}_1(n-d)$, the data for the user 1

$$\hat{a}_1(n-d) = \frac{1}{\tilde{\boldsymbol{g}}_1^H \tilde{\boldsymbol{g}}_1} \boldsymbol{f} \boldsymbol{Y}_L(n) = \frac{1}{\tilde{\boldsymbol{g}}_1^H \tilde{\boldsymbol{g}}_1} \boldsymbol{h}_1^H \left(\boldsymbol{T}_1 - \boldsymbol{Q}\boldsymbol{T}^{\perp} \right) \boldsymbol{Y}_L(n) \tag{9}$$

The covariance matrix of the prediction errors is then given by

$$\boldsymbol{R}_{ZZ} = \boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}_1^H - \boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}^{\perp H} \left(\boldsymbol{T}^{\perp} \boldsymbol{R}^d \boldsymbol{T}^{\perp H} \right)^{-1} \boldsymbol{T}^{\perp} \boldsymbol{R}^d \boldsymbol{T}_1^H, \tag{10}$$

From the above structure of the interference canceler, we observe that when $\boldsymbol{T}_1 \left(\boldsymbol{Y}_L - \tilde{\boldsymbol{g}}_1 a_1(n) \right)$ can be perfectly estimated from $\boldsymbol{T}^{\perp} \boldsymbol{Y}_L$, the matrix $\boldsymbol{R}_{ZZ}$ is rank-1 in the noiseless case! Using this fact, the desired user channel can be obtained (upto a scale factor) as the maximum eigenvector of the matrix $\boldsymbol{R}_{ZZ}$, since $\boldsymbol{Z}(n) = (\boldsymbol{C}_1^H \boldsymbol{C}_1) \otimes \boldsymbol{I}_{MJ} \boldsymbol{h}_1 \tilde{a}_1(n-d)$. It can further be shown easily that if $\boldsymbol{T}^{\perp} = \boldsymbol{T}_1^{\perp}$, then

$$\boldsymbol{T}_1 \boldsymbol{R}_{YY}^{-1} \boldsymbol{T}_1^H = \left(\boldsymbol{T}_1 \boldsymbol{T}_1^H \right) \boldsymbol{R}_{ZZ}^{-1} \left(\boldsymbol{T}_1 \boldsymbol{T}_1^H \right), \tag{11}$$

where, $\boldsymbol{R}_{ZZ}$ is given by (10), and $\boldsymbol{Q}$, given by (8), is optimized to minimize the estimation error variance. $\boldsymbol{R}^d$ replaces $\boldsymbol{R}_{YY}$ in the above developments. From this, we can obtain the propagation channel estimate for the desired user, $\hat{\boldsymbol{h}}_1$ as $\hat{\boldsymbol{h}}_1 = \mathrm{V}_{\max}\{ (\boldsymbol{T}_1 \boldsymbol{T}_1^H)^{-1} \boldsymbol{R}_{ZZ} (\boldsymbol{T}_1 \boldsymbol{T}_1^H)^{-1} \}$. The above structure results in perfect interference cancelation (both ISI and MAI) in the noiseless case, the evidence of which is the rank-1 estimation error covariance matrix, and a consequent distortionless response for the desired user.

5.1. Multicode Projection Receiver

In the two code case discussed in section 4, we can present an analogous treatment to the single code case. However, this time, the unbiasedness constraint of the single code problem becomes a ZF constraint expressed as

$$\boldsymbol{f} \left[\begin{array}{cc} \tilde{\boldsymbol{g}}_1 & \tilde{\boldsymbol{g}}_2 \end{array} \right] = \boldsymbol{f} \left[\begin{array}{cc} \boldsymbol{T}_1^H \boldsymbol{h}_1 & \boldsymbol{T}_2^H \boldsymbol{h}_1 \end{array} \right] = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right], \tag{12}$$

where, $\boldsymbol{f} = \left[\begin{array}{cc} \boldsymbol{f}_1^T & \boldsymbol{f}_2^T \end{array} \right]^T$ is now a matrix. We can further write the above as

$$\boldsymbol{f}\boldsymbol{T} = \boldsymbol{f} \left[\begin{array}{cc} \boldsymbol{T}_1^H & \boldsymbol{T}_2^H \end{array} \right] = (\mathcal{H}^H \mathcal{H})^{-1} \mathcal{H}^H, \tag{13}$$

where $\mathcal{H} = \left[\begin{array}{cc} \boldsymbol{h}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{h}_1 \end{array} \right]$. The set of receiver vectors, $\boldsymbol{f}$, can now be determined as from the constrained minimization of the output energy (MOE) as before [1] and the problem to be solved to be solved is given as

$$\min_{\boldsymbol{f} : \boldsymbol{f} [\begin{array}{cc} \tilde{\boldsymbol{g}}_1 & \tilde{\boldsymbol{g}}_2 \end{array}] = \boldsymbol{I}} \mathrm{tr} \left\{ \boldsymbol{f} \boldsymbol{R}_{YY} \boldsymbol{f}^H \right\}, \tag{14}$$

from where we obtain as solution for $\boldsymbol{f}$,

$$\boldsymbol{f} = (\mathcal{H}^H \mathcal{H})^{-1} \mathcal{H}^H \left(\boldsymbol{T}^H \boldsymbol{R}_{YY}^{-d} \boldsymbol{T} \right)^{-1} \boldsymbol{T}^H \boldsymbol{R}_{YY}^{-d}, \tag{15}$$

Note that a noiseless (and regularized) version of the received signal covariance matrix is employed in the derivations to account for the fact that the unbiasedness and zero-distortion constraint (the original constraint in the MVDR criterion) are equivalent only in the noiseless case.

It can be noticed that the channel impulse response, $\boldsymbol{h}_1$ is yet unknown in the above derivation. As discussed in [1], the MOE which is given by $\mathrm{MOE} = \mathrm{tr}\left\{ \boldsymbol{f} \boldsymbol{R}_{YY} \boldsymbol{f}^H \right\}$

$$= (\mathcal{H}^H \mathcal{H})^{-1} \mathcal{H}^H \left(\boldsymbol{T}^H \boldsymbol{R}_{YY}^{-1} \boldsymbol{T} \right)^{-1} \mathcal{H} (\mathcal{H}^H \mathcal{H})^{-1}, \tag{16}$$

can now be maximized over all possible values of $\boldsymbol{h}_1$ according to Capon's principle

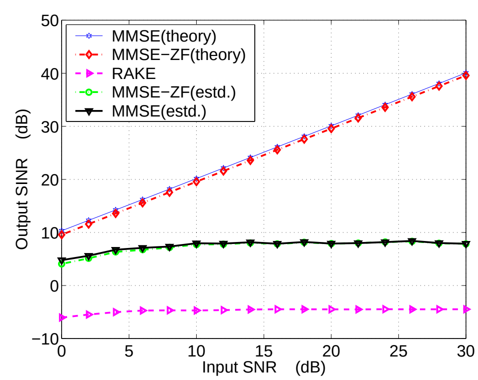
$$\max_{\hat{h}_1 : \|\hat{h}_1\| = 1} \mathrm{MOE}(\hat{\boldsymbol{h}}_1) = \max_{\hat{h}_1 : \|\hat{h}_1\| = 1} \mathrm{tr} \left\{ \mathcal{H}^H \left(\boldsymbol{T}^H \boldsymbol{R}_{YY}^{-d} \boldsymbol{T} \right)^{-1} \mathcal{H} \right\}, \tag{17}$$

to lead to a blind channel estimate obtained as

$$\widehat{\boldsymbol{h}}_1 = \mathrm{V}_{\max} \left\{ \left[\left(\boldsymbol{T}^H \boldsymbol{R}_{YY}^{-d} \boldsymbol{T} \right)^{-1} \right]_{11} + \left[\left(\boldsymbol{T}^H \boldsymbol{R}_{YY}^{-d} \boldsymbol{T} \right)^{-1} \right]_{22} \right\}. \tag{18}$$

where, $[\cdot]_{ij}$ stands for the square sub-matrix formed by the ith block row and jth block column of the matrix $[\cdot]$.

In a GSC implementation as shown in fig. 4, the main branch will be replaced by $\boldsymbol{T}^H$, the set of code correlators for both sub-users of the multicode user, and the second branch by $\boldsymbol{T}^{\perp H}$, a blocker ($\boldsymbol{T}^{\perp H} \boldsymbol{T} = \boldsymbol{0}$) for the desired signal components of both sub-users. A simulation example is presented in the figure below.



6. REFERENCES

[1] I. Ghauri and D. T. M. Slock, "Blind decentralized projection receiver for asynchonous CDMA in multipath channels," *Annals of Telecommunications*, pp. 379–391, Jul./Aug. 1999.

[2] C. Schlegel, S. Roy, P. Alexander, and Z. Xiang, "Multiuser projection receivers," *IEEE J. on Selected Areas in Communications*, vol. 14, pp. 1610–1618, October 1996.