AN ADAPTIVE RLS SOLUTION TO THE OPTIMAL MINIMUM POWER FILTERING PROBLEM WITH A MAX/MIN FORMULATION

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ABSTRACT

In signal processing, there are problems where the processed signal output energy is maximized while the noise component is minimized. This gives rise to a max/min problem, which is equivalent to a generalized eigenvalue problem. Exemplary applications of the max/min formulation have been seen in Capon's blind beamforming method and the blind minimum output energy (MOE) detection in CDMA wireless communications. The solution to such a problem involves eigen-decomposition of a transformed data covariance matrix inverse, which is computationally expensive to implement. This paper offers an adaptive RLS solution to the optimal minimum power filtering problem without involving eigen-decompositions. It is based on a new Recursive Least Square updating procedure that works for multiple linear constraints, and uses a one-dimensional subspace tracking method to update the filter weights. The performance is comparable with that of using the direct eigen-decomposition and matrix inversion.

1. INTRODUCTION

Adaptive constrained filtering occurs in many signal processing applications such as array processing, adaptive beamforming, spectral analysis, and telecommunications. A common approach to the linear filtering optimization problem consists of searching an optimal filtering whose parameters are subject to a set of linear equations. The constraints may be selected in order to improve the signal-to-noise ratio or to maintain some properties of the processed signal or of the filtering response. When there are unknown parameters in the linear equations due to system uncertainty, these unknown channel parameters may be optimized on top of the filtering optimization, resulting in a max/min optimization structure with data-dependent variable constraint parameters. The blind minimum variance beamformer designed by Capon [1] is a direct application of this principle. In CMDA wireless communications, Tsatsanis and Xu [3] proposed an optimal minimum output energy (MOE) detector [2] with variable constraints using the max/min method. It was shown that this MOE approach compared favorably with subspace based channel estimation methods and multichannel linear prediction methods.

Adaptive filters can be implemented in a sequential update algorithm such as Least Mean Squares (LMS) and Recursive Least Squares (RLS). LMS adaptation has a low computational complexity but can be slow to converge. RLS adaptation is quick to converge but is more computationally complex. In the literature, most of the solutions to the max/min method are given in the batch form [4] or the LMS form. We study the adaptive implementation issues for the max/min problem using RLS upating to achieve faster convergence rate.

The standard direct form RLS algorithm only applies to the single gain-only (distortionless) constrained case. Resende et al. [5] extended the RLS results to the more general multiple constrained case. On the other hand, it is convenient to implement the multiple constrained linear filters using the partitioned linear interface canceller (PLIC), or equivalently, the generalized sidelobe canceller (GSC) structure [6]. We propose a new exact least squares algorithm in the direct form for adaptive filtering with multiple fixed linear constraints. Based on this direct form RLS algorithm, we develop a RLS solution to the max/min optimization problem via subspace tracking, where the the filter weights are updated by tracking the principle eigenvector of a transformed data covariance matrix. This adaptive approach does not require matrix inverse or block eigen-decomposition, therefore is computationally simple. It is also derived for the PLIC structure.

This paper is organized as follows: The next section describes the optimal minimum power filtering problem using a max/min formulation. Section 3 presents a new direct-form RLS updating procedure that applies to multiple fixed linear constraint. In section 4, the PASTd subspace-tracking method [7] [8] is incorporated into the new RLS iterations to track the adaptive weights of the max/min filters. A summary is presented in section 5.

2. THE MINIMUM POWER FILTERING PROBLEMS

Let $\mathbf{x}(n)$ denote the $N \times 1$ vector of signals received by a transversal filter with N taps. In narrow-band filtering, a complex weight is applied to the signal at each tap and summed to form the filter output,

$$y(n) = \mathbf{w}^H \mathbf{x}(n). \tag{1}$$

The weights of a linearly constrained minimum power (LCMP) filter are chosen to minimize the output power of the filter subject to a set of m linear constraints of the form $\mathbf{C}^H\mathbf{w} = \mathbf{f}$, where $\mathbf{C}$ is the $N \times m$ constraint matrix, and $\mathbf{f}$ is the $m \times 1$ vector of constraint values. The constraint matrix $\mathbf{C}$ is designed from the knowledge of the filtering environment to achieve the desired filtering response. For example, in CDMA wireless communication systems using the minimum output energy detection, the matrix $\mathbf{C}$ contains the shifted versions of the spreading codes of the desired user in order to preserve its output energy.

The LCMP optimization problem can be formulated as

$$\mathbf{w} = \arg\min \; E\{|y(n)|^2\} = \mathbf{w}^H \mathbf{R}_\mathbf{x} \mathbf{w} \quad st. \; \mathbf{C}^H \mathbf{w} = \mathbf{f} \tag{2}$$

where $\mathbf{R}_\mathbf{x} = E\left\{\mathbf{x}(n)\mathbf{x}(n)^H\right\}$ is the data covariance matrix. The optimal solution is

$$\mathbf{w} = \mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\left[\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\right]^{-1}\mathbf{f}. \tag{3}$$

In the partitioned linear interface canceller, or the generalized sidelobe canceler structure [6] , $\mathbf{w}$ is decomposed into two components, one in the constraint subspace and one orthogonal to it. The weights are given by

$$\mathbf{w} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_a, \tag{4}$$

where the vector $\mathbf{w}_q$ is the fixed $N \times 1$ quiescent weight vector

$$\mathbf{w}_q = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{f}. \tag{5}$$

The vector $\mathbf{w}_a$ is the $(N-m) \times 1$ adaptive weight vector which can adapt freely to improve interference suppression in the $N-m$ dimensional orthogonal subspace. The normalized matrix $\mathbf{B}$ is the $N \times (N-m)$ blocking matrix which is orthogonal to $\mathbf{C}$, i.e. $\mathbf{B}^H\mathbf{C} = 0$ and $\mathbf{B}^H\mathbf{B} = \mathbf{I}$. Using this decomposition, the optimal solution to the LCMP optimization problem is

$$\mathbf{w}_a = (\mathbf{B}^H\mathbf{R}_\mathbf{x}\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}_\mathbf{x}\mathbf{w}_q. \tag{6}$$

Let $\mathbf{z}(n) = \mathbf{B}^H\mathbf{x}(n)$. The adaptive weights can also be expressed as

$$\mathbf{w}_a = \mathbf{R}_\mathbf{z}^{-1}\mathbf{p}_\mathbf{z}, \tag{7}$$

where $\mathbf{R}_\mathbf{z} = \mathbf{B}^H\mathbf{R}_\mathbf{x}\mathbf{B}$ is the $(N-m)\times(N-m)$ covariance matrix of $\mathbf{z}(n)$ and $\mathbf{p}_\mathbf{z} = \mathbf{B}^H\mathbf{R}_\mathbf{x}\mathbf{w}_q$ is the $(N-m) \times 1$ cross-correlation vector of $\mathbf{z}(n)$ and $\mathbf{w}_q^H\mathbf{x}(n)$.

Due to system uncertainties, the knowledge of the constraint parameters $\mathbf{f}$ may not be available to the filter. In this case, the unknown parameters $\mathbf{f}$ may be optimized using the max/min approach, resulting in optimization problem with data-dependent variable constraints. A general formulation of the the max/min approach can be expressed as

$$\max_{||\mathbf{f}||=1} \; \min_{\mathbf{w}} \quad E\{|\mathbf{w}^H\mathbf{x}(n)|^2\} = \mathbf{w}^H\mathbf{R}_\mathbf{x}\mathbf{w} \tag{8}$$

$$st. \quad \mathbf{C}^H\mathbf{w} = \mathbf{f}. \tag{9}$$

The optimal solution to $\mathbf{f}$ is $\mathbf{e}_1$, the principle eigenvector of $\mathbf{S} = (\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C})^{-1}$, and the optimal weight vector has the form

$$\mathbf{w} = \mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\right)^{-1}\mathbf{e}_1, \tag{10}$$

where λ_1 is the largest eigenvalue of $(\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C})^{-1}$, and $\mathbf{e}_1$ is the corresponding eigenvector.

3. A NEW DIRECT-FORM RLS SOLUTION WITH MULTIPLE LINEAR CONSTRAINTS

The LCMP problem in (2) can be implemented adaptively using RLS updating. However, the standard direct-form RLS method applies when there is only one linear constraint in (2). To extend the RLS updating method to multiple linear constraints, we first define $\mathbf{T} = \mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\right)^{-1}$. Note that for a generic constraint $\mathbf{f}$

$$\begin{aligned}
\mathbf{w} &= \mathbf{T}\mathbf{f} && (11)\\
&= \left[\mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H + \mathbf{B}\mathbf{B}^H\right]\mathbf{T}\mathbf{f} && (12)\\
&= \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{f} + \mathbf{B}\mathbf{B}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\right)^{-1}\mathbf{f} \\
&= \mathbf{w}_q - \mathbf{B}\mathbf{w}_a && (13)
\end{aligned}$$

Using Eq. (6), we have the identity

$$\begin{aligned}
\mathbf{w}_a &= -\mathbf{B}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\right)^{-1}\mathbf{f} && (14)\\
&= (\mathbf{B}^H\mathbf{R}_\mathbf{x}\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}_\mathbf{x}\mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{f} && (15)
\end{aligned}$$

which yields the identity

$$\mathbf{T} = \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1} - \mathbf{B}(\mathbf{B}^H\mathbf{R}_\mathbf{x}\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}_\mathbf{x}\mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}. \tag{16}$$

Defining

$$\tilde{\mathbf{z}}(n) = \left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H\mathbf{x}(n), \tag{17}$$

we can write

$$\begin{aligned}
\mathbf{T} &= \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1} - \mathbf{B}(\mathbf{B}^H\mathbf{R}_\mathbf{x}\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}_{\mathbf{x}\tilde{\mathbf{z}}^H} && (18)\\
&= \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1} - \mathbf{B}\mathbf{R}_\mathbf{z}^{-1}\mathbf{R}_{\mathbf{z}\tilde{\mathbf{z}}^H} && (19)\\
&= \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1} - \mathbf{B}\mathbf{T}_a. && (20)
\end{aligned}$$

We then have a straightforward procedure for updating $\mathbf{T}$ in the direct form

$$\begin{aligned}\bar{\mathbf{z}}_p(n) &= \mathbf{T}(n-1)^H\mathbf{x}(n) && (21)\\ \mathbf{T}(n) &= \mathbf{T}(n-1)+\mathbf{g}_B(n)\bar{\mathbf{z}}_p(n)^H && (22)\end{aligned}$$

and PLIC

$$\begin{aligned}\bar{\mathbf{z}}_p(n) &= \tilde{\mathbf{z}}(n)-\mathbf{T}_a(n-1)^H\mathbf{z}(n) && (23)\\ \mathbf{T}_a(n) &= \mathbf{T}_a(n-1)+\mathbf{g}_\mathbf{z}(n)\bar{\mathbf{z}}_p(n)^H, && (24)\end{aligned}$$

where $\mathbf{g}_B(n)$ and $\mathbf{g}_\mathbf{z}(n)$ are gain vectors that are updated by the standard RLS (Table 1). If the constraint is fixed, we get the standard updates $\mathbf{w}(n)=\mathbf{T}(n)\mathbf{f}$ and $\mathbf{w}_a(n)=\mathbf{T}_a(n)\mathbf{f}$. For optimized constraints, $\mathbf{f}$ can be adapted by tracking the principle eigenvector of $\mathbf{S}(n)$.

4. SUBSPACE TRACKING FOR LCMP FILTERING WITH VARIABLE CONSTRAINTS

A number of subspace tracking techniques have been developed in the literature, such as the rank-one signal eigenstructure updating (ROSE) algorithm [9], the projection approximation subspace tracking (PAST) approach [7] [8], and the low rank adaptive filtering (LORAF) method [10], etc. A good survey is available in [11]. Among these technique, the deflated PAST (PASTd) algorithm [8] is very effective and has low computational complexity. Moreover, it tracks the principle components sequentially, therefore is ideally suited when only the largest eigenvalue and corresponding eigenvector are desired. Its performance is insensitive to knowledge of the subspace dimension, and its computational complexity is linear in the length of the data vector. It tracks the principle component $\mathbf{e}_1(n)$ of a matrix $\mathbf{S}(n)=\sum_{i=1}^{n}\mu^{n-i}\mathbf{y}(n)\mathbf{y}(n)^H$ as follows

$$\begin{aligned}\rho_1(n) &= \mathbf{e}_1(n-1)^H\mathbf{y}(n) && (25)\\ \eta_1(n) &= \mu\eta_1(n-1)+|\rho_1(n)|^2 && (26)\\ \mathbf{e}_1(n) &= \mathbf{e}_1(n-1)+\\ &\quad [\mathbf{y}(n)-\mathbf{e}_1(n-1)\rho_1(n)]\,\frac{\rho_1^*(n)}{\eta_1(n)}. && (27)\end{aligned}$$

We now have to put $\mathbf{S}(n)=\left(\mathbf{C}^H\mathbf{R}_\mathbf{x}(n)^{-1}\mathbf{C}\right)^{-1}$ into the proper form. In a standard RLS update, we have

$$\begin{aligned}\mathbf{P}_\mathbf{x}(n) &= \mu^{-1}\mathbf{P}_\mathbf{x}(n-1)-\\ &\quad \frac{\mathbf{P}_\mathbf{x}(n-1))\mathbf{x}(n)\mathbf{x}^H(n)\mathbf{P}_\mathbf{x}(n-1)}{\mu+\mathbf{x}(n)^H\mathbf{P}_\mathbf{x}(n-1))\mathbf{x}(n)} && (28)\end{aligned}$$

Then

$$\begin{aligned}\mathbf{S}(n)^{-1} &= \mathbf{C}^H\mathbf{P}_\mathbf{x}(n)\mathbf{C} && (29)\\ &= \mu^{-1}\mathbf{C}^H\mathbf{P}_\mathbf{x}(n-1)\mathbf{C}-\\ &\quad \mu^{-1}\frac{\mathbf{C}^H\mathbf{P}_\mathbf{x}(n-1)\mathbf{x}(n)\mathbf{x}^H(n)\mathbf{P}_\mathbf{x}(n-1)\mathbf{C}}{\mu+\mathbf{x}(n)^H\mathbf{P}_\mathbf{x}(n-1)\mathbf{x}(n)}\\ &= \mu^{-1}\mathbf{S}(n-1)^{-1}-\alpha^{-1}\mathbf{q}(n)\mathbf{q}(n)^H && (30)\end{aligned}$$

where $\mathbf{q}(n)=\mathbf{C}^H\mathbf{P}_\mathbf{x}(n-1)\mathbf{x}(n)$ and $\alpha=\mu\left(\mu+\mathbf{x}(n)^H\mathbf{P}_\mathbf{x}(n-1)\mathbf{x}(n)\right)$. Using the matrix inversion lemma, we have

$$\begin{aligned}\mathbf{S}(n) &= \mu\mathbf{S}(n-1)+\frac{\mu^2\mathbf{S}(n-1)\mathbf{q}(n)\mathbf{q}(n)^H\mathbf{S}(n-1)}{\alpha-\mu\mathbf{q}(n)^H\mathbf{S}(n-1)\mathbf{q}(n)}\\ &= \mu\mathbf{S}(n-1)+\mathbf{y}(n)\mathbf{y}(n)^H && (31)\end{aligned}$$

with

$$\mathbf{y}(n)=\frac{\mathbf{S}(n-1)\mathbf{q}(n)}{\sqrt{\mu^{-2}\left(\alpha-\mu\mathbf{q}(n)^H\mathbf{S}(n-1)\mathbf{q}(n)\right)}} \qquad (32)$$

Now note that $\mathbf{T}(n)=\mathbf{P}_\mathbf{x}(n)\mathbf{C}\mathbf{S}(n)$ and $\mathbf{P}_B(n)=\mathbf{P}_\mathbf{x}(n)-\mathbf{P}_\mathbf{x}(n)\mathbf{C}\mathbf{S}(n)\mathbf{C}^H\mathbf{P}_\mathbf{x}(n)$, therefore

$$\begin{aligned}\mathbf{y}(n) &= \frac{\mathbf{T}(n-1)^H\mathbf{x}(n)}{\sqrt{1+\mu^{-1}\mathbf{x}(n)^H\mathbf{P}_B(n-1)\mathbf{x}(n)}} && (33)\\ &= \frac{\bar{\mathbf{z}}_p(n)}{\sqrt{1+\mu^{-1}\mathbf{x}(n)^H\mathbf{P}_B(n-1)\mathbf{x}(n)}} && (34)\end{aligned}$$

The PASTd update can be accomplished with

$$\begin{aligned}\rho_1(n) &= \frac{\mathbf{e}_1(n-1)^H\bar{\mathbf{z}}_p(n)}{\sqrt{(1+\mu^{-1}\mathbf{x}(n)^H\mathbf{P}_B(n-1)\mathbf{x}(n))}} && (35)\\ \eta_1(n) &= \mu\eta_1(n-1)+|\rho_1(n)|^2 && (36)\\ \mathbf{e}_1(n) &= \mathbf{e}_1(n-1)+\\ &\quad [\mathbf{y}(n)-\mathbf{e}_1(n-1)\rho_1(n)]\,\frac{\rho_1^*(n)}{\eta_1(n)} && (37)\end{aligned}$$

and the weight vector is $\mathbf{w}(n)=\mathbf{T}(n)\mathbf{e}_1(n)$. Note that the denominator in (35) is not needed in (37) to update $\mathbf{e}_1(n)$. Therefore the procedure can be simplified by replacing $\rho_1(n)$ and $\eta_1(n)$ by their numerator parts, $\rho(n)$ and $\eta(n)$, respectively,

$$\begin{aligned}\rho(n) &= \mathbf{e}_1(n-1)^H\bar{\mathbf{z}}_p(n), && (38)\\ \eta(n) &= \mu\eta(n-1)+|\rho(n)|^2. && (39)\end{aligned}$$

Equation (37) should be adjusted accordingly to

$$\mathbf{e}_1(n)=[\bar{\mathbf{z}}_p(n)-\mathbf{e}_1(n-1)\rho(n)]\,\frac{\rho^*(n)}{\eta(n)}. \qquad (40)$$

In the PLIC, the subspace tracking is accomplished in the same manner, with the term $\mathbf{x}(n)^H\mathbf{P}_B(n-1)\mathbf{x}(n)$ replaced with $\mathbf{z}(n)^H\mathbf{P}_\mathbf{z}(n-1)\mathbf{z}(n)$.

To initialize, $\mathbf{e}_1(0)$ and $\eta(0)$ may be chosen to be the principle eigenvector and eigenvalue of $\mathbf{S}(0)=(\mathbf{C}^H\mathbf{P}_\mathbf{x}\mathbf{C})^{-1}=\sigma_o^2(\mathbf{C}^H\mathbf{C})^{-1}$. In CDMA applications, $\mathbf{C}^H\mathbf{C}$ is assumed to be very close to $L\mathbf{I}$ where L is the length of the spreading code. Therefore, $\eta(0)$ will be close to $\frac{1}{L}\sigma_o^2$ and $\mathbf{e}_1(0)$ will be close to $\mathbf{1}=[1\,0\cdots0]^T$.

The above implementations for the optimized minimum power filtering with variable constraints are summarized in Table 1.

Table 1. RLS Implementations of Optimized Linearly Constrained MOE Detectors

	Direct-form	PLIC
Init.	$\mathbf{T}(0) = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1},\ \mathbf{e}_1(0) = \mathbf{1},$ $\mathbf{w}(0) = \mathbf{w}_q,\ \mathbf{P}_B(0) = \frac{1}{\sigma_o^2}\mathbf{P}_c^{\perp}$	$\mathbf{T}_a(0) = \mathbf{0},\ \mathbf{e}_1(0) = \mathbf{1},$ $\mathbf{w}(0) = \mathbf{w}_q,\ \mathbf{P}_{\mathbf{z}}(0) = \frac{1}{\sigma_o^2}\mathbf{I}$
Input Data		$\bar{\mathbf{z}}(n) = (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{x}(n)$ $\mathbf{z}(n) = \mathbf{B}^H\mathbf{x}(n)$
RLS Update	$\mathbf{g}_B(n) = \dfrac{\mathbf{P}_c^{\perp}\mathbf{P}_B(n-1)\mathbf{x}(n)}{\mu + \mathbf{x}^H(n)\mathbf{P}_B(n-1)\mathbf{x}(n)}$ $\mathbf{P}_B(n) = \mu^{-1}\left[\mathbf{P}_B(n-1) - \mathbf{g}_B(n)\mathbf{x}^H(n)\mathbf{P}_B(n-1)\right]$	$\mathbf{g}_{\mathbf{z}}(n) = \dfrac{\mathbf{P}_{\mathbf{z}}(n-1)\mathbf{z}(n)}{\mu + \mathbf{z}^H(n)\mathbf{P}_{\mathbf{z}}(n-1)\mathbf{z}(n)}$ $\mathbf{P}_{\mathbf{z}}(n) = \mu^{-1}\left[\mathbf{P}_{\mathbf{z}}(n-1) - \mathbf{g}_{\mathbf{z}}(n)\mathbf{z}^H(n)\mathbf{P}_{\mathbf{z}}(n-1)\right]$
Subspace Tracking	$\bar{\mathbf{z}}_p(n) = \mathbf{T}(n-1)^H\mathbf{x}(n)$ $\rho(n) = \mathbf{e}_1(n-1)^H\bar{\mathbf{z}}_p(n)$	$\bar{\mathbf{z}}_p(n) = \bar{\mathbf{z}}(n) - \mathbf{T}_a^H(n-1)\mathbf{z}(n)$ $\rho(n) = \mathbf{e}_1(n-1)^H\bar{\mathbf{z}}_p(n)$
	$\eta(n) = \mu\eta(n-1) + \lvert\rho(n)\rvert^2$ $\mathbf{e}_1(n) = \mathbf{e}_1(n-1) + \left[\bar{\mathbf{z}}_p(n) - \mathbf{e}_1(n-1)\rho(n)\right]\rho^*(n)/\eta(n)$	
Weight Update	$\mathbf{T}(n) = \mathbf{T}(n-1) + \mathbf{g}_B(n)\bar{\mathbf{z}}_p^H(n)$ $\mathbf{w}(n) = \mathbf{T}(n)\mathbf{e}_1(n)$	$\mathbf{T}_a(n) = \mathbf{T}_a(n-1) + \mathbf{g}_{\mathbf{z}}(n)\bar{\mathbf{z}}_p^H(n)$ $\mathbf{w}_a(n) = \mathbf{T}_a(n)\mathbf{e}_1(n)$ $\mathbf{w}(n) = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{e}_1(n) - \mathbf{B}\mathbf{w}_a(n)$

5. SUMMARY

In the paper, we develop a recursive least square updating method for the minimum power filtering problem with a max/min formulation. A new direct-form RLS solution is derived. It not only applies to the general case of multiple linear constraints, but also inherently updates a transmformed data covariance matrix inverse that has been explicitly used in the max/min approach. A subspace-tracking method is then incorporated into this RLS recursion to update the optimized filter weights. This approach avoids direct eigen-decomposition and matrix inversion, therefore is computationally attractive. It is presented in both the direct form and the PLIC/GSC structure. Its effectiveness has been proved in solving the blind minimum output energy detection problem in CDMA wireless communications [13].

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