TURBO SPACE-TIME EQUALIZATION OF TCM FOR BROADBAND WIRELESS CHANNELS

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ABSTRACT

This paper presents a turbo (iterative) equalization method for complex TCM signals over broadband wireless channels based on receiver antenna array measurements. The channel is highly dispersive at high data rates causing a severe intersymbol intereference (ISI) effect and making the direct application of any trellis based equalization algorithm infeasible. The problem of reducing this excess interference is solved by receiver diversity combining, i.e. using a linear antenna array and a broadband beamformer in the receiver. The beamformer output that contains less ISI is viewed as the output of a serial concatenated coding system and optimum symbol detection is achieved by a turbo equalization and decoding scheme. The proposed receiver structure is simulated for two dimensional TCM signals such as 8-16 PSK and 16-QAM and the results indicate an improved performance of the diversity receiver.

1. INTRODUCTION

The advantages of space-time processing and turbo decoding can be combined for equalization of TCM encoded multipath fading channels as in [1]. Here, a linear antenna array is deployed at the end of a stationary frequency selective channel which takes interleaved complex TCM symbols as its input. The channel effects along with that of the transmitter and receiver are described by an equivalent impulse response at the output of each array element. The discrete vector signal obtained by the array is viewed as the output of a serial concatenated encoder and is decoded by a turbo equalizer that employs two concatenated soft-output decoders iteratively. The equalizer is employed either to the vector array output or to the scalar sequence obtained by spatial combining of array measurements through a narrowband beamformer.

In this work, the space-time turbo equalization approach of [1] is extended to the fixed-broadband case. The stationary broadband channel assuming inputs from a TCM encoder is described by an impulse response at each receiver array element, but severe multipath dispersion causes the equivalent finite state machine model to have a large memory. This makes the direct application of any trellis based decoding algorithm for equalization infeasible. The excess interference must be reduced to such a level that the equivalent channel can be described by a few tap coefficients. This is achieved by a broadband beamformer that combines the finite impulse response (FIR) filtered responses of individual array observations. The filter coefficients are selected in a way to reduce the mean square error between the beamformer output and the desired signal [2]. Since the beamformer output is fed to a turbo equalizer, it must preserve the whiteness of the channel noise, which can be achieved by requiring that the vector broadband equalizer should have the structure of a power complementary FIR filter bank [3]. The channel observed at the beamformer output can then be described by a small number of coefficients and the turbo equalization can be applied to perform optimum symbol detection.

The organization of this note is as follows: In section 2, a channel and signal model for diversity receiver is developed. Then the design of the broadband beamformer is described in section 3, followed by the operation of the iterative equalizer in section 4. Section 5 is devoted to the simulation results of the iterative equalizer for two-dimensional TCM constellations such as 8/16-PSK and 16-QAM. Finally, this paper is ended with some conclusive remarks and review of further work in section 6.

2. CHANNEL AND SIGNAL MODEL

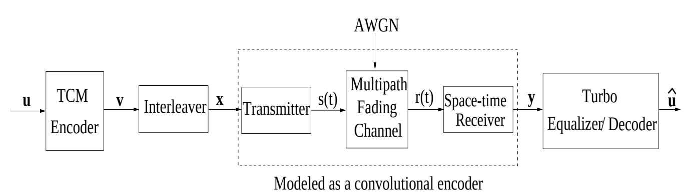

Fig. 1. Transmission Scheme

The transmission system under consideration is shown in Fig. 1 where the elements of a binary data sequence $\mathbf{u}$ are TCM encoded and mapped onto the points of the PSK or QAM signal constellation as described by Ungerboeck [4]. The complex output sequence of the TCM encoder $\mathbf{v}$, is reordered by an interleaver and the resultant sequence $\mathbf{x}$ is passed through the pulse shaping filter, $p(t)$ with signaling interval T. The complex baseband modulated signal $s(t)$ is expressed as

$$s(t) = \sum_m x_m p(t - mT). \quad (1)$$

The multipath fading channel is described by an L-ray channel impulse response

$$h(t) = \sum_{l=1}^{L} a_l \delta(t - \tau_l) \quad (2)$$

where a_l are the complex fading coefficients giving the amplitude and phase of each ray and τ_l the corresponding delay. At the receiver side, an N-element linear antenna array is deployed where the first antenna is assumed to be the reference point of all observations. Assuming that $h(t)$ is the channel impulse response seen by the first antenna, the noisy discrete observation at the i^{th} antenna after sampling is described as

$$r_i(kT_s) = \sum_m x_m h_i(kT_s - mT) + n_i(kT_s) \quad 1 \le i \le N \quad (3)$$

where each $n_i(kT_s)$ is an additive white Gaussian noise with zero mean and intensity σ^2,

$$h_i(kT_s) = \sum_{l=1}^{L} a_l e^{-j(i-1)\varphi_l} p(kT_s - \tau_l) \qquad 1 \le i \le N \qquad (4)$$

is the equivalent impulse response seen at the output of each sensor and

$$\varphi_l = 2\pi \frac{d Sin\ \theta_l}{\lambda} \qquad (5)$$

is the inter-antenna phase factor that is determined by the inter-element spacing d, the incident path angle θ_l of the l^{th} ray and the carrier wavelength λ.

In narrowband systems where multipath dispersion is small compared to the signaling interval the ISI in equation (3) is limited to a few symbols and hence the multipath channel effects can be observed as that of a convolutional encoder with rate $\frac{1}{n}$ where $n = \frac{NT}{T_s}$. Since the equivalent ISI encoder is concatenated to the TCM encoder through an interleaver, the received signal is treated as produced by a serial concatenated encoder and turbo equalization is applied to the array output for optimum symbol detection. However, large multipath dispersion of wideband channels causes more severe ISI and a tremendous increase in the number of states of the equivalent channel trellis diagram and as a result it is difficult to apply trellis based equalization schemes. In order to employ a turbo equalizer, the excess ISI has to be removed by prepocessing so that the equivalent channel is again limited to a small number. This is done by a broadband beamformer whose operation is described in the next section.

3. BROADBAND BEAMFORMER

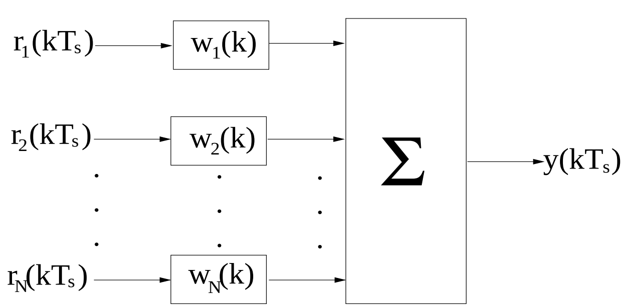

Fig. 2. Broadband beamformer

The broadband beamformer is shown in Fig. 2. where each antenna observation is passed through an FIR filter and the output is the spatial combination of filter responses. The filter coefficients are selected such that the ISI due to secondary paths is subsided at the beamformer output. Different criteria can be used for coefficient optimization but the minimization of mean square error (MMSE) is considered in this work. Once the tap coefficients are computed, the beamformer output can be written in terms of the antenna array observations and FIR filter coefficients as

$$\begin{aligned} y(kT_s) &= \sum_{i=1}^{N} w_i(k) \otimes r_i(kT_s) \\ &= \sum_{i=1}^{N} \sum_{d=0}^{M-1} w_{i,d}\, r_i((k-d)T_s) \\ &= \sum_m x_m f(kT_s - mT) + \hat{n}(kT_s) \qquad (6) \end{aligned}$$

where $\otimes$ denotes the convolution operation, $M-1$ is the order of each FIR filter, $w_{i,d}$ is the d^{th} coefficient of the i^{th} filter and

$$f(kT_s) = \sum_{i=1}^{N} \sum_{d=0}^{M-1} w_{i,d} h_i((k-d)T_s) \qquad (7)$$

$$\hat{n}(kT_s) = \sum_{i=1}^{N} \sum_{d=0}^{M-1} w_{i,d} n_i((k-d)T_s) \qquad (8)$$

are the spatio-temporal combinations of the equivalent channel impulse responses and the noise terms, respectively.

When used with oversampling, the beamformer output in equation (6) contains a level of ISI equivalent to that of the narrowband case and the equivalent system can be described by a finite state machine with a trellis diagram of reasonable complexity. Since the input of this finite state machine is the interleaved TCM encoder symbols, beamformer output is observed as the output of a serial concatenated coding scheme which is suitable for the use of iterative equalizer/decoder for symbol detection.

As mentioned before, the turbo equalizer employs two soft-input soft-output decoders both operating in the presence of white noise. Therefore, the beamformer output noise $\hat{n}(kT_s)$ must be white, meaning that the beamforming filters must also form a power complementary filter bank satisfying

$$\sum_{i=1}^{N} \tilde{W}_i(z) W_i(z) = 1 \qquad (9)$$

where $W_i(z)$ is the transfer function of the i^{th} filter $w_i(k)$ and the tilde accent on the function $W_i(z)$ is defined such that $\tilde{W}_i(z) = W_i^*(z^{-1})$. It is possible to find an approximate numerical solution to the minimization problem with the quadratic constraint given in (9). Another alternative considered here is to use the lattice structure to implement the power complementary beamformer to perform a recursive minimization in each stage of the lattice [5]. In the following, this approach is described first for a two component filter bank and then the results are generalized to the N filter case.

Suppose we have two m^{th} order power complementary FIR filters, $W_1^m(z)$ and $W_2^m(z)$, satisfying

$$\left[\tilde{W}_1^m(z), \tilde{W}_2^m(z)\right] \begin{bmatrix} W_1^m(z) \\ W_2^m(z) \end{bmatrix} = 1. \qquad (10)$$

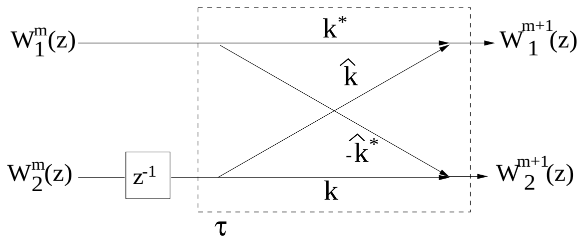

Fig. 3. Two branch butterfly

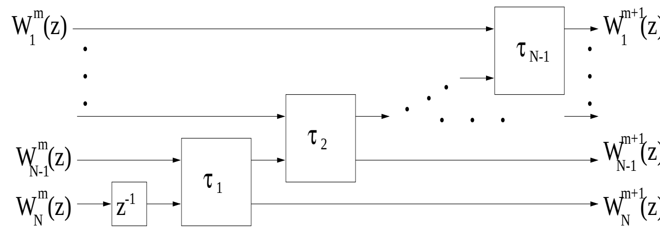

Fig. 4. Implementation of (m+1)th order lattice filter

The filters can be augmented to $(m+1)^{th}$ order using the lattice butterfly shown in Fig.3 and the new filters can be written in terms of lattice coefficients as

$$\begin{bmatrix} W_1^{m+1}(z) \\ W_2^{m+1}(z) \end{bmatrix} = \begin{bmatrix} k^* & \hat{k} \\ -\hat{k}^* & k \end{bmatrix} \begin{bmatrix} W_1^m(z) \\ z^{-1}W_2^m(z) \end{bmatrix} \quad (11)$$

$$= \tau \begin{bmatrix} W_1^m(z) \\ z^{-1}W_2^m(z) \end{bmatrix} \quad (12)$$

where τ is the rotation matrix. Furthermore, the augmented filters preserves the power complementary structure if the lattice coefficients are chosen on the unit circle, i.e., $\mid k \mid^2 + \mid \hat{k} \mid^2 = 1$. Therefore, once initialized, it is possible to obtain any order of power complementary FIR filter banks by successive use of cascaded-lattice realization [3].

In each stage, the information from the previous stages is used to compute the lattice coefficients as the solution of the MMSE problem. Suppose $x_1^m(k)$ and $x_2^m(k)$ are the m^{th} stage filter outputs, and $x_1^{m+1}(k)$ and $x_2^{m+1}(k)$ are the $(m+1)^{th}$ stage filter outputs. Then the $(m+1)^{th}$ stage beamformer output is written as

$$\begin{aligned} y^{m+1}(k) &= x_1^{m+1}(k) + x_2^{m+1}(k) \\ &= \left[k - \hat{k},\ k^* + \hat{k}^*\right]^* \begin{bmatrix} x_1^m(k) \\ x_2^m(k-1) \end{bmatrix} \quad (13) \\ &= \mathbf{k}^* \cdot \mathbf{x}^m(k). \end{aligned}$$

Then $\mathbf{k}$ minimizing the MSE between the beamformer output and the desired signal $s_k = s(kT_s)$ is the solution to the equation

$$\mathbf{k} = argmin\ E \parallel \mathbf{k}^* \cdot \mathbf{x}^m(k) - s_k \parallel^2 \quad (14)$$

subject to the constraint $\frac{1}{2}\mathbf{k}^*\mathbf{k} = 1$ in order to preserve the power complementary structure. An elegant method to solve this second order problem is presented in [6] and used in this work.

As described above the global optimization problem for two filters can be broken into successive and less complex minimization problems. In fact, the same algorithm can also be generalized to the case where more than two filters are used. When there are N filters present, the augmentation can be implemented as a cascade of $N-1$ butterflies as shown in Fig. 4. Then the coefficients of each butterfly is computed by minimizing the cost function given in equation (14).

As mentioned before, the broadband beamformer is used only to reduce the ISI in such a way that the equivalent channel response observed at its output has a few taps. Once this is done, the signal detection is performed by the iterative equalizer presented in the following section.

4. TURBO EQUALIZATION

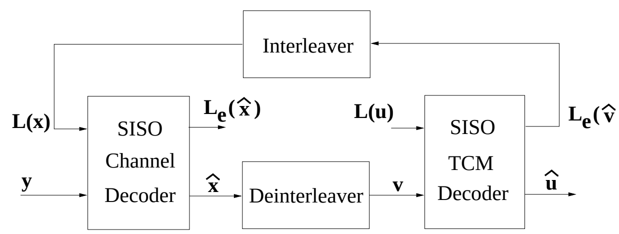

Fig. 5. Iterative equalizer

The turbo equalizer using a serially concatenated decoding scheme is shown in Fig. 5. Each soft-in/soft-out (SISO) decoding module is a four-port device with two inputs and two outputs. The inputs are the observation sequence and the a priori information denoted by $L(\cdot)$. The outputs are the decoder decision sequence and the extrinsic information either on the data symbols or on the code symbols, both denoted by $L_e(\hat{\cdot})$.

Both decoders make their soft decisions on nonbinary constellations based on the a posteriori symbol probabilities which are generated by the symbol-by-symbol MAP algorithm [7]. The computation of the soft-outputs by the MAP algorithm for two dimensional TCM constellations is described in [1], therefore only the operation of the iterative equalizer is presented in the following.

The inner SISO decoder in Fig. 5 takes the channel values as input with no a priori information at the first iteration and outputs the a posteriori information on the interleaved TCM symbols. This soft information is used to generate a decision sequence that is fed forward to the TCM decoder through a de-interleaver. The TCM encoder trellis constructed according to Ungerboeck rules differs from the ISI trellis, but the SISO TCM decoder follow similar steps to generate the soft outputs on TCM input sequence. Since all TCM inputs are equally likely the a priori information for the TCM decoder is set to a constant value. In addition to the input sequence, the TCM decoder has to produce a posteriori probabilities on the output symbols. Finally this information is interleaved and sent back to the channel decoder as the a priori information to start the subsequent iteration.

5. NUMERICAL RESULTS

The iterative equalizer is simulated for 8-PSK, 16-PSK, and 16-QAM through a broadband channel where 15000 data bits are transmitted in each constellation. A stationary broadband channel with complete state information is considered in simulations and is described by five paths with random fading coefficients and approach angles. The pulse shaping filter is described by the raised-cosine function with roll-off factor 0.2. The TCM codes are generated according to Ungerboeck rules over 8 state trellises for all three constellations.

For each TCM scheme three simulations are performed with the turbo equalizer. First a single antenna receiver is considered for comparison purposes where the antenna output is applied directly to the equalizer. Then the turbo equalizer is simulated for two and three antenna array receivers, respectively. The array observations are combined via a broadband beamformer whose coefficients are computed as described in section 3.In each case, five iterations are carried by the turbo equalizer and the performances are shown in figures 6-8.

The use of an extra element in the antenna array increases the capacity of the channel by about 3 dB in all three TCM schemes. Further improvement is achieved by the turbo equalizer, where at a 10^{-4} BER level, the equalization gain of the turbo equalizer/decoder is 2-2.5 dB for 8-PSK and 16-PSK, and around 1 dB for 16-QAM.

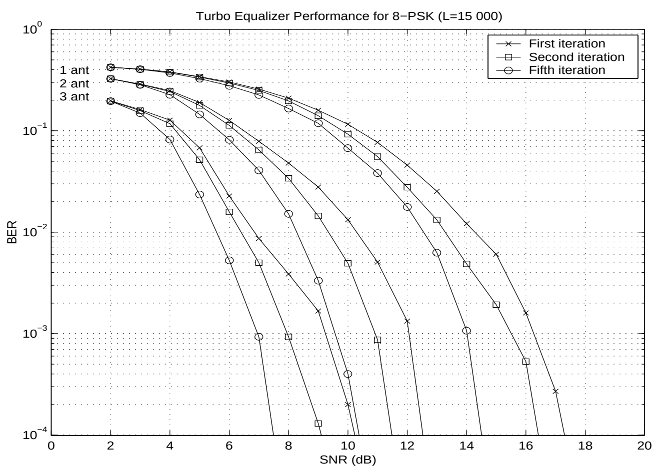

Fig. 6. Equalization performance for 8-PSK

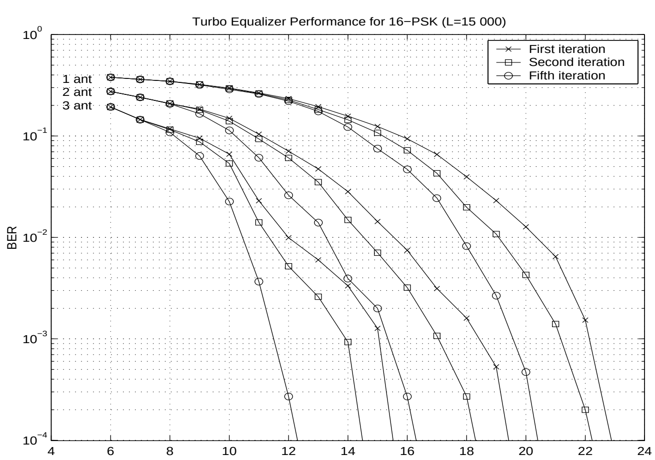

Fig. 7. Equalization performance for 16-PSK

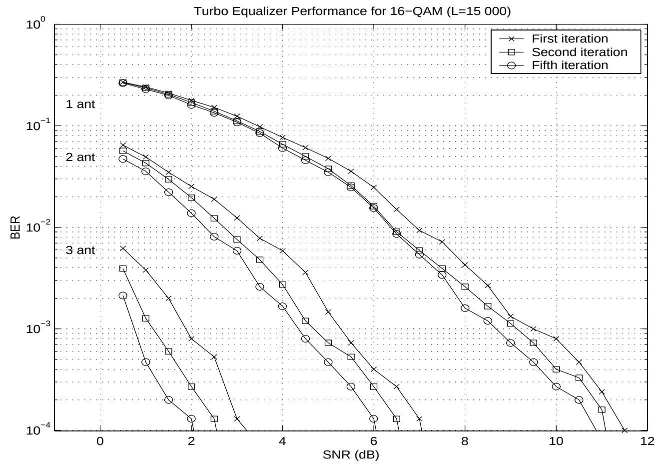

Fig. 8. Equalization performance for 16-QAM

6. CONCLUSION

In this work, receiver diversity and spatio-temporal processing are utilized in order to use the turbo decoding principle for equalization of TCM for broadband wireless channels. This new receiver differs from its narrowband counterpart only in its use of a broadband beamformer whose contribution to the overall computational complexity is not very significant.

7. REFERENCES

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