ON THE USE OF MATCHING PURSUIT TIME-FREQUENCY TECHNIQUES FOR MULTIPLE-CHANNEL DETECTION

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ABSTRACT

In situations where the presence of a signal is to be detected in several noisy channels, often one channel will have higher signal-to-noise ratio (SNR) than the others. When the SNR on one channel is sufficiently high that the signal can be extracted from that channel, it may be possible to use the extracted signal to aid in detecting the presence of the signal on the other channels. In this paper, the matching pursuit time-frequency method with matched signal dictionaries is used to extract a chirp signal from a noisy channel. The extracted signal is used in one channel of a generalized coherence (GC) detector with the goal of detecting the presence of the signal on other, even noisier, channels. This approach is compared via simulation to a GC detector that does not pre-process the highest SNR channel to extract the signal. Detector performance is shown to be significantly enhanced by matching pursuit signal extraction prior to coherence estimation.

1. INTRODUCTION

In numerous applications, signals received at multiple sensors are used for source detection and localization. In particular, multiple-channel detection of the same signal from different noisy channels can provide useful information in locating the source of the signal. The generalized coherence (GC) estimate has been established as an effective statistic for multiple-channel detection which naturally extends the widely used magnitude-squared coherence approach for two-channel detection [1, 2]. As additional channels are considered in a GC-based detector, improved performance depends on the signal-to-noise ratio (SNR) of each channel [3, 4]. In particular, as the SNR increases on one channel, the performance of the GC detector increases even though the SNR on the other channels remains low. Hence, if one could obtain a good estimate of the signal from a high SNR channel and substitute this estimate for the actual channel data in the GC-based detector, thenthe resulting detector is expected to provide improved performance.

In this paper, we propose to use a modified version of the time-frequency based matching pursuit [5, 6] in order to extract the signal of interest from a known high SNR channel. In essence, we will pre-process the signal from the high SNR channel in order to obtain a noise-free output. As we will demonstrate, this will provide an increased performance in the multiple-channel detection problem.

2. GC MULTIPLE CHANNEL DETECTION

The GC estimate has been used to detect the presence of a signal that is common on multiple noisy channels [1]. A block representation of a detector that utilizes the GC is shown in Figure 1 with M channels. Thus, M complex

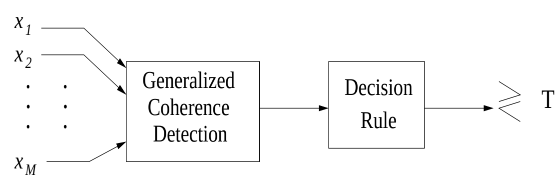

Fig. 1. Multiple channel detection using the GC estimate.

sequences, $x_i(t)$, $i = 1, \ldots, M$, each of length N, are observed. The common transmitted signal is assumed deterministic and embedded in additive white Gaussian noise. The resulting GC estimate is given by [1]

$$\lambda^2_{M,N}(x_1, \ldots, x_M) = 1 - \frac{g(x_1, \ldots, x_M)}{||x_1||^2 \ldots ||x_M||^2} \tag{1}$$

where $g(x_1, ..., x_M)$ is the determinant of the Gram matrix

$$\mathbf{G}(x_1, ..., x_M) = \begin{bmatrix} \langle x_1, x_1 \rangle & \ldots & \langle x_1, x_M \rangle \\ \vdots & \ddots & \vdots \\ \langle x_M, x_1 \rangle & \ldots & \langle x_M, x_M \rangle \end{bmatrix}.$$

With $M = 2$ channels, the GC in (1) simplifies to the magnitude squared coherence estimate (MSC),

$$\lambda^2_{2,N}(x_1, x_2) = \frac{\langle x_1, x_2 \rangle}{||x_1||^2 ||x_2||^2} = \lambda^2 . \tag{2}$$

Here, $\langle x, y \rangle = \int x(t) y^*(t) dt$ and $||x||^2 = \langle x, x \rangle$. The distribution function of the MSC, with the assumption that the

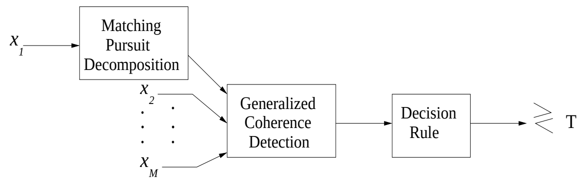

Fig. 2. GC detection using a pre-processing matching pursuit algorithm with TF dictionaries. Here, $x_1(t)$ corresponds to the output signal of the highest SNR channel.

two independent received vectors $x_1(t)$ and $x_2(t)$ are embedded in complex white Gaussian noise, is given by [1]

$$Pr\{\lambda^2 \leq T\} = 1 - (1-T)^{N-1}, \quad 0 \leq T \leq 1, \qquad (3)$$

where Pr denotes probability. Equation 3 follows a beta-distribution of the statistic λ^2, and T is the detection threshold. Considering Equation (3) under the only-noise hypothesis, the probability of false alarm P_{FA} for the MSC is given by

$$P_{FA} = Pr(\lambda^2 > T) = (1-T)^{N-1}\,. \qquad (4)$$

Thus, for a fixed P_{FA}, the detection threshold is derived as

$$T = 1 - (P_{FA})^{\frac{1}{N-1}}\,.$$

As the number of channels increases, new expressions for the P_{FA} and the corresponding thresholds can be obtained as shown in [1]. For example, for three channels ($M = 3$),

$$P_{\text{FA}} = (1-T)^{N-1} + (N-1)(N-2)(1-T)^{N-1} \cdot \log(1-T) + (N-1)^2[(1-T)^{N-2} - (1-T)^{N-1}]\,.$$

3. TF BASED MULTIPLE-CHANNEL DETECTOR

In high noise environments, poor signal quality on all channels dramatically reduces the performance of the GC detector. We propose a method that will provide a good estimate of the signal from a high SNR channel. We make the assumption that efforts were made a priori to reduce the noise level of one channel. For a fair detection performance comparison, the same assumption is maintained for the GC estimate. The proposed technique uses the matching pursuit algorithm [5] with TF dictionaries to obtain a noise-free estimate of the highest SNR channel output. The extracted signal can then be used to obtain better estimates from the low SNR channels using the GC.

3.1. Modified Matching Pursuit Algorithm

The original matching pursuit algorithm [5] decomposes a signal iteratively using elements from a matched dictionary. The elements consist of time-frequency shifted and scaled versions of one basic atom. This atom is in general chosen to be a Gaussian signal as it is highly localized in the TF plane. Our modified matching pursuit, following [6], forms its dictionary using waveforms that are matched, in TF structure, to the signal of interest. For example, in a sonar multi-channel detection application, linear frequency modulated (FM) chirps would be chosen to form the dictionary. In order for the dictionary to be complete, we transform the signal via TF shifts as well as a transformation that causes a constant shift to the instantaneous frequency of the received waveform. In the case of linear FM chirps, this corresponds to a constant shift in the FM chirp rate. Following [5], we expand a signal $x(t)$ as

$$x(t) = \sum_{n=-\infty}^{\infty} a_n\, h\big(t - \tau_n\big)\, e^{j2\pi\nu_n t} e^{j2\pi\frac{\beta_n}{2}t^2},$$

where $h(t) = e^{-j2\pi t^2}$ is a linear FM chirp, (τ_n, ν_n) is the TF shift at the nth iteration, β_n is the change in FM rate, and a_n is a weighting coefficient factor [5]. For any received signal $x(t)$ of appreciable SNR, signal components are more localized than noise components, and will have the highest correlation with matched dictionary atoms. The matching pursuit uses this property to extract signal atoms until some iteration when nearly all signal atoms have been extracted or when the residual signal energy approximately equals the energy of the noise.

3.2. Detection using Matching Pursuit

Our multiple-channel detection method proposes to extract the desired transmitted signal from the highest SNR channel using the modified matching pursuit algorithm. The GC method can then be used as depicted in Figure 2 to detect the presence of the signal using the extracted signal and signals from the remaining low SNR channels.

The matching pursuit can be used to filter noise, even at low SNRs, provided a small number of iterations is used. Thus, the matching pursuit can be used to pre-process more than one channel. However, the corresponding gain in performance will come at the expense of computational time. We seek to minimize computational time while maximizing detector performance by pre-processing only the channel with the highest SNR, which is assumed here for simplicity to be the channel with output $x_1(t)$ (see Figure 2). Note that the matching pursuit is not suitable for pre-processing very low SNR channels as noise will be included in the expansion, and the probability of detection may substantially decrease if the channel output is decomposed incorrectly.

In order to demonstrate the successful performance of the matching pursuit method in extracting a noise-free signal from a high SNR signal, we consider a linear FM signal in noise with SNR = 0 dB. The highly localized Wigner distribution (WD) [7] of the high SNR signal is plotted in Figure 3(a), and the WD of the corresponding decomposed signal using the matching pursuit is shown in Figure 3(b). Figure 3(c) shows the WD of the output of a channel with SNR = -19 dB to be detected using the proposed method.

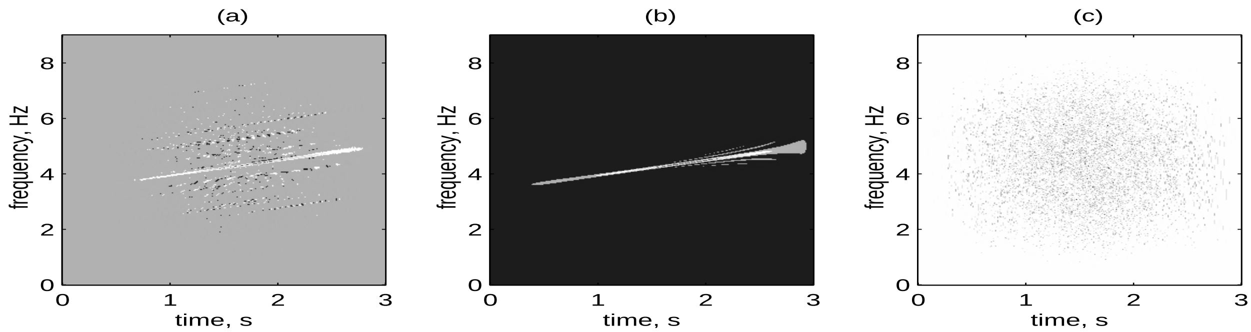

Fig. 3. Wigner distribution of the (a) channel 1 output with SNR = 0 dB, (b) matching pursuit decomposed output of channel 1, and (c) channel 2 output with SNR = -19 dB.

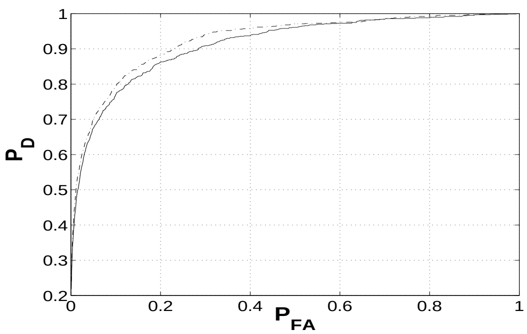

Fig. 4. MSCD (solid line) and MSCMPD performances with 10 dB SNR in channel 1 and -19 dB SNR in channel 2.

Thus, the GC performance is expected to be higher when the noise-free extracted signal in Figure 3(b) is used instead of the high SNR signal in Figure 3(a).

4. SIMULATIONS AND PERFORMANCE

In order to demonstrate our improved performance, we simulate an MSC estimate detection problem with complex data sequences of length $N = 300$ and for varying SNRs on both channels. The MSC estimate is calculated using Equation (2), and the probability of false alarm P_{FA} using Equation (4). We also simulate an MSC detector using the channel 1 decomposed signal and the channel 2 output signal. The results of both methods using Monte Carlo simulations are compared in Figures 4 through 8. These figures correspond to receiving operator characteristic (ROC) curves of probability of detection P_D versus P_{FA}. Note that we obtain the P_D from the Monte Carlo simulations. The matching pursuit algorithm provides a good approximation of the signal from the high SNR channel to improve detection on the remaining channels. The following can be observed from the results of our simulations:

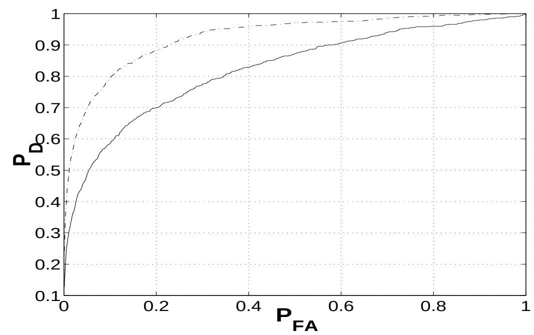

Fig. 5. MSCD (solid line) and MSCMPD performances with 0 dB SNR in channel 1 and -19 dB in channel 2.

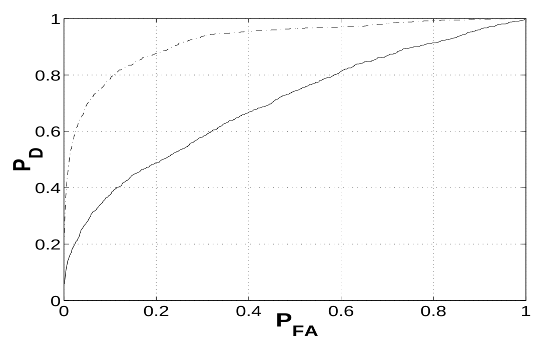

Fig. 6. MSCD (solid line) and MSCMPD performances with -10 dB SNR in channel 1 and -19 dB in channel 2.

- The MSC that uses the matching pursuit decomposition (MSCMPD) performs better than the MSC that directly uses the channel 1 output (MSCD). However, both methods produce comparably good performance when channel 1 has a very high SNR (see Figure 4), and comparably poor performance when channel 2 has very low SNR (see Figure 7).
- With a fixed SNR on channel 2, the MSCD detector performance deteriorates for decreasing SNR on channel 1 whereas the MSCMPD detector remains approximately unchanged. For a fixed channel 2, Figures 5 and 6 illustrate the resulting change in performance of the MSCD when the SNR on channel 1 decreased from 0 dB to -10 dB. As the signal on channel 1 is always extracted via the matching pursuit, the MSCMPD detector performance will remain

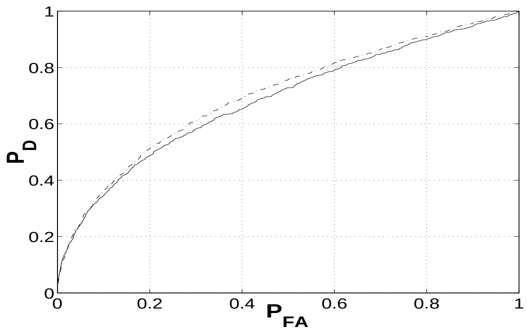

Fig. 7. MSCD (solid line) and MSCMPD performances with 0 dB SNR in channel 1 and -25 dB in channel 2.

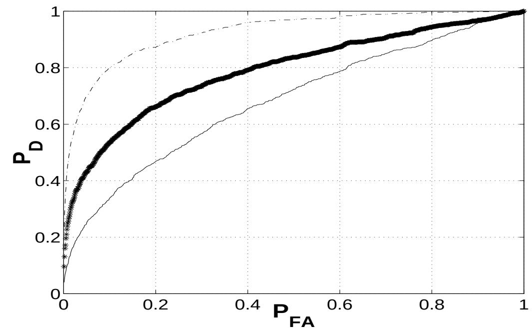

Fig. 8. Performance of the MSCD (thin solid line), MSCMPD with one iteration (dash-dotted line) and MSCMPD with six iterations (thick solid line) with -10 dB SNR in channel 1 and -19 dB SNR in channel 2.

unchanged until the SNR is so low that the signal will no longer be extracted or noise will be included in the extracted signal.

- After all localized signal components have been extracted using the matching pursuit, the correlation between noise and dictionary atoms becomes significant. Further matching pursuit iterations will now introduce noise terms back into the decomposed signal, and the performance of the corresponding MSCMPD will degrade accordingly. An example is shown in Figure 8 where an MSCMPD with one iteration (dash-dotted line) is compared to an MSCMPD with six iterations (thick solid line). It can be seen that the MSCMPD performance decreases with increasing iteration numbers. Prior knowledge of the signal SNR or number of signal components can be used here to control the number of iterations for the matching pursuit algorithm as described in Section 3.1. The spectrogram [7] can be used, for instance, to pre-process the high SNR data to obtain the number of signal components. It can also be used to identify a signal's band of interest. Bandpass filtering the signal would then reduce the computation for the matching pursuit and also provide a fair advantage to the MSCD as noise outside the band of interest is not considered.

5. CONCLUSION

In multiple-channel detection problems, it is advantageous to obtain as high a performance as possible in order to derive useful signal information. In this paper, we demonstrated that our proposed method outperforms the classical magnitude squared coherence estimate detector. The new method uses matching pursuit decomposition with similar, in TF structure, dictionaries elements in order to obtain a noise-free signal estimate of the output of a high SNR channel before using it in a GC detector. An important issue that needs to be further addressed is the trade-off between computational complexity and performance of the two methods. The matching pursuit approach is computationally intensive depending on the size of the dictionary. Thus, its use might not be preferable if a channel exists with either very high or very low SNR. However, the gain in performance in all other cases makes it worthwhile, especially when some preprocessing reduces the amount of computation. The application of an optimal matched filter to the low SNR channels using the decomposed noise-free deterministic signal is currently under investigation.

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