Refractivity Estimation from Radar Clutter by Sequential Importance Sampling with a Markov Model for Microwave Propagation

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Abstract

This paper addresses the problem of estimating range-varying parameters of the height-dependent index of refraction over the sea surface in order to predict ducted microwave propagation loss. Refractivity estimation is performed using a Markov model for microwave radar clutter returns from the sea surface. Specifically, the parabolic approximation for numerical solution of the wave equation is used to formulate the problem within a non-linear recursive Bayesian state estimation framework. Solution for the conditional expectation of range-varying refractivity, given log-amplitude clutter versus range data, is achieved using a sequential importance sampling technique. Simulation results are presented which demonstrate the ability of this approach to synoptically estimate range-varying refractivity parameters by "through-the-sensor" remote sensing.

1. INTRODUCTION

The refractivity structure associated with the capping inversion of the marine atmospheric boundary layer often causes ducted microwave propagation [1], [2]. Synoptic monitoring of ducting conditions by direct measurement of the three-dimensional humidity and temperature profiles, which determine refractivity, is difficult and expensive [3]. Thus this paper addresses the problem of estimating refractivity from clutter (RFC). In previous work, simple global parameterizations of the range and height dependent refractivity profile have been fitted to clutter returns, producing some promising real data results in several instances [4]. However, in more complex range-varying scenarios, the number of global parameters required becomes too large to handle efficiently. In this paper, the parabolic approximation for numerical solution of the wave equation is used to formulate the more general range-varying refractivity estimation problem within a non-linear recursive Bayesian state estimation framework. The potential advantage of this state-space formulation of RFC is that it can be solved efficiently using sequential importance sampling methods. This recursive Bayesian approach also imposes smoothness constraints on physically-realizable refractivity parameters. As with other RFC methods, the final objective is to predict propagation loss as a function of range and height which can be achieved by numerical solution of the wave equation using the estimated refractivity profile. Such propagation loss predictions are known as "coverage diagrams" and are often used as tactical decision aids to naval radar operators.

2. MODEL FORMULATION

Numerical solution for the electromagnetic field at range, *x,* and height, *z,* due to ducted propagation in inhomogeneous tropospheric conditions is commonly performed by using the parabolic equation (PE) approximation of the wave equation. In particular, the split-step Fourier PE solution [5] recursively computes the field, $u(x_{k+1}, z)$, at range, $x_{k+1} = x_k + \delta x$, as a function of height, *z,* given the solution at range, *x*, using a linear transformation given by:

$$u(x_{k+1}, z) = \exp\left\{ j\frac{k}{2}\left(\eta^2 + \frac{2z}{a_e} - 1\right)\delta x \right\} \times \mathbf{F}^{-1}\left(\exp\left\{ -j\frac{p^2 \delta x}{2k} \right\} \mathbf{F}\left\{ u(x_k, z) \right\} \right) \quad (1)$$

which amounts to assuming Fresnel diffraction through a thin phase-screen at each range step. The height-dependent refractivity profile between x_k and $x_k + \delta x$, is denoted, η, which enters into the phase screen term, which is the first complex exponential in (1). Other symbols in (1) are the radius of the earth, a_e, and the spatial Fourier transform operator, *F*, taken with respect to height, *z*. Consider now the case where the refractivity profile, $\eta(z, x_k, \mathbf{g}_k)$, is modeled as being a non-linear function of an uncertain random parameter vector, $\mathbf{g}_k$, whose range dependence is Markovian, i.e.

$$\mathbf{g}_{k+1} = \mathbf{A}\mathbf{g}_k + \mathbf{w}_k \quad (2)$$

where the known transition matrix **A** constrains the smoothness of the parameter variation across small range steps and the independent random vectors, $\mathbf{w}_k$, model uncertain variations between ranges. Now defining the vector of complex field values over height,

$\mathbf{u}_k = [u(z_1, x_k), ... u(z_N, x_k)]^T$, at range step *k,* equation (1) can be written as:

$$\mathbf{u}_{k+1} = \mathbf{f}(\mathbf{u}_k, \mathbf{g}_k) \quad (3)$$

where the vector-valued function $\mathbf{f}(\cdot,\cdot)$ represents the split-step Fourier solution for the field. Putting $\mathbf{u}_k$ and $\mathbf{g}_k$ of (2) and (3) into a single state vector, $\mathbf{x}_k = [\mathbf{g}_k, \mathbf{u}_k]^T$, the electromagnetic field and range-varying refractivity parameters are constrained by a non-linear set of equations given by:

$$\begin{bmatrix} \mathbf{g}_k \\ \mathbf{u}_k \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{g}_{k-1} \\ \mathbf{f}\left(\mathbf{u}_{k-1}, \mathbf{g}_{k-1}\right) \end{bmatrix} + \begin{bmatrix} \mathbf{w}_k \\ \mathbf{0} \end{bmatrix} \quad (4)$$

In this paper, $\mathbf{w}_k$ is modeled as zero-mean, Gaussian with covariance matrix $\sum_W$. In the above formulation, the process noise is only used in the model for the refractivity variables. Propagation of a weak forward random scattered field could, in principle, be handled by also including an additive process noise component in the state equations for $\mathbf{u}_k$. In the current application, the initial condition, $\mathbf{u}_0$, can assumed to be known from the antenna pattern of the radar. Historical observations and possibly an *in situ* measurement of refractivity at the radar can be used to form a prior distribution on $\mathbf{g}_o$.

Clutter returns from the sea surface can be expressd in terms of $\mathbf{x}_k$ by letting the matched-filtered radar return of the n^{th} pulse at the k^{th} slant range, denoted by $f_n(k)$, be expressed as:

$$f_n(k) = a_n(k) L(\mathbf{x}_k) + v_n(k) \quad (5)$$

where $a_n(k)$ is a complex, zero-mean, white Gaussian process with variance, $\sigma_a^2(k)$, representing local surface backscatter and $L(\mathbf{x}_k)$ is the magnitude of the field calculated at a nominal sea surface height. The receiver noise, $v_n(k)$, is modeled here as additional zero-mean complex white Gaussian noise process with variance, σ_v^2. In effect, (5) models the clutter return as the propagation loss modulated by a random "speckle noise", whose variance is the backscatter cross-section of the sea surface, in additive noise. In the forward problem, the range-dependent refractivity parameter sequence, $\mathbf{g}_k$, could be used as input to a PE propagation model to compute the propagation loss. The goal here, however, is to estimate the sequence of refractivity parameters, $\mathbf{g}_k$, given an observation of microwave radar clutter return statistics.

A common statistic of the received data that is available in many radars is the pulse-position indicator (PPI) output, y_k. The PPI is typically formed in the radar by averaging N matched-filtered, log-amplitude pulses such that $y_k = \frac{20}{N} \sum_{n=1}^{N} \log|f_n(k)|$. For the model of (5), it can be shown [4] that the PPI output for large *N*, conditioned on $\mathbf{x}_k$, is approximately Gaussian distributed with mean $10 \log\left[\sigma_a^2(k) L(\mathbf{x}_k) + \sigma_v^2\right] + 0.116$ and variance, which is a known constant, σ_y^2. Thus using (5) and noting that $L(\mathbf{x}_k) = \mathbf{e}_1^H \mathbf{x}_k \mathbf{x}_k^H \mathbf{e}_1$ where $\mathbf{e}_1 = [1, 0, ..., 0]^T$, the PPI clutter return can be modeled as:

$$y_k = \beta(\mathbf{x}_k) + \varepsilon_k \quad (6)$$

where $\beta(\mathbf{x}_k) = \frac{10}{\ln(10)} \ln\left(\mathbf{e}_1^H \mathbf{x}_k \mathbf{x}_k^H \mathbf{e}_1 \sigma_a(k) + \sigma_v^2\right) - const.$ and the ε_k are Gaussian random variables with constant variance, σ_y^2. Given the non-linear state-space formulation of (4) and (6), the objective is now to estimate the sequence of refractivity parameters, $\mathbf{g}_k$, given an observation of microwave radar returns, y_k.

3. SIS ESTIMATION OF REFRACTIVITY

A classical solution to the non-linear RFC state estimation problem would involve linearization of equations (4) and (6) and solution using the extended Kalman filter (EKF). Unfortunately, however, the appearance of η in the complex exponential of (1) makes the linearized model prone to instability. In this paper, therefore, the minimum mean-square error (MMSE) estimate of range-dependent refractivity is computed using a sequential importance sampling (SIS) approach. The basic idea behind SIS is that the posterior distribution of the state given the data can be represented by a set of random realizations (or "particles") instead of a continuous high-dimensional function. This approach was originally developed in Bayesian statistics literature [6, 7], but is beginning to receive attention in the signal processing literature [8,9].

To describe the SIS approach taken here, suppose at range step, *k,* random realizations, $\mathbf{x}_{k-1}(i)$, $i = 1, ..., M$, are available from probability density, $p(x_{k-1} | y_1, ..., y_{k-1})$. Then realizations or particles, $x_k^*(i)$, from $p(\mathbf{x}_k | y_1, ..., y_{k-1})$ can be obtained by using each of these particles as input to the state equation of (4) together with random samples $\mathbf{w}_k(i)$ drawn from the Normal distribution, $N(0,\Sigma_w)$. The PPI clutter measurement, y_k, at range bin, k, is then used to update the prior for the current range cell by evaluating the likelihood of each particle:

$$q_i = \frac{p(y_k \mid x_k^*(i))}{\sum_{j=1}^{N} p(y_k \mid x_k^*(i))} \qquad (7)$$

which is a discrete approximation to the *a posteriori* probability density, $p(\mathbf{x}_k \mid y_1,...y_k)$, i.e. the probability mass at the sample points, $\mathbf{x}_k^*(i)$. Samples from $p(\mathbf{x}_k \mid y_1,...y_k)$ can now be approximated by bootstrap resampling M times from this discrete distribution such that $\Pr\{x_k(j) = x_k^*(i)\} = q_i$ [6]. For the RFC problem formulation:

$$p(y_k \mid \mathbf{x}_k) \propto \exp\left\{ -\frac{\left(y_k - \beta\left(\mathbf{x}_k\right)\right)^2}{2\sigma_\gamma^2} \right\} \qquad (8)$$

since the log-amplitude PPI data is nearly Gaussian for large *N*. State prediction and update are repeated for each range-step in the PPI data resulting in a recursive Bayesian estimate of $p(\mathbf{x}_k \mid y_1,...y_k)$, albeit in the form of discrete particles with approximate probability masses. Calculation of the conditional mean estimate of the refractivity parameters at range k is obtained by taking:

$$\hat{\mathbf{g}}_k = \frac{1}{M}\sum_{i=1}^{M} \mathbf{g}_k(i) q_i \qquad (9)$$

where $\mathbf{g}_k(i)$ are the refractivity components of $\mathbf{x}_k(i)$.

The SIS method described above was originally referred to by [6] as the bootstrap filter. It is used here as a simple recursive method for approximating the conditional mean refractivity estimator. A difficulty with bootstrap filters, however, is that they are generally not the most efficient with respect to the number of particles which must be used. More efficient SIS methods are described in [8,9] and involve sampling from an importance function of the form:

$$\pi(x_k \mid x_{k-1}, y_k) = p(x_k \mid x_{k-1}, y_k) \qquad (10)$$

designed to ensure the majority of particles are samples from non-zero regions of the posterior distribution. For the model of (4) and (6), which does not include a random forward scattering term for the field, it can be shown that $\pi(x_k \mid x_{k-1}, y_k) = p(x_k \mid x_{k-1})$ which in turn results in an SIS method equivalent to the bootstrap filter.

4. RESULTS USING SIMULATED PPI DATA

To test the proposed refractivity estimation method, simulated PPI clutter data was generated assuming a shipboard radar operating at 2.85 GHz. with an antenna at a height of approximately 30 meters. The elements of $\mathbf{g}_k$ to be estimated were, respectively, the so-called base-height and M-deficit of a standard tri-linear refractivity profile commonly used to model refractivity [cf. e.g. 4]. The $\mathbf{g}_k$ were range-varying over $\delta x = 200$ m. increments according to (4). The simulated PPI clutter return, y_k, versus range with *N=128* snapshots is shown as the solid line in Figure 1. Ducting over the sea is responsible for the significant clutter observed at ranges beyond 20 km. The prior distribution for the base-height in SIS RFC was a Gaussian distribution centered at 35 with standard deviation of 5m. The prior for the M-deficit was also Gaussian with mean 25 M-units and standard deviation of 5 M-units. These priors are intended to model the scenario where a single noisy refractivity profile measurement is available at the ship. The backscatter cross-section is assumed to be constant and known over the entire range of interest. The transition matrix, **A**, in (2) is the identity with process noise covariance, $\Sigma_w = diag([1, 0.5])$ so as to be able to track base height changes of a couple of meters per range step. In this simulation example, the true base-height and M-deficit profiles are illustrated by the solid lines in Figures 2 and 3, respectively. SIS RFC estimation of (9) was performed using simulated clutter data and *M=2000* particles, resulting in base height and M-deficit estimates represented by the dashed lines in Figures 2 and 3, respectively. Note the estimates closely track the true values. The clutter estimate computed using, $\hat{\mathbf{g}}_k$, is shown to compare favorably with clutter generated using the true, $\mathbf{g}_k$, in Figure 1. Finally, the predicted propagation loss versus range and height, obtained using the SIS RFC estimate is illustrated in Figure 4. Ducted propagation is clearly evident by the low loss near the sea surface and this agrees very well with a coverage diagram produced using the true range-varying tri-linear refractivity profile.

5. CONCLUSION

This paper has shown how sequential importance sampling can be coupled with the parabolic equation method for solving the wave equation in order to estimate range-varying refractivity from clutter. Although simulation results show promise, further work is required to evaluate the performance of the method with real clutter data.

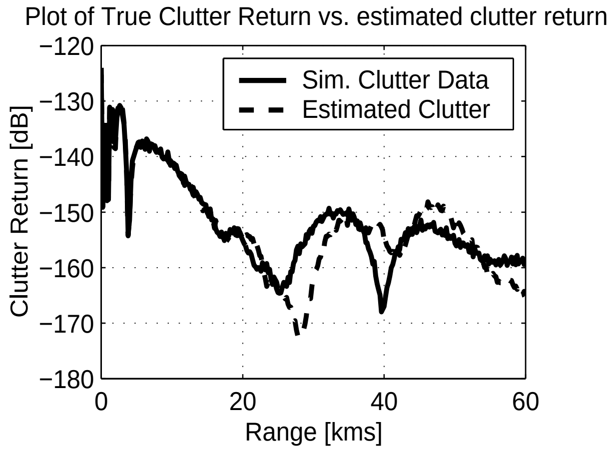

Figure 1: Plot of measured and estimated clutter

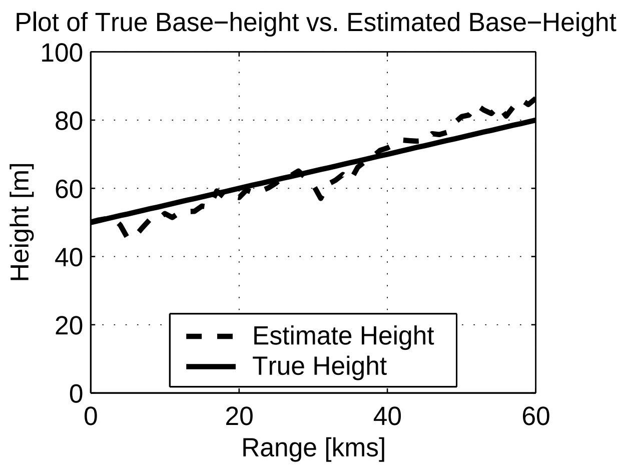

Figure 2: Plot of estimated and true height

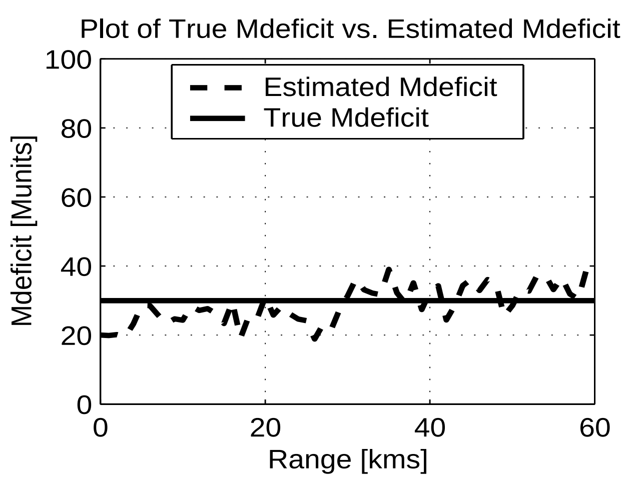

Figure 3: Plot of estimated and true Mdeficit

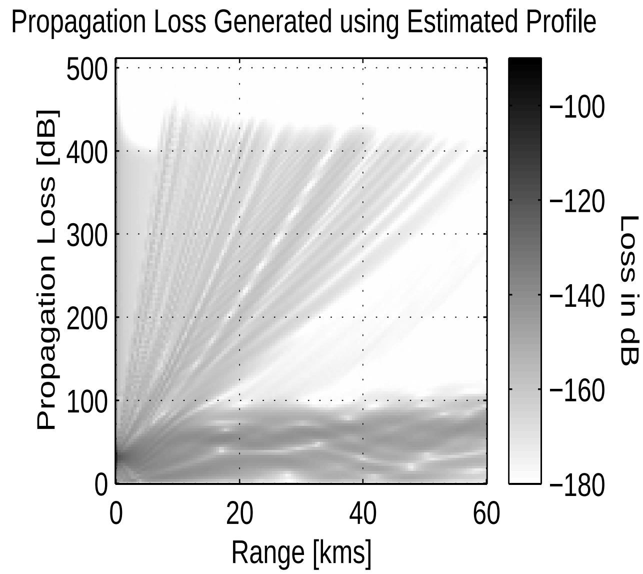

Figure 4: Plot of Propagation Loss corresponding to estimated profile.

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