STRUCTURED MMSE EQUALIZATION FOR SYNCHRONOUS CDMA WITH SPARSE MULTIPATH CHANNELS

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ABSTRACT

We present chip-level MMSE equalizers for the forward link in CDMA that exploit the underlying channel structure, specifically the fact that the channel impulse response is sparse. The assumption we make is that the channel vector lies in the subspace associated with the pulse shaping filter convolution matrix. We can then project the chip-level MMSE equalizers onto a much lower dimensional subspace due to the sparseness of the channel. The simulations demonstrate that this low-rank MMSE equalizer converges very quickly to the asymptotic SINR, even where the underlying assumption is not valid.

1. INTRODUCTION

For high-speed wireless links, the multi-path delay spread may span a significant portion of the symbol period or several symbol periods, so that the orthogonality of the Walsh-Hadamard spreading codes on the downlink is lost and there is significant multi-user access interference (MAI). When many or all users are active in the cell, the BER curve of the standard RAKE receiver has been shown to flatten out at higher SNR [1]. Chip-rate MMSE equalizers that minimize the mean-square error between the synchronous sum signal of all active users and its estimate were proposed independently by Ghauri [2] and Frank [3], and were shown to significantly outperform both Zero-Forcing and RAKE receivers [1].

For high-speed CDMA, the vector of multipath channel coefficients is typically sparse, but the chip-rate linear equalizers will not be sparse in general. Under certain conditions, the overall channel coefficients lie in a subspace associated with the time-shifted versions of the pulse shaping filter. Direct equalization that utilizes this structure was presented in [4]. An added advantage is that, due to the sparse nature of the multipath arrivals, the total channel vector lies in a subspace spanned by only a few columns of the pulse shaping convolution matrix [5]. In this paper, we construct chip-level equalizers that utilize the structure of the sparse multipath channel and the information from the training symbols, and show their convergence to be superior to reduced-rank adaptive equalizers based on training alone.

The results presented herein are for CDMA forward link with synchronous users, saturated loading, frequency selective fading, long code scrambling and two antennas at the receiver. The channel is assumed to be invariant with time, which is only true over a short time interval.

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2. DATA AND CHANNEL MODEL

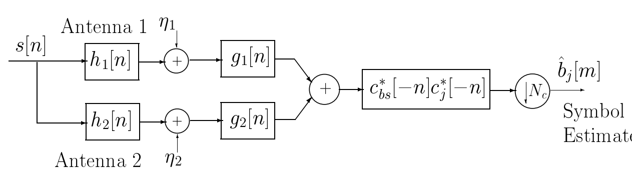

Fig. 1. Chip-level MMSE equalization for CDMA downlink with 1 base-station.

The channel model is shown in Fig. 1. For the sake of simplicity, we assume the mobile unit is receiving transmission from only one base-station. The impulse response for the i-th antenna channel between the transmitter and receiver is given by

$$h_i(t) = \sum_{k=0}^{N_m-1} h_{c_i}[k] p_{rc}(t - \tau_k) \qquad i = 1, 2. \qquad (1)$$

where $p_{rc}(t)$ is the composite chip waveform (including the matched low-pass filters on the transmit and receive end), assumed to have a raised cosine spectrum. N_m is the total number of delayed paths within the maximum delay spread, some of which may have zero or negligible power, so that the channel impulse response is sparse. In other words, if L_c is the actual number of multipath arrivals, then $L_c \ll N_m$.

The transmitted 'sum' signal may be described as

$$s[n] = c_{bs}[n] \sum_{j=1}^{N_u} \sum_{m=0}^{N_s-1} b_j[m] c_j[n - N_c m] \qquad (2)$$

where $c_{bs}[n]$ is the base-station dependent long code, $b_j[m]$ is the bit/symbol sequence of the j-th user, $c_j[n]$ is the j-th user's channel (short) code of length N_c, N_u is the total number of active users, N_s is the number of bit/symbols transmitted during a given time window.

The signal received at the i-th antenna (after convolving with a matched filter) is given by

$$y_i(t) = \sum_n s[n] h_i(t - nT_c) \qquad i = 1, 2 \qquad (3)$$

where T_c is the chip period.

3. CHIP-LEVEL MMSE ESTIMATOR

The chip-level MMSE equalizer is designed to minimize the mean-squared error between the multi-user synchronous sum signal, $s[n]$ and the sum of the equalizer outputs, as depicted in Fig. 1. Krauss and Zoltowski [1] modeled the base-station scrambling code as an i.i.d. sequence. Under this assumption, the MMSE equalizer is of the form

$$\mathbf{g}_c = \{\sigma_s^2 \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H + \boldsymbol{\mathcal{R}}_{\eta\eta}\}^{-1} \boldsymbol{\mathcal{H}} \boldsymbol{\delta}_{D_c} \qquad (4)$$

where $\boldsymbol{\delta}_{D_c}$ is a column vector of all zeros except 1 in the (D_c+1)-th position, D_c is the combined delay of the equalizer and channel, σ_s^2 is the signal power, $\boldsymbol{\mathcal{R}}_{\eta\eta}$ is the noise covariance matrix and $\boldsymbol{\mathcal{H}}$ is the $2N_g \times (L+N_g-1)$ channel convolution matrix, with N_g being the length of the equalizer, and

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix}; \; \mathbf{H}_i = \begin{bmatrix} h_i[0] & 0 & \cdots \\ \vdots & \ddots & \ddots \\ h_i[L-1] & h_i[L-2] & \cdots \\ \vdots & \ddots & \ddots \\ 0 & \cdots & h_i[L-1] \end{bmatrix}^T .$$

Equation (4) has the form of the Wiener-Hopf solution

$$\mathbf{w} = \mathbf{R}_{xx}^{-1} \mathbf{r}_{dx} \qquad (5)$$

where $\mathbf{R}_{xx}$ is the channel covariance matrix and $\mathbf{r}_{dx}$ is the cross-correlation vector.

3.1. Adaptive Equalization for CDMA Downlink

Given the orthogonality of the channel codes, an estimate of the symbol, $\hat{b}_j[m]$ can be obtained via a correlate and sum with the channel code and the base-station long code at the output of the chip-level MMSE equalizer, once per symbol.

$$\hat{b}_j[m] = \sum_{i=0}^{N_c-1} \left\{ \mathbf{g}_c^H \mathbf{y}[n+i] \right\} c_{bs}^*[n+i] c_j^*[i] \qquad (6)$$

$$\equiv \mathbf{g}_c^H \mathbf{C}_j^H[m] \tilde{\mathbf{y}}[m], \qquad (7)$$

where $n = mN_c + D_c$,

$$\mathbf{y}[n] = \begin{bmatrix} y[n] \\ \vdots \\ y[n - N_g + 1] \end{bmatrix} \quad \tilde{\mathbf{y}}[m] = \begin{bmatrix} y[n + N_c - 1] \\ \vdots \\ y[n] \\ \vdots \\ y[n - N_g + 1] \end{bmatrix}$$

and

$$\mathbf{C}_j[m] = \begin{bmatrix} c_{bs}[mN_c + N_c - 1] c_j[N_c - 1] & 0 & \cdots \\ \vdots & \ddots & \ddots \\ c_{bs}[mN_c] c_j[0] & \cdots & \cdots \\ \ddots & \ddots & \ddots \\ 0 & \cdots & c_{bs}[mN_c] c_j[0] \end{bmatrix}$$

It is not possible to train the MMSE equalizer on the chip-rate synchronous sum signal as that would require knowledge of all of the active channel codes and the transmitted symbols for all users. Instead, we train the equalizer on the symbols of the pilot channel of CDMA downlink, which employs a code of all 1's.

4. STRUCTURED EQUALIZER IN SPARSE MULTIPATH

From section 3, the channel cross-correlation vector is –

$$\mathbf{r}_{dx} = \boldsymbol{\mathcal{H}} \boldsymbol{\delta}_{D_c} \qquad (8)$$

If we restrict ourselves to the following conditions

1. $N_g \geq L$ and 2. $N_g - 1 \geq D_c \geq L - 1$

then it can be easily seen that $\mathbf{r}_{dx}$ contains all the elements of the channel impulse response [4]. In particular, if we choose $N_g = L$ and $D_c = L - 1$, we get

$$\mathbf{r}_{dx} = \begin{bmatrix} & \tilde{\mathbf{I}} \\ \tilde{\mathbf{I}} & \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix} \qquad (9)$$

where $\tilde{\mathbf{I}} = \begin{bmatrix} 0 & \cdots & 1 \\ & \ddots & \\ 1 & \cdots & 0 \end{bmatrix}$

If we assume a high enough chip rate so that the multipath delays are integer multiples of the chip period T_c, then we can write (cf. equation (1))

$$\mathbf{h}_i = \mathbf{G}\,\mathbf{h}_{c_i} \qquad (10)$$

where $\mathbf{G}$ is the convolution matrix corresponding to the chip pulse shaping waveform, and $\mathbf{h}_{c_i}$ is a sparse vector containing the multipath coefficients. In this case, the vector channel $\mathbf{h}_i$ lies in a subspace spanned by only a few columns of $\mathbf{G}$ and we can simplify equation (8) as

$$\mathbf{r}_{dx} = \begin{bmatrix} & \mathbf{G}_p \\ \mathbf{G}_p & \end{bmatrix} \begin{bmatrix} \mathbf{h}_{p_1} \\ \mathbf{h}_{p_2} \end{bmatrix} = \mathcal{G}\,\mathbf{h}_p \qquad (11)$$

where $\mathbf{G}_p$ contains only the L_c columns of $\tilde{\mathbf{I}}\mathbf{G}$ corresponding to the L_c dominant multipath arrivals, and $\mathbf{h}_p$ is the vector of corresponding complex gains.

Thus the chip-level MMSE equalizer has the form

$$\mathbf{g}_c = \mathbf{R}_{xx}^{-1}\mathcal{G}\,\mathbf{h}_p \qquad (12)$$

This structure due to the sparse multipath channel can be utilized to increase the convergence speed of adaptive MMSE equalizer. We project the observed data vector onto a rank $2L_c \ll 2L$ subspace by forming

$$\mathbf{x}_r[m] = \mathcal{G}^H\mathbf{R}_{xx}^{-1}\mathbf{C}_1^H[m]\tilde{\mathbf{y}}[m] \qquad (13)$$

We refer to the MMSE equalizer residing in this low-rank subspace as the 'semi-blind projected' equalizer.

4.1. Generalized Arrival Times

When the multipath arrival times are not exact multiples of the chip period, equation (10) is only an approximate relation. In this case we form $\mathbf{G}_p$ by taking those columns of $\tilde{\mathbf{I}}\mathbf{G}$ corresponding to the two nearest integer multiples of T_c for each multipath arrival.

4.2. Delay Estimation

Note that we do not require any knowledge of the multipath coefficients, and the multipath delays will change relatively slowly as compared to the complex gains even in a time-varying situation. Furthermore, the use of multiple antennas at the receiver should provide enough diversity to yield fast, accurate delay estimates. Typically in CDMA mobile receivers, a serial or block serial search is performed (perhaps over 512 chips), where the received sequence is correlated with the base-station long code. These short coherent correlations are combined in energy to obtain the delay estimates.

In this paper, we assume that the receiver has already formed estimates of the multipath arrival times, and we invert the estimate of the covariance matrix obtained after processing the entire block to find the low-rank subspace via equation (13).

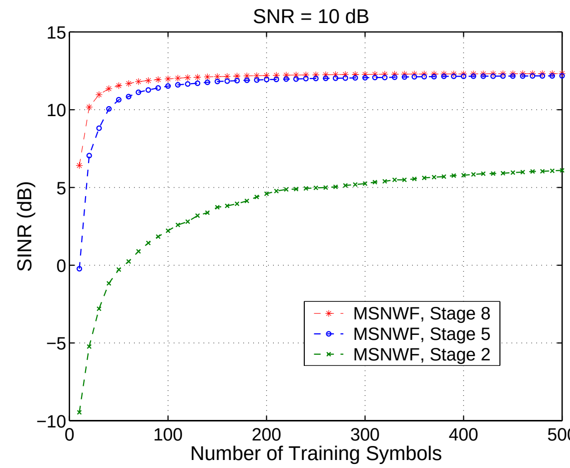

Fig. 2. SINR for Semi-blind Projected Equalizers, Arrivals at Chip-period

5. SIMULATION RESULTS

A wideband CDMA forward link was simulated similar to one of the options in the US cdma2000 proposal. The chip rate was 3.6864 MHz ($T_c = 0.27\mu s$), the spreading factor was 64, all 64 channels were active and of equal power. The channels were modeled to have four equal-power multipaths, arrival times at antenna 1 and 2 are the same, but the fading was independent.

First, we simulate frequency-selective channels where the multipath arrivals are at exactly the chip-rate, the first one arriving at 0, the last at 37 chips (corresponding to about $10\mu s$) and the other two randomly in between. The 'unstructured' equalizer length is 57 at both antennas (taking tails of the chip pulse waveform to be 5 chips at both ends), so the dimension of the full space is 114.

Fig. 2 shows the output SINR vs. time (in symbols) at a fixed SNR of 10 dB for 'semi-blind projected' chip-level equalizers, which are of dimension 8. After the projection, we use different stages of the multi-stage nested Wiener filter (MSNWF) algorithm [6] to further reduce the rank of the solution space. The stage 2 MSNWF does not perform very well, but the convergence rate for stages 5 and 8 (maximum) is very good.

To illustrate the superior performance of the 'semi-blind projected' equalizers, in Fig. 3 we compare with 'unstructured' chip-level equalizers, which operate in a reduced-rank subspace using MSNWF (see [7],[8] for details). After training with 100 symbols there is approximately 5 dB difference in output SINRs of the 'semi-blind projected' MMSE equalizer and an unstructured MSNWF solution of rank 10.

Next, we simulate frequency-selective channels where the first multipath arrival is at 0, and the other three are uniformly distributed within $10\mu s$, with the only constraint

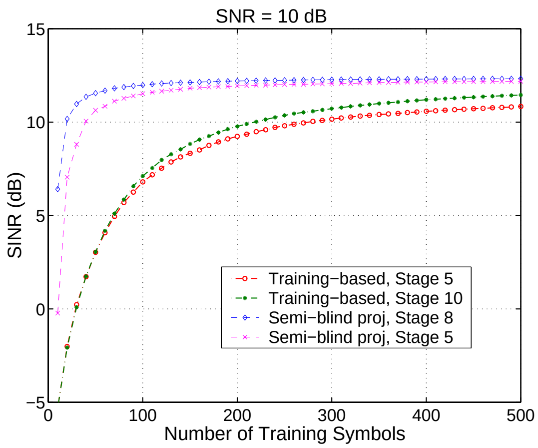

Fig. 3. SINR for Chip-level Equalizers using MSNWF

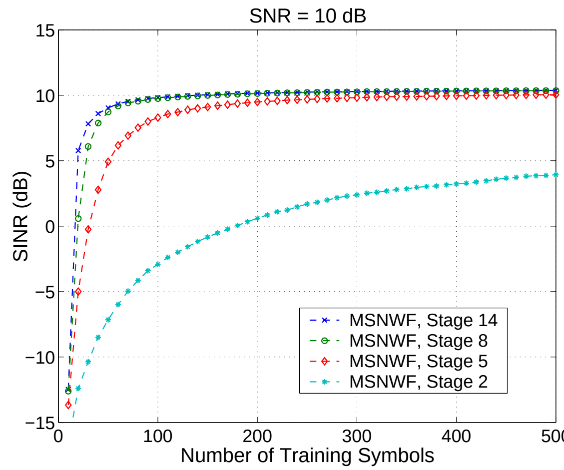

Fig. 4. Semi-blind Projected Equalizers, Random Arrivals

being that the multipath delays are spaced at least one chip-period apart. In this case the 'semi-blind projected' equalizers are of dimension 14, as we form $\mathbf{G}_p$ by taking two consecutive columns of $\tilde{\mathbf{I}}\mathbf{G}$ corresponding to each multipath arrival that is in between two chip-periods. Fig. 4 shows the SINR plot after various stages of the MSNWF, and we see that 8 stages of the MSNWF are sufficient. There is a loss of about 2 dB in asymptotic SINR compared to Fig. 2 due to the arrival times not being at exact chip periods. Most of this loss might be recouped by equalizing at twice chip-rate.

6. CONCLUSIONS

We developed a structured MMSE equalizer that exploits the sparse nature of the multipath channel to substantially reduce the number of parameters that need to be adapted. The convergence rate for this projected MMSE equalizer was significantly better than unstructured MSNWF operating in a subspace of similar rank. Furthermore, the structured equalizer showed excellent convergence even when the underlying assumption was not accurate, i.e. arrival times were not integer multiples of the chip-rate. In future, we will incorporate delay estimation, and low-complexity, real-time estimation of the inverse of the covariance matrix into an adaptive low-rank solution.

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