IMPORTANCE SAMPLING EVALUATION OF DIGITAL PHASE DETECTORS BASED ON EXTENDED KALMAN-BUCY FILTERS

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ABSTRACT

This paper proposes an *importance sampling* methodology for the performance evaluation of a class of open-loop receivers with random carrier phase tracking in additive white Gaussian noise channels. The receivers, consisting of a bank of extended Kalman-Bucy filters and a decision algorithm based on the filters' innovations processes, perform symbol-by-symbol phase detection while kepping track of the random phase process within the symbol interval. We use a large deviations approach to start a stochastic importance sampling optimization, both for the irreducible error floor and for the general noisy operation of the receiver. Our simulations show a practical coincidence with conventional Monte Carlo results, with considerable simulation time gains.

1. INTRODUCTION

Digital phase detection and random carrier phase tracking in *additive white Gaussian noise* (AWGN) channels are problems considered in reference [1]. The receivers therein proposed consist of a bank of 'matched' stochastic *nonlinear filters* (NLF) and a decision algorithm driven by the filters innovations processes. Modeling random carrier phase as Brownian motion, the NLF developed in [1] represents the conditional densities as *Tikhonov* functions whose parameters are recursively propagated. The resulting NLF-based receiver is then evaluated and compared with an alternative structure where the estimation units are *extended Kalman-Bucy filters* (EKBF). This comparative evaluation has been performed by conventional, time-consuming, Monte Carlo (MC) simulation. To further test and refine the approach and the algorithms developed, fast simulation tools have to be devised. This was the purpose of the work reported in [2], dealing only with the NLF-based receiver. With the same objective, this paper considers the *importance sampling* (IS) assessment of EKBF-based receiving structures.

The paper is organized as follows: In Section 2 we present the adopted communications model and receiver and some IS considerations relevant to the analysis. In section 3 we derive the ideal error set and show how to take advantage of its characteristics to support the IS optimization process. In section 4 we address some implementation issues and present simulations results.

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2. PROBLEM FORMULATION

2.1. Model and receiver description

Consider the discrete base-band phase modulated received signal consisting of N samples per k^{th} symbol interval, $[kT_s, (k+1)T_s]$, of duration T_s:

$$\mathbf{s}_n = \exp\left[j\left(\theta_n^{(s)} + \phi_n\right)\right] + \mathbf{v}_n, \qquad n = 1, \ldots, N$$

where $\theta_n^{(s)}$ is the digital phase sequence associated to one of the M symbols, $\alpha_s \in \{\alpha_1, \cdots, \alpha_M\}$, ϕ_n is a discrete Brownian motion described by $\phi_n = \phi_{n-1} + \delta_n$, where δ_n is a zero mean white Gaussian sequence of variance σ_ϕ^2; $\mathbf{v}_n$ is a complex zero mean white Gaussian sequence of variance σ_v^2. The purpose is to analyze a receiver consisting of a bank of EKBF driven by the same input $\mathbf{s}_n$ and a decision algorithm driven by the filters innovations processes as sketched in Fig. 1. The detector decides, at the end of the current symbol interval, according to a minimum Euclidean metric computed from those innovations. Parameters of the selected EKB filter are used as initial conditions to all EKB filters for the next symbol interval (see [1] for details).

Matching to symbol α_s is achieved by eliminating the modulating sequence from the observation vector giving rise to observations $\mathbf{z}_n^{(s)}$. The EKBF equations, implementing density mean ($\hat{\phi}$) and variance (Σ) propagation, are, for the filtering (F) and prediction (P) steps:

- Filtering

$$\hat{\phi}_n^F = \hat{\phi}_n^P + \frac{\Sigma_n^P}{\Sigma_n^P + \sigma_v^2}\left[\begin{array}{cc} -sin\hat{\phi}_n^P & \cos\hat{\phi}_n^P \end{array}\right] \times \left[\begin{array}{c} z_{1,n} - \cos\hat{\phi}_n^P \\ z_{2,n} - \sin\hat{\phi}_n^P \end{array}\right] \qquad (1)$$

$$\Sigma_n^F = \frac{\sigma_v^2 \Sigma_n^P}{\Sigma_n^P + \sigma_v^2} \qquad (2)$$

- Prediction

$$\hat{\phi}_{n+1}^P = \hat{\phi}_n^F \qquad (3)$$

$$\Sigma_{n+1}^P = \Sigma_n^F + \sigma_\phi^2. \qquad (4)$$

In the next symbol interval, all the EKBF are initialized with parameters $\left(\hat{\phi}_{N+1}^P, \Sigma_{N+1}^P\right)$ from the previously selected branch.

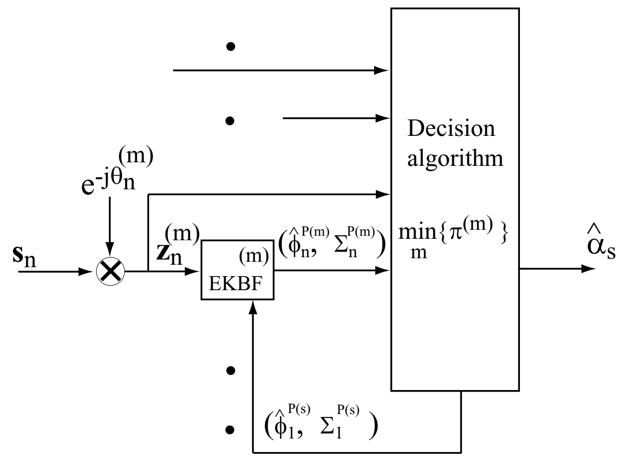

Figure 1: Diagram of receiver with branch matched to α_m

2.2. Modeling for importance sampling

Fig. 2 is a schematic representation of the above described communication model, with $A_k = \alpha_s$ being the transmitted symbol with $\alpha_s \in \{\alpha_1, \cdots, \alpha_M\}$, and Y_{N_k} and S_{N_k} being the transmitted and received signal vectors respectively containing N samples each within symbol interval $[kT_s, (k+1)T_s]$. V_{N_k} is the AWGN vector $V_{N_k} = [\mathbf{v}_1, \ldots, \mathbf{v}_N]_k$, and $\Delta_{N_k} = [\delta_1, \ldots, \delta_N]$ the phase increment vector. In our IS simulation model, an additional variable I_k, normally 'hidden' under MC simulation, will be generated to model the influence of estimate $\hat{A}_{k-1}$ on $\hat{A}_k$. This variable will replace the internal decision feedback mechanism, allowing the implementation of an *event simulation* (see [3] and [4]), (as opposed to *stream simulation* in MC), using a record of N samples (one symbol) plus one initialization variable I_k. The unbiased IS P_e estimator will be in this context

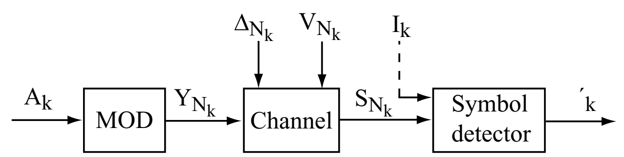

Figure 2: Model for the communication system

$$\hat{P}_e^* = \frac{1}{N_{IS}} \sum_{i=1}^{N_{IS}} 1_E\left([Y,U]_i^*\right) W\left([Y,U]_i^*\right)$$

where $1_E(\cdot)$ is the error set indicator function, $U = [\Delta_N; V_N; I]$ and $W\left([Y,U]_i^*\right) = p\left([Y,U]_i^*\right) / p^*\left([Y,U]_i^*\right)$ is the likelihood ratio between the unbiased simulation density $p(\cdot)$ and the biased one $p^*(\cdot)$. With $p^*(\cdot)$, we generate the records $[Y,U]^*$ for a higher error rate in order to minimize the estimator variance

$$\sigma_{IS}^2 = \frac{1}{N_{IS}} \left[\int 1_E(y,u) W(y,u) p(y,u) dy du - P_e^2 \right].$$

We will seek its minimization for a given N_{IS}, not considering the infeasible unconstrained solution for $p^*(\cdot)$ which depends on the unknown P_e (see Lemma 1 in [5]).

In practice we will take advantage of the event simulation system through the implementation of a conditional IS (CIS) scheme in which the biasing of $p(\cdot)$ will be conditioned on the symbol under test and the variable I_k. As I_k derives from the decision process at symbol $k-1$, there is always a discrete component of I_k allowing conditioning.

In our analysis we seek the minimization of σ_{IS}^2 using a stochastic technique that consists in estimating the characteristics of the optimal multimodal density. The major problem is to start this search and keep it yielding useful results in the presence of a complex unknown error set.

For the purpose of starting and guiding the minimization of σ_{IS}^2 we use the results established in large deviations theory [5].

3. IMPORTANCE SAMPLING METHODOLOGY

3.1. The ideal error set derivation

Error set analysis becomes very difficult due to non-linear recursion in equations (1) to (4) and the adopted decision algorithm. Restricting our analysis to the N dimensional space of the random phase increments denoted by $\mathcal{D}$, $\left(\sigma_\phi^2 \neq 0, \sigma_v^2 = 0\right)$, we obtain the simplified filter equations:

$$\begin{aligned} \hat{\phi}_n^F &= \hat{\phi}_n^P + \sin\left(\arg\left(\mathbf{z}_n\right) - \hat{\phi}_n^P\right) \qquad (5) \\ \Sigma_n^F &= 0 \\ \hat{\phi}_{n+1}^P &= \hat{\phi}_n^F \\ \Sigma_{n+1}^P &= \sigma_\phi^2, \end{aligned}$$

where we still find a degree of recursion that prevents an useful analysis of the error set. However, if we assume an ideal situation where the EKB filter operates in close tracking, we may replace the $\sin\left(\arg\left(\mathbf{z}_n\right) - \hat{\phi}_n^P\right)$ term in equation (5) by its argument, obtaining $\hat{\phi}_n^F = \left(\phi_n\right)_{2\pi}$. With this approximation, and considering the binary case $M = 2$, the decision metric for a generic branch t in the receiver, when symbol α_s is transmitted, is

$$\pi_{t|s} = \sum_{n=1}^{N} \left\| \exp j\left(\theta_n^{(s)} - \theta_n^{(t)} + \phi_n\right) - \exp j\hat{\phi}_n^{P(t)} \right\|^2,$$

where

$$\begin{aligned} \hat{\phi}_n^{P(t)} &= \theta_{n-1}^{(s)} - \theta_{n-1}^{(t)} + \phi_{n-1}, \ \ n > 1 \\ \hat{\phi}_1^{P(t)} &= I_{i|j} + \phi_0. \end{aligned}$$

The receiver initialization $\hat{\phi}_1^{P(t)}$ is in the present context the sum of the random phase ϕ_0 with an error term $I_{i|j} = \theta_N^{(j)} - \theta_N^{(i)}$ resulting from the $(k-1)^{th}$ election of branch i given the transmitted symbol α_j. We now define $\Delta_{N_k} = [\delta_1, \ldots, \delta_N]$ and

$$E_{t|s, I_{i|j}}^D = \left\{ \Delta_N \in R^N : \pi_{s|s} \geq \pi_{t|s} \right\}$$

as the error set in $\mathcal{D}$ conditioned on transmission of symbol α_s and initialization error $I_{i|j}$. Although equation $\pi_{s|s} = \pi_{t|s}$ defining the decision boundary can not be solved in general, we may identify a denumerable set $\partial\triangle$ of infinite solutions satisfying

$$\cos\left(\delta_1 - I_{i|j}\right) = \cos\left(\epsilon_1^{t|s} + \delta_1 - I_{i|j}\right)$$

$$\cos\delta_n = \cos\left(\epsilon_n^{t|s} + \delta_n\right), \quad n = 2, \ldots, N$$

where

$$\epsilon_n^{t|s} = \theta_n^{(s)} - \theta_n^{(t)} - \left(\theta_{n-1}^{(s)} - \theta_{n-1}^{(t)}\right), \qquad n = 2, \ldots, N$$
$$\epsilon_1^{t|s} = \theta_1^{(s)} - \theta_1^{(t)}.$$

A finite set of solutions $\partial\triangle_{2\pi} \subset \partial\triangle$ containing only 2^N elements is obtained by the intersection

$$\partial\triangle_{2\pi} = \partial\triangle \cap \left\{\Delta_N \in [-\pi, \pi] \times \cdots \times [-\pi, \pi]_{C_{\Delta_N}}\right\}.$$

There are 2^N disjoint quadrants $Q_i \in \mathcal{D}$ defined wrt the center of symmetry C_{Δ_N}. C_{Δ_N} is derived from the points in $\partial\triangle_{2\pi}$. An example of the shape of such error set for $N = 2$ is shown in Fig. 3 where the solution set $\partial\triangle_{2\pi} = \{\Delta_{s_1}, \Delta_{s_2}, \Delta_{s_3}, \Delta_{s_4}\}$ is also represented. The error region exhibits a periodic structure generally non-connected and extending all over $\mathcal{D}$.

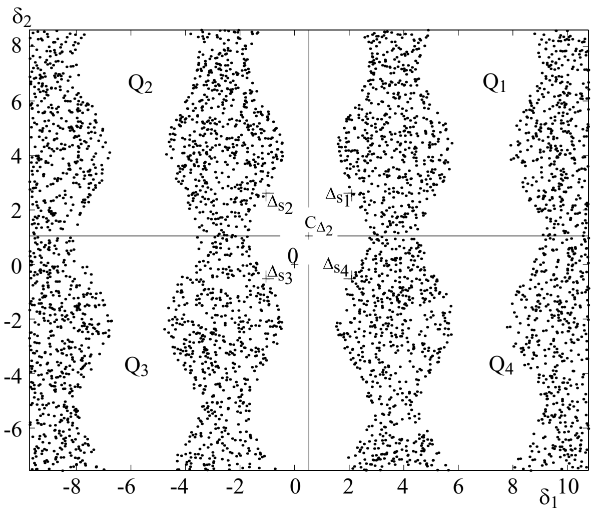

Figure 3: Ideal error set example. N=2, $I = 0$, $\epsilon_1 = 2/3\pi$, $\epsilon_2 = \pi/3$

3.2. Large deviations aspects

If $E^D_{t|s,I_{i|j}}$ was the true error set, we would be concerned with the identification of the unique dominating point $\nu \in \partial E^D_{t|s,I_{i|j}}$ (see [5]) for the optimal biasing of the Gaussian density

$$p\left(\Delta_N\right) = \left(\frac{1}{\sqrt{2\pi}\sigma_\phi}\right)^N \exp\left(-\sum_{n=1}^{N} \delta_n^2/(2\sigma_\phi^2)\right),$$

of the independent phase increments $\Delta_N \in \mathcal{D}$. The dominating point for $E^D_{t|s,I_{i|j}}$ does not exists, as can be inferred by the periodicity of the solution for $\pi_{s|s} = \pi_{t|s}$. Besides, as we are not dealing with the true error set, the use of the dominating point for density biasing (or any equivalent procedure based on the minimum rate points as required in [5] - Theorem 2) would lead to a non-efficient simulation density. Regarding the statements in the referred theorem, and the symmetry exhibited by $E^D_{t|s,I_{i|j}}$, we synthesize a biased simulation density consisting of a finite combination of Gaussian terms for the sampling of each one of the disjoint subsets $E_{Q_i} = E^D_{t|s,I_{i|j}} \cap Q_i$. Then we perform a stochastic search (based on the technique presented in [6]) of the shift terms for the biased simulation density. We estimate

$$\mu_i = E\left\{\Delta_N | \Delta_N \in E_{Q_i}\right\}$$

for each quadrant of interest considering that only a small number of them contribute significantly for the total mass of the error probability in $E^D_{t|s,I_{i|j}}$. Only the most important shift terms, namely $\{\mu_1, \ldots, \mu_{N_m}\}$ from a total of 2^N are considered. We start the quadrant-wise estimation of the N_m biasing solutions μ_i in the minimum rate point (see Definition 2 in [5]) of each $E_{Q_i} \subset E$, which are found by a constrained quadrant-wise minimization of the large deviations *rate function* starting in the solutions contained in $\partial\triangle_{2\pi}$.

3.3. Biasing for full range analysis

In the preceding section we dealt with biasing in $\mathcal{D}$. This refers to IS simulation for the error floor assessment. As we are interested in the full range analysis of the receiver $\left(\sigma_\phi^2 \neq 0, \sigma_v^2 \neq 0\right)$, we define the $3N+1$ dimensional product space $\mathcal{D} \times \mathcal{V} \times \mathcal{I}$ containing the error set E^{DVI}. This error set contains all the vectors of phase increment Δ_N, noise V_N and initialization error $I_{i|j}$ samples (we are now modeling $I_{i|j}$ as a continuous rv) that jointly produce an error event. Estimation of density biasing terms as explained in the previous subsection apply to this more general case *mutatis mutandis*. We start the optimization process in the product space $\mathcal{D} \times \mathcal{V} \times \mathcal{I}$ in the points $\{(\mu_i; \mathbf{0}; 0), \ldots, (\mu_{N_m}; \mathbf{0}; 0)\}$.

4. SIMULATION ASPECTS AND RESULTS

4.1. Practical considerations

For $M > 2$, (the m-ary case), the error set conditioned on transmission of α_s and initialization error $I_{i|j}$ is the union $E^D_{s,I_{i,j}} = \cup_{\substack{t=1\\t\neq s}}^{M} E^D_{t|s,I_{i|j}}$ of generally non-disjoint error sets similar to the one derived above. This implies the introduction of another level of multiple biasing with $M-1$ terms - one bias term for each error target symbol when α_s generation occurs.

For the error floor analysis (ideal case) $I_{i|j}$ is a discrete rv taking values on a finite set with unknown $P\left(I_{i|j}\right)$, while for the full range analysis, it becomes a continuous rv lacking characterization. Our tests showed that all the $I_{i|j}$ values have the same impact on $\hat{P}_e$ as $I_{i|i}$. Although our simulators are prepared to recursively estimate $P\left(I_{i|j}\right)$, we simulated only $I_{i|i} = 0$ (as conditioning event) with $P\left(I_{i|i}\right) = 1$. For the full range analysis we used $I \sim \mathcal{N}\left(0, s_I^2\right)$ instead of $I_{i|i} = 0$, with s_I^2 estimated in a short preamble due to its dependence from σ_ϕ^2 and σ_v^2.

Our stopping criterion consists of using the traditional test on the empirical precision (see [4] for instance) with the threshold set to 10%. This corresponds to stopping the MC simulation after the occurrence of 100 independent errors.

4.2. Results

Results were obtained for orthogonal 4-FSK signaling with $\Delta f = 1/T_s$ rads^{-1} between adjacent symbols. The number of samples per symbol was set to $N = 10$. The simulation gain - γ_s - is defined as the ratio between the *MC* simulation time, T_{MC}, and the corresponding *IS* time, T_{IS}, $(\gamma_s = T_{MC}/T_{IS})$. Simulations were stopped at an empirical precision value lower than 10%.

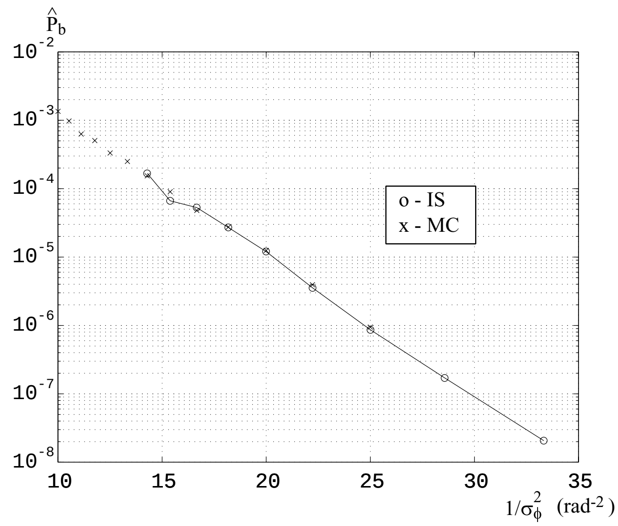

Figure 4: BER estimates comparison (IS with MC) in the error floor $\left(\hat{P}_b\right)$ versus σ_ϕ^{-2}.

σ_ϕ^{-2}	14.3	15.4	16.7	18.2	20.0	22.2	25
γ_s	10	17	24	77	171	514	2142

Table 1: Simulation time gains in the error floor

In Fig. 4, simulation results are presented for the error floors, with σ_ϕ^2 comprised between 0.1 rad^2 $\left(1/\sigma_\phi^2 = 10\right)$ and 0.03 rad^2 $\left(1/\sigma_\phi^2 = 33.3\right)$. Notice the coincidence between MC (mark x) and IS (mark o) $\hat{P}_b$ estimates. In Table 1, the simulation gains are presented for the error floor MC/IS estimates. The value of T_{MC} for $\hat{P}_b \left(\sigma_\phi^{-2} = 25\right)$ is 13.7 hours for a PIII@450MHz computer.

The IS results in Fig. 5, show the receiver performance for a wide range of operating conditions (including the error floor). MC values for $\hat{P}_b$ are also presented for $\sigma_\phi^2 = 0.04$rad^2. In this curve, the observed simulation time gains $(E_b/N_0; \gamma_s)$ were (15dB;14), (18dB;52), (21dB;142), (24dB;226), (27dB;309) and (30dB;544). For 12dB, the multimodal stochastic optimization did not achieve convergence. For this estimate $T_{MC} \simeq 16min$.

Points on the lower curve $\left(\sigma_\phi^2 = 0.035rad^2\right)$ took 12 minutes while the corresponding *MC* points would take about 11.7 days.

5. CONCLUDING REMARKS

In this study we have assumed that the random carrier phase is a Brownian motion. It would be important to extend the simulation

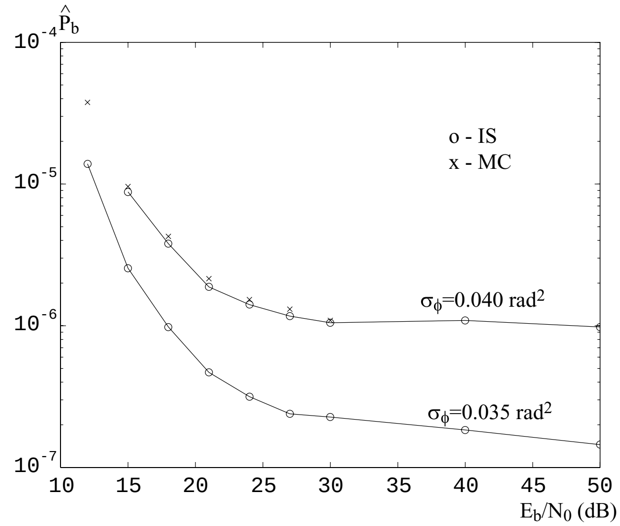

Figure 5: $\hat{P}_b$ versus E_b/N_0 for two values of σ_ϕ^2

techniques herein developed to higher order carrier phase dynamics with simultaneous channel fading. We are currently working toward that objective.

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