OPTIMAL TRANSCEIVERS FOR DMT BASED MULTIUSER COMMUNICATION

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ABSTRACT

This paper considers discrete multitone (DMT) modulation for multiuser communications where different users are supported by the same system. These users may have differing quality of service (QoS) requirements, as quantified by their respective bit rate and symbol error rate specifications. Our goal is to minimize the transmitted power given the QoS specifications for the different users, subject to the knowledge of colored interference at the receiver input. In particular we find an optimum bit loading scheme that distributes the bit rate transmitted across the various subchannels belonging to the different users, and subject to this bit allocation, determine an optimum transceiver.

1. INTRODUCTION

The discrete multitone (DMT) modulation channel coding scheme has established itself as an effective high rate data communication technique in both wired and wireless environments and is used for example in ADSL and HDSL, [1]. We consider DMT in a multiuser environment. Thus the DMT system studied here supports multiple users, with varying quality of service (QoS) requirements, quantified by their respective bit rate and symbol error rate (SER) specifications.

Specifically, consider the DMT system as in fig. 1 which depicts an M-subchannel filter bank model of a DMT system. We consider an overinterpolated ($N > M$) filter bank as the transceiver. We assume that the channel $C(z)$ is FIR of lenth κ (preequalization is assumed to have been done), and $v(n)$ is additive colored noise with known spectrum. Thus for example $v(n)$ could represent co-channel interference. Note, [8] provides models for cochannel interference in a variety of settings. To mitigate intersymbol interference (ISI), a form of redundancy is incorporated by choosing $N = M + \kappa$. The transmitting filters, $F_k(z)$, and the receiving filters, $H_k(z)$, are constrained to length N, and act as modulating and demodulating transforms respectively. In a DFT based DMT implementation [1], the IDFT and DFT are used as the modulating and demodulating transforms respectively.

In this paper, as in [5] we will consider more general transformations leading to a *generalized DMT* system. To capture a multiuser environment, we assume that there are L-users each having been assigned M/L subchannels. Further the k-th user requires a bit rate of t_k, and an SER of no more than η_k. Our goal is to select F_i and H_i, and distribute the bit rates among the various sub-channels to achieve the above specifications with the *minimum possible transmitted power*. The problem addressed here thus directly generalizes that in [5], which also addresses the same power minimization issue, but assuming a single user subject to only one bit rate and SER constraint. The multiuser setting renders the optimization problem highly nontrivial in comparison to the single user case. Further we show as much as 8 dB and 12 dB savings in transmit power in our simulations with general DMT systems over DFT based DMT systems with optimal bit allocation and no bit allocation respectively.

Related literature includes [4] which develops fast loading algorithms using table lookups and a fast Lagrange bisection method for a single user setting. [7] considers a single user optimization of the transceiver mutual information. [3], considers the optimum bit loading problem when two users are present.

Section 2, defines the generalized DMT system and formulates a precise mathematical problem. Sections 3 and 4 respectively consider the bit rate allocation and filter selection problems. Section 5 gives simulations.

2. DMT BASED MULTIUSER SYSTEM MODEL

In this Section we give some preliminaries. Specifically, in Section 2.1, we recount the details of the generalized DMT system provided in [5]. Section 2.2 provides a precise optimization problem.

2.1. Polyphase representation of the DMT system

Consider the filter bank based DMT model in fig. 1. $v(n)$ is a zero mean wide sense stationary additive noise. As

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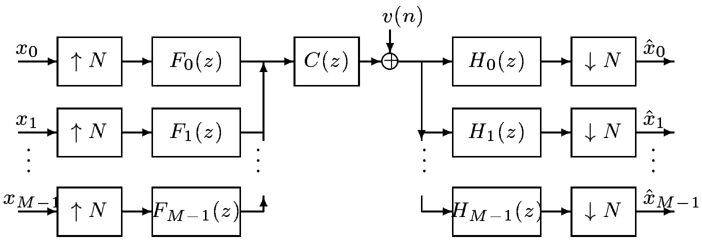

Fig. 1. Filter bank based DMT model.

the filters $F_k(z)$ and $H_k(z)$ have lengths $\leq N$, we may write the following polyphase decompositions: $F_k(z) = \sum_{i=0}^{N-1} z^{-i} G_{i,k}$, and $H_k(z) = \sum_{i=0}^{N-1} z^{i} S_{k,i}$, with constant $G_{i,j}$ and $S_{i,j}$. Define the $N \times M$ matrix G with ij-th element $G_{i,j}$ and the $M \times N$ matrix S with elements $S_{i,j}$. Call the constant matrices G and S the transmitting and receiving matrix repectively. Then with $\mathbf{x}$ and $\hat{\mathbf{x}}$ the vector of the signals x_i and $\hat{x}_i$, repectively, $\tilde{\mathbf{v}}$, the blocked version of $v(n)$, one has the equivalent system in fig. 2. Here the pseudocirculant matrix $\mathbf{C}(z)$ [9], is formed by the coefficients of the FIR channel $C(z) = c_0 + c_1 z^{-1} + \ldots + c_\kappa z^{-\kappa}$. It obeys:

$$\mathbf{C}(z) = \left[\begin{array}{cc} \mathbf{C_0} & \mathbf{C_1}(z) \end{array} \right] \tag{2.1}$$

where $\mathbf{C_0}$ is constant, $N \times M$, and $\mathbf{C_1}(z)$ is $N \times \kappa$. Note the knowledge of the autocorrelation of v, yields the autocorrelation matrix of $\tilde{\mathbf{v}}$.

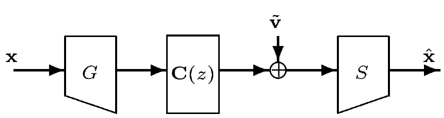

Fig. 2. Polyphase representation of the DMT system.

For DMT systems using zero padding, the transmitting and receiving matrices are respectively given by

$$G = \left[\begin{array}{c} W^\dagger \\ 0 \end{array} \right], \quad S = \Gamma^{-1} \left[\begin{array}{cc} W & W_0 \end{array} \right] \tag{2.2}$$

where W is the $M \times M$ unitary DFT matrix with $[W]_{l,m} = \frac{1}{\sqrt{M}} e^{-\frac{j2\pi lm}{M}}$, $l, m = 0, \ldots, M-1$, W_0 is the $M \times \kappa$ submatrix of W having the first κ columns of W, and Γ is the $M \times M$ diagonal matrix with elements that are the M-point DFTs of the channel impulse response, [1]. We consider more general DMT systems that can lead to reduction in sidelobes and better noise rejection properties of the filters. The transmitting matrix of such a general DMT is given by

$$G = \left[\begin{array}{c} G_0 \\ 0 \end{array} \right] \tag{2.3}$$

where G_0 is an arbitrary $M \times M$ unitary matrix. The condition for perfect reconstruction (PR) is given as

$$S\mathbf{C}(z)G = I \tag{2.4}$$

Using (2.1) and (2.3), the PR condition reduces to

$$S\mathbf{C_0}G_0 = I \tag{2.5}$$

Using singular value decomposition, $\mathbf{C_0}$ can be written as

$$\mathbf{C_0} = \underbrace{\left[\begin{array}{cc} U_0 & U_1 \end{array} \right]}_{U} \left[\begin{array}{c} \Lambda \\ 0 \end{array} \right] V^T = U_0 \Lambda V^T \tag{2.6}$$

where U and V are respectively $N \times N$ and $M \times M$ unitary matrices whose columns are the eigenvectors of $\mathbf{C_0}\mathbf{C_0}^T$ and $\mathbf{C_0}^T\mathbf{C_0}$. Λ is the $M \times M$ diagonal matrix with diagonal elements that are the singular values of $\mathbf{C_0}$.

Using (2.6), one clear choice for S satisfying (2.5) is

$$S = G_0^T V \Lambda^{-1} U_0^T \tag{2.7}$$

2.2. Problem definition

The optimum bit loading problem is to find the best bit rate allocation scheme to minimize the transmit power, under different bit rate and SER budgets of the users. The optimal transceiver is then designed to minimize the power subject to optimum bit loading.

Assume there are r users, with each user being allocated L subbands (in fig. 1, $M = rL$ and $N = M + \kappa$). Let the input power in the j-th subband of the k-th user be $\sigma^2_{x_{j,k}}$. Due to PR, this is also the output signal power $\sigma^2_{\hat{x}_{j,k}}$ in the j-th subband of the k-th user. Let the output noise power in this subband be $\sigma^2_{e_{j,k}}$, and $b_{j,k}$ be the number of bits allocated in this subchannel. Due to different QoS requirements, we may have different bit rate constraints for the users. The average number of bits for the k-th user is $b_k = \frac{1}{L}\sum_{j=0}^{L-1} b_{j,k}$. However we need to account for the reduction in bit rate due to the zero padding. The average bit budget for the k-th user is then $t_k = \frac{L}{N} b_k = \frac{1}{N}\sum_{j=0}^{L-1} b_{j,k}$.

With a high bit rate assumption made on the modulation system, we have, [5], for the k-th user

$$\sigma^2_{x_{j,k}} = c_k 2^{2b_{j,k}} \sigma^2_{e_{j,k}}$$

where the constant c_k depends on the SER η_k. We seek to minimize the average transmission power given by

$$f = \frac{1}{M} \sum_{k=1}^{r} \sum_{j=0}^{L-1} \sigma^2_{x_{j,k}} \tag{2.8}$$

$$= \frac{1}{M} \sum_{k=1}^{r} \sum_{j=0}^{L-1} c_k 2^{2b_{j,k}} \sigma^2_{e_{j,k}} \tag{2.9}$$

subject to the bit rate budgets

$$t_k = \frac{1}{N}\sum_{j=0}^{L-1} b_{j,k}, \quad k = 1, \ldots, r, \tag{2.10}$$

and the PR requirement (2.7).

3. OPTIMUM BIT ALLOCATION

The problem of minimizing (2.9) under the set of constraints (2.10) is a constrained optimization problem. Using the AM-GM[1] inequality and (2.10),

$$f = \frac{1}{M}\sum_{k=1}^{r}\sum_{j=0}^{L-1} c_k 2^{2b_{j,k}} \sigma^2_{e_{j,k}} \tag{3.11}$$

$$\geq \frac{L}{M}\sum_{k=1}^{r} c_k (\prod_{j=0}^{L-1} 2^{2b_{j,k}} \sigma^2_{e_{j,k}})^{1/L} \tag{3.12}$$

$$= \frac{1}{r}\sum_{k=1}^{r} c_k (2^{2Nt_k} \prod_{j=0}^{L-1} \sigma^2_{e_{j,k}})^{1/L} \tag{3.13}$$

with equality holding iff for all j, k:

$$b_{j,k} = \frac{N}{L}t_k + \frac{1}{2}log_2(\prod_{j=0}^{L-1} \sigma^2_{e_{j,k}})^{1/L} - \frac{1}{2}log_2(\sigma^2_{e_{j,k}}). \tag{3.14}$$

This is the *optimum bit allocation strategy*. The optimal transceiver design is to find matrices S, G so as to minimize

$$J = \sum_{k=1}^{r} (\alpha_k \prod_{j=0}^{L-1} a_{j,k})^{1/L} \tag{3.15}$$

where

$$\alpha_k = c_k^L 2^{2Nt_k} \quad a_{j,k} = \sigma^2_{e_{j,k}}. \tag{3.16}$$

Observe, if one chooses $L = 2$, and $\alpha_k = \alpha$ for all k, then (3.15) reduces to the optimization function considered in [2], for the subband coding of cyclostationary signals.

Optimal arrangement: Observe, [2, 6], that given a set of positive numbers $\{\delta_k\}_{k=1}^{2l}$, $\delta_k \geq \delta_{k+1}$ the minimum among all possible $\sum \delta_{k_i}\delta_{k_j}$ is $\sum_{k=1}^{l} \delta_k \delta_{2l-k+1}$. Thus among the various permutations of $a_{j,k}$, any that minimizes (3.15) must have the following property:

$$a_{m,k_1} \geq a_{n,k_2} \Rightarrow \alpha_{k_1} \prod_{j \neq m}^{L-1} a_{j,k_1} \leq \alpha_{k_2} \prod_{j \neq n}^{L-1} a_{j,k_2} \tag{3.17}$$

and

$$\alpha_m \geq \alpha_n \Rightarrow \prod_{j=0}^{L-1} a_{j,m} \leq \prod_{j=0}^{L-1} a_{j,n}. \tag{3.18}$$

[1] The arithmetic mean (AM) of a set of positive numbers is greater than or equal to their geometric mean (GM), with equality iff all the numbers are equal.

4. OPTIMUM TRANSCEIVER DESIGN

In this Section we address the problem of filter selection to minimize (3.15). This reduces to selecting a unitary matrix G_0. Given that the matrices in (2.6) are known, (2.7) fixes S. Observe that the situation in fig. 3 prevails, and $R_{\tilde{e}}$, the autocorrelation of $\tilde{\mathbf{e}}$, is known. Further the autocorrelation matrix of $\mathbf{e}$ is given by

$$R_e = G_0^T R_{\tilde{e}} G_0, \tag{4.19}$$

and that $a_{j,k}$ in (3.15), are simply the diagonal elements of R_e.

We need a few results from the theory of majorization that will be used in solving the optimization problem at hand. We will first introduce the notion of majorization and Schur concavity [6].

Definition 4.1 *Consider two sequences $x = \{x_i\}_{i=1}^n$ and $y = \{y_i\}_{i=1}^n$ with $x_i \geq x_{i+1}$ and $y_i \geq y_{i+1}$. Then we say that y majorizes x, denoted as $x \prec y$, if the following holds with equality at $k = n$*

$$\sum_{i=1}^{k} x_i \leq \sum_{i=1}^{k} y_i, \quad 1 \leq k \leq n.$$

Definition 4.2 *A real valued function $\phi(z) = \phi(z_1, \ldots, z_n)$ defined on a set $\mathcal{A} \subset R^n$ is said to be Schur concave on $\mathcal{A}$ if*

$$x \prec y \quad on\ \mathcal{A} \quad \Rightarrow \quad \phi(x) \geq \phi(y).$$

ϕ is strictly Schur concave on $\mathcal{A}$ if strict inequality $\phi(x) > \phi(y)$ holds when x is not a permutation of y.

We will now state a theorem that results in a test for strict Schur concavity. We denote

$$\phi_{(k)}(z) = \frac{\partial\phi(z)}{\partial z_k} \quad and \quad \phi_{(i,j)}(z) = \frac{\partial^2\phi(z)}{\partial z_i \partial z_j}.$$

Theorem 4.1 *Let $\phi(z)$ be a scalar real valued function defined and continuous on $\mathcal{D}$, and twice differentiable on the interior of $\mathcal{D}$. Then $\phi(z)$ is strictly Schur concave on $\mathcal{D}$ iff*

(i) *ϕ is symmetric in its arguments,*

(ii) *$\phi_{(k)}(z)$ is increasing in k, and*

(iii) $\phi_{(k)}(z) = \phi_{(k+1)}(z) \Rightarrow \phi_{(k,k)}(z) - \phi_{(k,k+1)}(z) - \phi_{(k+1,k)}(z) + \phi_{(k+1,k+1)}(z) < 0.$

Theorem 4.2 *If H is an $n \times n$ hermitian matrix with diagonal elements $h_1, \ldots, h_n$ and eigenvalues $\lambda_1, \ldots, \lambda_n$, then $h \prec \lambda$ on R^n.*

To connect the results from majorization theory developed to our optimization problem, we state the following theorem.

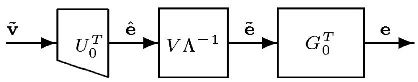

Fig. 3. Receiver block diagram.

Theorem 4.3 *The real valued scalar function J as defined in (3.15) under the optimality conditions (3.17-3.18) is strictly Schur concave.*

In particular as the search of G_0 is restricted to unitary matrices, if one chooses G_0 to be Ω a matrix of orthonormal eigenvectors of $R_{\tilde{e}}$, then R_e is a diagonal matrix containing the eigenvalues of $R_{\tilde{e}}$. Note that diagonal elements of R_e are $\sigma^2_{e_{j,k}}$. Thus from theorem 4.2, this choice of G_0 yields a sequence of $\sigma^2_{e_{j,k}}$ that majorizes all other achievable sequences. Consequently if arranged optimally, Theorem 4.3 holds, that such a sequence will minimize (3.15). It remains simply to arrange the eigenvalues of $R_{\tilde{e}}$ among the $\sigma^2_{e_{j,k}}$, through exhaustive search if need be, so that an arrangement that minimizes (3.15) is obtained. Thus for a suitable permutation matrix, P, the optimizing G_0 is

$$G_0 = P\Omega. \tag{4.20}$$

5. SIMULATION RESULTS

In this section, we compare the trasmitting power of the DFT based DMT under no bit allocation and optimum bit allocation with our optimum transceiver. We assume the channel to be $C(z) = 1 + 0.5z^{-1}$, and a noise source $v(n)$ whose power spectral density is shown in fig. 4. The plot shows that there is an 8 dB saving in transmit power with our design over the DFT based DMT under optimum bit allocation, and a 12 dB improvement over the conventional DMT with no optimum bit allocation. We however note that there may exist noise environments where the DFT based DMT performs as well as our optimal design.

6. CONCLUSIONS

In this paper, we have presented an optimum bit allocation strategy and transceiver design for minimizing the transmit power when different users have varied QoS requirements. Simulations confirm the efficacy of our results.

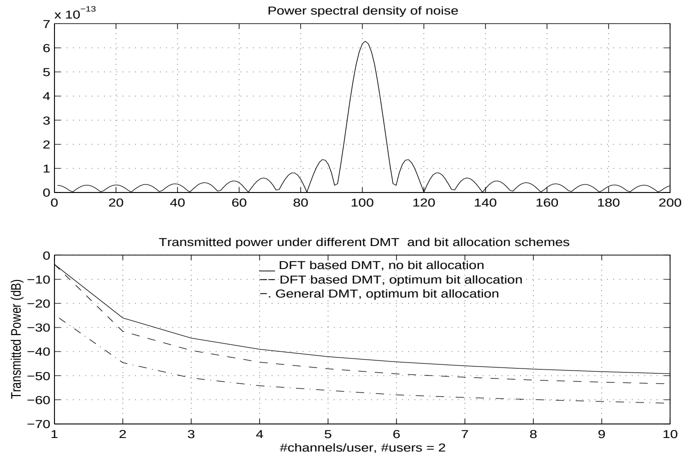

Fig. 4.

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