PREDICTION ERROR BASED FEEDBACK FOR DOWNLINK TRANSMIT BEAMFORMING

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ABSTRACT

This paper examines a downlink transmit beamforming scheme recently proposed in [1]. The idea is based on the use of an adaptive channel estimator at the base-station and requires the mobile to selectively feed back the value of the *prediction error*. This paper proposes a joint maximum-likelihood and set-membership filtering algorithm for adaptive channel estimation that provides robustness against incorrect feedback due to mobile decision errors. The amount of power and bandwidth-saving possible with this scheme is quantified via an uplink capacity analysis.

1. INTRODUCTION

A novel scheme for facilitating downlink beamforming over fading channels is considered in this paper. There has been interest in incorporating feedback of side information from the mobile to the base-station, especially for certain 3G systems [2, 3, 4, 5]. However, such feedback usually causes a serious degradation to uplink information capacity. Recently, the authors have introduced a feedback scheme that is motivated by the idea that the amount of feedback should BE *independent* of the number of transmitting elements. This scheme features a selective feedback, wherein the feedback information is sent intermittently and infrequently [1].

To compensate for imperfections in the feedback information, a novel joint Maximum Likelihood (ML) and Set-Membership Filtering (SMF) method for adaptive channel estimation is proposed in Section 3 and is shown via simulations to achieve robustness with respect to such errors. Section (4) quantifies the power and bandwidth saving that is possible with the proposed feedback mechanism. Finally, Section 5 concludes the paper.

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2. PREDICTION ERROR FEEDBACK

The proposed feedback is based on an adaptive filtering approach to downlink transmit beamforming. By employing an adaptive Set-Membership filtering algorithm, it was shown in [1] that significant savings in feedback requirement is possible without sacrificing performance. This concept utilizes an adaptive channel estimator and beamformer at the base-station.

We consider a single-path fading wireless channel with multiple transmit antennas. Assuming that there are a total of K users in the system, denote the information bit streams of the k^{th} user by $\{b_k(i) \in \{+1, -1\}\}$. The sent signal for that user is given by $s_k(t) = \sum_i b_k(i)\phi_k(t-iT_s)$, where $\phi_k(\cdot)$ are a set of orthogonal square-integrable (possibly complex-valued) functions. The beamformer is characterized by a set of K vectors, $\{\mathbf{w}_k \in \mathbb{C}^M\}$ and the output of the beamformer is $\mathbf{x}(t) = \sum_{m=1}^{K} \mathbf{w}_m s_m(t);\ \forall t$, with the received signal at the k^{th} user given by:

$$r_k(t) = \mathbf{a}_k^T \mathbf{x}(t) + n(t) \tag{1}$$

where $\mathbf{a}_k$ is the channel vector and $n(t)$ is a zero-mean white Gaussian noise process.

The proposed feedback mechanism has been motivated by the requirement that the amount of feedback information should not grow with M, the number of sensors [1]. The idea is that the channel can be identified at the base-station with the input-desired output pairs $(\mathbf{x}_k(i), d_k(i))$ where, $\mathbf{x}_k(i) \triangleq \int \mathbf{x}(t)\phi_k^*(t-iT_s)dt = \mathbf{w}_k(i)b_k(i)$, and,

$$d_k(i) = \mathbf{a}_k^T \mathbf{x}_k(i) + n_k(i) \ \ \forall\, i$$

where $n_k(i)$ is *i.i.d.* Gaussian noise.

Now, assume that the base-station has, by adaptively estimating the channel, an estimate $\hat{\mathbf{a}}_k(i-1)$ at time i, which is used to design the beamformer as

$$\mathbf{w}_k = \frac{\hat{\mathbf{a}}_k^*(i-1)}{\|\hat{\mathbf{a}}_k(i-1)\|^2}$$

Then, we have the special property that the prediction error needed by the base-station for updating the channel estimate, is given by

$$\delta_k(i) \stackrel{\triangle}{=} d_k(i) - \hat{\mathbf{a}}_k^T(i-1)\mathbf{x}_k(i) = d_k(i) - b_k(i)$$

Thus, if the mobile has a good hard estimate of $b_k(i)$, denoted by $\hat{b}_k(i)$, it can form an empirical estimate of the prediction error as $\hat{\delta}_k(i) \stackrel{\triangle}{=} d_k(i) - \hat{b}_k(i)$ and feed this quantity back to the base-station, which, in turn, can use this value to update its channel estimate to $\hat{\mathbf{a}}_k(i)$. Further, it was shown in [1] that if an adaptive Set-Membership Filtering (SMF) algorithm [6] is used for channel estimation, the prediction error need not be fed back all the time since the algorithm updates its estimate if and only if $|\delta_k(i)| > \gamma_k$, where $\gamma_k > 0$ is a pre-specified error bound. Thus the mobile can decide not to feed anything back if $|\hat{\delta}_k(i)| \leq \gamma_k$. This further decreases the feedback requirement.

Note that this scheme is better than transmitting the actual value of $d_k(i)$ on two counts: • if the updating condition is not met, then no signal is fed back, resulting in a gain in the uplink capacity, and, • the mean-square value of $\hat{\delta}_k(i)$ is much less than that of $d_k(i)$, which results in significant savings in power requirements, leading to an increase in the battery life of the mobile handset. In essence, at most one complex-valued quantity (the prediction error) needs to be fed back as opposed to M channel coefficients, as in conventional schemes for feedback, see *e.g.*, [3, 4]. Thus the basic feedback power requirement is independent of M.

3. ROBUSTNESS TO DECISION FEEDBACK ERRORS

A critical assumption made in the feedback approach [1] was on the perfect bit estimates available at the mobile. The effects of these feedback errors on the performance of the proposed feedback scheme is examined in this section.

The empirical estimate of the prediction error being accurate is a reasonable assumption for high SNR situations. However, it is desirable in practice for the adaptive filtering based feedback scheme to work even in moderate to low SNR regions, wherein the assumption of perfect bit estimates cannot be made.

Based on the model (1), the base-station reconstructs, upon receiving the value of $\hat{\delta}_k(i)$, an estimate of $d_k(i)$, denoted by $\hat{d}_k(i)$, as:

$$\hat{d}_k(i) = \hat{\delta}_k(i) + b_k(i) = d_k(i) + \varepsilon_k(i) \qquad (2)$$

where $\varepsilon_k(i) = b_k(i) - \hat{b}_k(i)$. If the mobile's estimate of $b_k(i)$ was correct, then $\varepsilon_k(i) = 0$. It can be seen from simulations that these errors, when not compensated for in the adaptive filtering step, can lead to severe degradation. In the following, we develop a joint adaptive maximum likelihood and set-membership algorithm that explicitly accounts for the statistics of the noise plus errors, while preserving the selective updating property.

The problem here is to estimate $\mathbf{a}_k$ from the *corrupted* observations $\{\hat{d}_k(i)\}$, which are given by the model:

$$\hat{d}_k(i) = \mathbf{a}_k^T\mathbf{x}_k(i) + v_k(i) = \alpha_k(i)b_k(i) + v_k(i) \qquad (3)$$

where $v_k(i) \stackrel{\triangle}{=} n_k(i) + \varepsilon_k(i)$ and $\alpha_k(i) \stackrel{\triangle}{=} \frac{\mathbf{a}_k^T\hat{\mathbf{a}}_k(i-1)}{\|\hat{\mathbf{a}}_k(i-1)\|^2}$.

Because of the nature of the input, $\mathbf{x}_k(i)$, which depends on $\hat{\mathbf{a}}_k(i-1)$, it can be seen that $\varepsilon_k(i)$ is not an independent sequence. This is due to the fact that $\hat{b}_k(i)$, which forms a part of $\varepsilon_k(i)$, is dependent on $\hat{\mathbf{a}}_k(i-1)$, which in turn, is dependent on the noise $n_k(i-1)$. Intuitively, this is to be expected because if, say, $\hat{b}_k(i-1)$ were incorrect, then $\hat{\mathbf{a}}_k(i-1)$ would be updated with corrupted data. Further, an inaccurate estimate $\hat{\mathbf{a}}_k(i-1)$ increases the chances that $\hat{b}_k(i)$ is incorrect, thus influencing the value of $\varepsilon_k(i)$, and hence, of $v_k(i)$.

Unfortunately, characterizing this dependence is not straightforward. The approach here is to characterize the marginal *pdf* of the errors, and design an adaptive ML estimator such that the exact marginal statistics of the data are at least accounted for, while making an *i.i.d.* assumption on the data.

The marginal *pdf* of $\hat{d}_k$ given $\mathbf{a}_k$ is given by $f_{v|b}(\hat{d}_k(i) - \mathbf{x}_k^T(i)\ \mathbf{a}_k)$, where $f_{v|b}(\cdot)$ is the conditional noise+error *pdf*, given knowledge of the transmitted bit, $b_k(i)$. The *weighted* ML estimate can be formulated as one of maximizing:

$$J_{\mathbf{a}_k}(i) = \omega(i)J_{\mathbf{a}_k}(i-1) + \nu(i)\ \log f_{v|b}(\hat{d}_k(i) - \mathbf{x}_k^T(i)\mathbf{a}_k)$$

which needs to be solved adaptively by making certain approximations [6].

With the following definitions: $\psi(e) = -\partial \log f_{v|b}(e)/\partial e^*$ and $\dot{\psi}(e) = -\partial^2 \log f_{v|b}(e)/\partial e \partial e^*$, a general update equation for adaptively estimating a parameter by the maximum likelihood cost function is given by [6]:

$$\hat{\mathbf{a}}_k(i) = \hat{\mathbf{a}}_k(i-1) + \nu(i)\ \mathbf{P}_k(i)\ \mathbf{x}_k^*(i)\ \psi(\hat{\delta}_k(i)) \qquad (4)$$

where

$$\mathbf{P}_k^{-1}(i) = \omega(i)\ \mathbf{P}_k^{-1}(i-1) + \nu(i)\ \dot{\psi}(\hat{\delta}_k(i))\ \mathbf{x}_k^*(i)\ \mathbf{x}_k^T(i) \qquad (5)$$

The sequence of weights $\omega(i)$ and $\nu(i)$ can be chosen according to the type of weighting desired.

By characterizing the marginal *pdf* of the noise, we have that the ML estimation based $\psi(\cdot)$ function is of the form:

$$\psi(e) = \begin{cases} e - 2b_k(i)\ ; & \text{if } b_k(i)\Re\{e\} \in (-\infty, -A_k(i)) \\ e - \frac{2\exp(\eta_b)}{1+\exp(\eta_b)}\ ; & \text{if } b_k(i)\Re\{e\} \in [-A_k(i), 2 - A_k(i)) \\ e\ ; & \text{if } b_k(i)\Re\{e\} \in [2 - A_k(i), \infty) \end{cases}$$

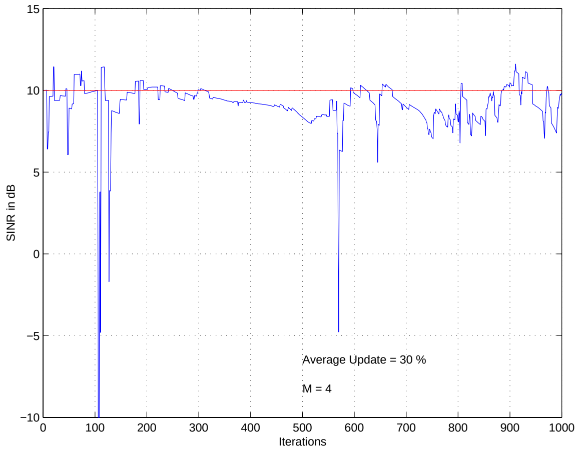

Fig. 1. Typical performance a joint ML-SMF algorithm under feedback errors for a target SNR of 10 dB with an initial training sequence of 50 bits.

where $\eta_b \triangleq 2(\Re\{e\} - b)/\sigma_v^2$, $\sigma_v^2 = E[|n_k(i)|^2]/2$, and $A_k(i) \triangleq \Re\{\alpha_k(i)\}$. From this, $\dot{\psi}(e)$ is evaluated easily. The following interpretation can be made of the assignment to $\psi(\cdot)$: if the prediction error is more negative (for $b_k(i) = +1$ and vice-versa for $b_k(i) = -1$) than a certain threshold , then the the mobile's bit decision is assumed to be in error and the value of $\hat{\delta}_k(i)$ is adjusted by subtracting $2b_k(i)$ from it, in order to correct for the likely mistake. One of the problems with using this ML-based function for adaptive estimation is the non-monotonicity of $\psi(e)$, which leads to ill-convergence behavior. Typically, when $\dot{\psi}(e)$ is negative, the matrix update equations (5) can become unstable because the positive-definiteness of $\mathbf{P}_k^{-1}(i)$ is not guaranteed for all i. This problem can be remedied by replacing the *optimum* $\psi(\cdot)$ function by an approximation. This forces $\psi(e) = e$ even when $b_k(i)\Re\{e\} \in [-A_k(i), 2 - A_k(i))$, thus ignoring the second term (which leads to the non-monotonicity). Further, for some error bound $\gamma_k > 0$, selective updating (and thus, an SMF-like bounded error constraint), can easily be incorporated in this framework by imposing a dead zone region [6] such that $\psi(e) = 0$ whenever $|e| < \gamma_k$.

It can also be shown that the sequence $A_k(i)$, which is required in the adaptation, can be accurately estimated by a simple averager-based least-squares estimator [6].

The typical performance of an exponentially weighted, selectively updating algorithm is plotted in Figure (1) for a slow-fading wireless channel, assuming uncorrelated scattering paths from each element, with $M = 4$ elements. The weight $\lambda(i) = 0.9 \;\forall\; i$. It is seen that the algorithm updates less than 30% of the time while having the capability to come out of large error events, which demonstrates the robustness of the algorithm.

4. UPLINK POWER AND BANDWIDTH SAVING

In this section, we quantify, via an uplink capacity analysis, the amount of achievable power saving via selective feedback. Consider an uplink channel without fading. In the first analysis, we do not consider any particular channel-sharing (or multiple-access scheme) between the information and feedback symbols as well as between all the users.

Then the received signal at the base-station for a K user system can be written as:

$$z = \sum_{k=1}^{K} y_k + \xi = \sum_{k=1}^{K} (y_{k,I} + y_{k,F}) + \xi \qquad (6)$$

where y_k is the total signal transmitted by user k and ξ is AWGN with a power spectral density height of $N_0/2$. The subscripts, I and F, on y_k, refer to the *information* and the *feedback* symbol sequences that are transmitted. Further, assume that

$$E[y]_k^2 = \bar{P}_k = P_{I_k} + P_{F_k} \;;\; P_{I_k} \triangleq E[y_{I_k}^2] \;;\; P_{F_k} \triangleq E[y_{F_k}^2]$$

for some set of $\bar{P}_k > 0, k = 1, 2, \ldots, K$. We make the reasonable assumption that the random variables corresponding to the I and F symbols are independent of each other. Let R_{I_k} and R_{F_k} denote the rates of transmission of the respective symbols for the k^{th} user. Then, we can view the uplink model (6) as a 2K-user multiple access channel. The capacity region is defined via the well-known set of inequalities

$$\sum_{m \in S_1} R_{I_m} + \sum_{j \in S_2} R_{F_j} \leq B \log\left(1 + \frac{\sum_{m\in\mathcal{S}_1} P_{I_m} + \sum_{j\in\mathcal{S}_2} P_{F_j}}{N_o B}\right) \qquad (7)$$

for all S_1 and S_2, which are index sets that belong to the power set of $\{1, 2, \ldots, K\}$, denoted by $\mathcal{S}_K$. The total bandwidth available is $2B$, the height of the noise power spectral density is $N_0/2$ and denote $\sigma^2 = N_0 B$.

Assuming that the powers of the users are ordered as $P_{I_1} > P_{I_2} > \ldots > P_{I_K} > P_{F_1} > P_{F_2} > \ldots > P_{F_K}$, then it is straightforward to show that the rates achieved by a successive decoding strategy lies on the boundary of the capacity region. The successive decoding rates are as follows:

$$\begin{aligned} R_{F_j} &= B \log\left(1 + \frac{P_{F_j}}{\sum_{m>j} P_{F_m} + \sigma^2}\right) \\ R_{I_k} &= B \log\left(1 + \frac{P_{I_k}}{\sum_{m>k} P_{I_m} + P_F + \sigma^2}\right) \end{aligned} \qquad (8)$$

The above equations hold for all $j, k = 1, 2, \ldots, K$. Further, $P_F \triangleq \sum_{m=1}^{K} P_{F_m}$.

Assume that the K different users have an updating fraction given by $\tau_1, \tau_2, \ldots, \tau_K$, where $\tau_j \in (0, 1]$. Denote the powers and the rates being used by the super-script c for the case of continuous feedback and by the super-script s for selective feedback. The basic equations governing the two situations are that the information symbol rate remains the same while the feedback rate is decreased by a factor of τ_j for the user j. That is, $R^s_{I_j} = R^c_{I_j}$ & $R^s_{F_j} = \tau_j\, R^c_{F_j}\ \forall\, j$, which can be solved for any general K. To exactly quantify the saving possible, let us consider a single-user system, *i.e.*, *K = 1*. These conditions then reduce to

$$P^s_F = \sigma^2 \left[\left(1 + \frac{P^c_F}{\sigma^2}\right)^{\tau} - 1 \right] \; ; \; P^s_I = P^c_I \left(1 + \frac{P^c_F}{\sigma^2}\right)^{-(1-\tau)} \tag{9}$$

and the total power required in the selective feedback scheme would be given by $\bar{P}^s = P^s_I + P^s_F$. It can be shown [6] that for an updating fraction of $10 - 30\%$, which is typical for SMF algorithms, the savings in total transmitted power is in the range of $45 - 55\%$.

Now we analyze the case when the two streams use frequency-division for sending the signals. The saving in *feedback bandwidth* can be quantified in this scenario. The capacity equation with FDMA for $K = 1$, *i.e.*, a two-user channel, is given by

$$R_I = B_I \log\left(1 + \frac{P_I}{B_I N_0}\right) \; ; \; R_F = B_F \log\left(1 + \frac{P_F}{B_F N_0}\right) \tag{10}$$

where B_I and B_F denote the bandwidths allocated to the I and F streams with the constraint that $B_I + B_F \leq B$. $P^s_I = P^c_I \triangleq P_I$; & $P^s_F = P^c_F \triangleq P_F$. With this, and from (10) and the rate relations, we obtain the relation between feedback bandwidth requirement as

$$B^s_F \log\left(1 + \frac{P_F}{B^s_F N_0}\right) = \tau B^c_F \log\left(1 + \frac{P_F}{B^c_F N_0}\right) \tag{11}$$

The above equation does not have a closed-form solution to it, although an approximate solution can be obtained as [6]

$$\frac{B^s_F}{B^c_F} \approx \frac{\frac{S}{2}\,\tau^2}{\left(\tau(\frac{S}{2} - 1) + 1\right)} \tag{12}$$

where $S \triangleq P_F / N_0 B^c_F$ is the SNR on the continuous feedback channel. This indicates that the bandwidth fraction scales linearly with τ, which is intuitively expected. Figure (2) shows the amount of bandwidth saving possible by explicitly solving (11) as well as the value obtained from the approximation given in (12). It is seen that the approximation is reasonably accurate for both $S = 0$ dB and $S = 7$ dB.

5. CONCLUSIONS

This paper analyzed a novel methodology for feedback-based transmit beamforming. The power and bandwidth savings due to selective feedback was quantified via an uplink capacity analysis. Further, a selectively updating adaptive maximum likelihood algorithm was developed to provide robust performance in the face of errors due to incorrect detection at the mobile.

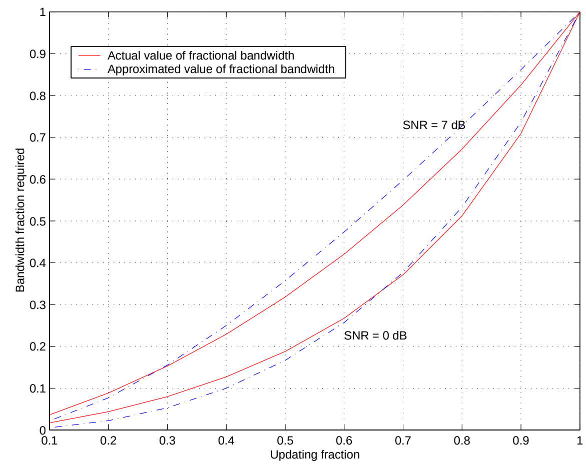

Fig. 2. Bandwidth saving with selective feedback as a function of τ for different values of feedback SNR.

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