EDGE ADAPTIVE RESTORATION OF NOISY, BLURRED IMAGES

George J. Foster, Member, IEEE and Nader M. Namazi, Senior Member, IEEE

Department of Electrical Engineering and Computer Science
The Catholic University of America
Washington, DC 20064
george.foster@ieee.org, namazi@cua.edu

ABSTRACT

We present a method to find the mean square estimate of the original image using a Gauss-Markov image model and a known point spread function. The performance of the edge adaptive technique compares favorably to the Wiener filter on synthetic and real images with mild linear (motion) blur and additive white Gaussian noise.

1. INTRODUCTION

There are many techniques for restoring noisy, blurred images; however, the major drawback with most of these techniques is the performance in the vicinity of edges. Because most noise reduction techniques take advantage of the low pass nature of most images to reduce the noise at higher frequencies. Thus, when high frequency content is present in images (such as edges), then image restoration filters do not restore the edges, but instead generate additional error. A simple example of this performance can be illustrated by observing Gibbs' phenomena: When a square wave is passed through a low pass filter, the loss of high frequency content actually causes ripples in the waveform emanating from the edge.

Prior to the consolidation of image restoration techniques in Lagendijk's monograph [7], the existing techniques fell into three main categories: stochastic restoration, algebraic restoration, and multiple constraints restoration.

Stochastic techniques use a stochastic image model to determine the appropriate level of noise filtering to apply during restoration [1, 11]. Algebraic techniques adjust the convergence factor or the regularization filter of iterative solutions to the inverse filtering problem based upon the presence of edges [3, 5, 6]. The methods using constraints apply the technique of projection onto convex sets (POCS) [8, 10, 12, 13]. Newer techniques use wavelet transforms to preserve edge information in the detail channels [2].

We develop the edge adaptive signal model first for one dimensional signals in Section 2 and the corresponding one dimensional estimator in Section 3. After a brief presentation of a one dimensional example in Section 4, we extend the signal model to two dimensional images in Section 5. We present our results using both a synthetic image and a real image in the presence of linear (motion) blur and white noise in Section 6.

2. ADAPTIVE SIGNAL MODEL

Many signals can be characterized by a Gauss-Markov process which states that the current value of a signal can be determined as a linear combination of past values of the signal and a Gaussian random signal [9]. For discrete signals, a first order Gauss-Markov process combines the immediate past value of the signal with a Gaussian random signal to produce the current signal. In mathematical notation, this is expressed as

$$s(k) = as(k-1) + bv(k) \tag{1}$$

where *s(k)* is the value of the signal at step *k*, *v(k)* is a random variable, and *a* and *b* are the weights. If the system is time invariant, then the signal can be represented in vector form as

$$\mathbf{s} = \mathbf{As} + \mathbf{Bv} \tag{2}$$

where **A** is a diagonal matrix with the first subdiagonal populated with the value *a*, and **B** is a diagonal matrix equal to *b***I**.

To create a system model that more closely models edges or discontinuities, we have to include the edge in the model. To accomplish this we simply add a hypothesis to our Gauss-Markov model:

$$s(k) = \begin{cases} as(k-1) + b_1 v(k), & k \notin edge \\ b_2 v(k) & , k \in edge \end{cases} \tag{3}$$

In vector form, we get the same equation (2), but with some values in **A** equal to zero, and **B** no longer a multiple of the identity matrix. Many methods exist to determine the location of edges in signals. For images, a two dimensional edge detector can be found in [4]. Another method could be to classify any location that falls outside the non-edge model by three standard deviations as an edge. For the purpose of this development, we will assume that the edges are known.

3. MEAN SQUARED ESTIMATOR

Now that we have a signal model, we can explore how our knowledge of the measurement system can be exploited to provide an optimal estimate of the signal. If we consider that our system is a typical AWGN system consisting of noisy measurements of the signal that have passed through a time invariant filter, then the system can be written as

$$\mathbf{r} = \mathbf{C}\mathbf{s} + \mathbf{D}\mathbf{w} \tag{4}$$

where **C** is the system matrix, and **D** is a diagonal matrix that allows for time varying noise contributions. We would like to find the optimal mean square estimate of the signal given the entire signal frame.

From the fundamental theorem of estimation theory [9], we know that the mean squared estimator is

$$\hat{\mathbf{s}}_{\mathbf{MS}} = E\left\{\mathbf{s}\middle|\mathbf{r}\right\} \tag{5}$$

If **r** and **s** are multivariate Gaussian random variables, then the estimator can be expressed as

$$\hat{\mathbf{s}}_{\mathbf{MS}} = \mathbf{m}_{\mathbf{s}} + \mathbf{P}_{\mathbf{sr}}\mathbf{P}_{\mathbf{r}}^{-1}\left(\mathbf{r} - \mathbf{m}_{\mathbf{r}}\right) \tag{6}$$

where $\mathbf{P}_{\mathbf{r}}$ is the covariance of **r** and $\mathbf{P}_{\mathbf{sr}}$ is the cross-covariance between **s** and **r**.

First, we examine the statistical nature of the signal **s**. Solving (2) for **s**, we write

$$\mathbf{s} = \left(\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B}\mathbf{v} \tag{7}$$

Because **v** is defined as a zero mean Gaussian random variable with variance σ_v^2, then we can determine the mean and covariance matrix of **s**:

$$\mathbf{m}_{\mathbf{s}} = E\left\{\left(\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B}\mathbf{v}\right\} = \mathbf{0} \tag{8}$$

$$\mathbf{P}_{\mathbf{s}} = \left(\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B}\mathbf{B}^T\left(\mathbf{I} - \mathbf{A}\right)^{-T}\sigma_v^2 \tag{9}$$

Next, substituting (7) into (4), we write

$$\mathbf{r} = \mathbf{C}\left(\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B}\mathbf{v} + \mathbf{D}\mathbf{w} \tag{10}$$

We can determine by inspection that the mean of **r** is also zero. From our knowledge that **w** is a zero mean white Gaussian independent of **v** and the definitions of **s** and **r** in (7) and (10), we can solve for the cross-covariance matrix between **s** and **r** as

$$\mathbf{P}_{\mathbf{sr}} = \left(\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B}\mathbf{B}^T\left(\mathbf{I} - \mathbf{A}\right)^{-T}\mathbf{C}^T\sigma_v^2 \tag{11}$$

The covariance of **r** can be as easily solved as

$$\mathbf{P}_{\mathbf{r}} = \mathbf{C}\left(\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B}\mathbf{B}^T\left(\mathbf{I} - \mathbf{A}\right)^{-T}\mathbf{C}^T\sigma_v^2 + \mathbf{D}\mathbf{D}^T\sigma_w^2 \tag{12}$$

Finally, by substituting (11) and (12) into (6), we can write the mean squared estimator of the signal as

$$\hat{\mathbf{s}}_{\mathbf{MS}} = \left[\mathbf{C}^T\mathbf{C} + \frac{\sigma_w^2}{\sigma_v^2}\mathbf{C}^T\mathbf{D}\mathbf{D}^T\mathbf{C}^{-T}\left(\mathbf{I} - \mathbf{A}\right)^T\mathbf{B}^{-T}\mathbf{B}^{-1}\left(\mathbf{I} - \mathbf{A}\right)\right]^{-1}\mathbf{C}^T\mathbf{r} \tag{13}$$

This estimator may be simplified slightly by taking advantage of the diagonal matrices **B** and **D**. For the special case that **B**=**D**=**I**, this estimator reduces to the vector-matrix form of the Wiener filter found in [7], which we write as

$$\hat{\mathbf{s}}_{\mathbf{MS}} = \left[\mathbf{C}^T\mathbf{C} + \frac{\sigma_w^2}{\sigma_v^2}\left(\mathbf{I} - \mathbf{A}\right)^T\left(\mathbf{I} - \mathbf{A}\right)\right]^{-1}\mathbf{C}^T\mathbf{r} \tag{14}$$

We present a simple example in the next section.

4. 1-DIMENSIONAL EXAMPLE

To illustrate the benefits of edge adaptive estimation, we restored a noisy and smeared square wave using both the traditional Wiener filter and the edge adaptive filter. The impulse response of the smearing function was [0.1 0.5 0.4] and the signal to noise ratio (SNR) was 30 dB. Figure 1 contains the original and noisy signals. Figure 2 demonstrates the superior performance of the edge adaptive method.

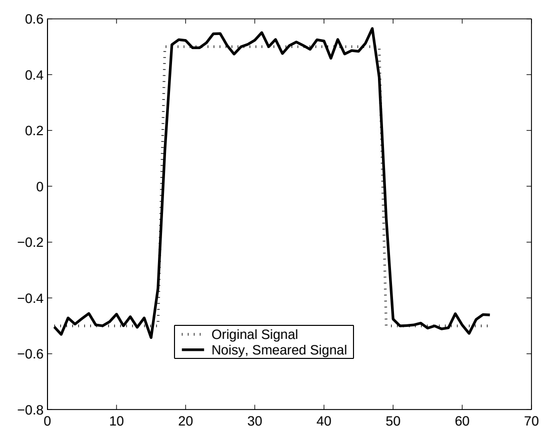

Figure 1. 1-D Original and Noisy Signals

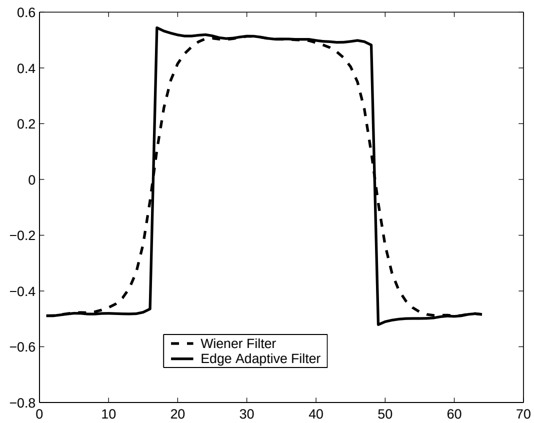

Figure 2. 1-D Restoration Results

5. ADAPTIVE IMAGE RESTORATION

The application of the one dimensional method can be easily adapted for a two dimensional image by representing the image as a vector of lexicographically ordered pixels. With this representation, the same equations apply directly as in the one dimensional case. The only change required is to the Gauss-Markov signal model to add dependence on the pixels in the neighboring row above the pixel of interest.

We use the quarter plane model to model the relationship between a pixel and it's neighbors. With the image in lexicographical vector form, the hypothesis in (3) is then modified such that non-edge pixels satisfy the equation

$$s(k) = a_1 s(k-n-1) + a_2 s(k-n) + a_3 s(k-1) + b_1 v(k) \quad (15)$$

where n is the length of a row. The use of (15) results in a slightly different form of the **A** matrix.

6. RESULTS

Before we can evaluate the performance of the edge adaptive technique against other techniques, we must define a performance measure. The signal to noise ratio improvement (ISNR) is the measure we choose. The ISNR is calculated as

$$ISNR = 10\log_{10}\frac{(\mathbf{r}-\mathbf{s})^T(\mathbf{r}-\mathbf{s})}{(\hat{\mathbf{s}}-\mathbf{s})^T(\hat{\mathbf{s}}-\mathbf{s})} \quad (16)$$

and, in this form, is written in units of decibels (dB).

We applied the edge adaptive filter to the simulated image shown in Figure 3 that had been blurred by horizontal linear blur of two pixels and with additive white Gaussian noise with variance in a range from -60 dB to 0 dB

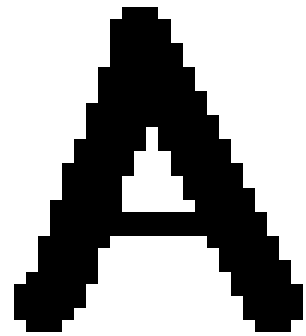

Figure 3. Synthetic Image

the edge adaptive technique is compared to that of the Wiener filter in Figure 4. The technique was also applied

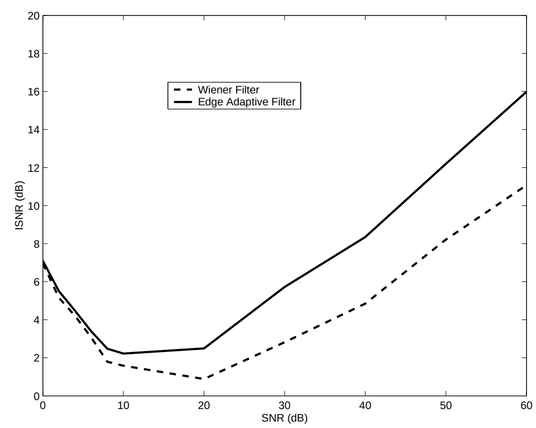

Figure 4. ISNR for synthetic image.

to part of the cameraman image shown in Figure 5. The

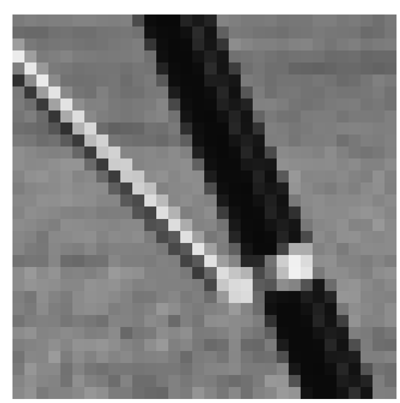

Figure 5. Part of cameraman image.

performance for the two techniques applied to part of this image is shown in Figure 6. For both images, the edge

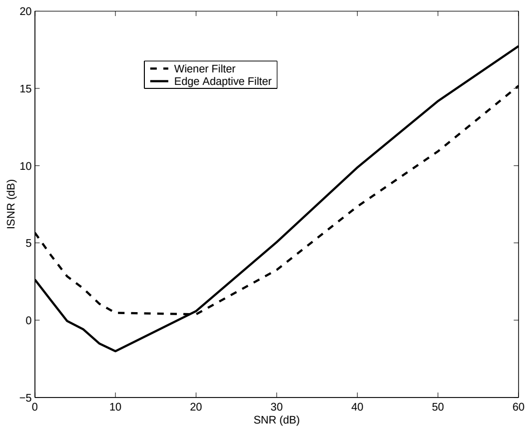

Figure 6. ISNR for cameraman image.

adaptive technique was able to further improve the image quality beyond that attainable by the Wiener filter, in some cases by 2-5 dB for SNR values of 30-60 dB.

7. CONCLUSIONS

We have developed an edge adaptive image restoration technique that provides a significant improvement over the Wiener filter. This technique, as demonstrated in the development of the estimator, can also be applied to one dimensional signals that contain discontinuities.

Further investigations include expansion of the signal model to include different models for the possible edge orientations, which should provide additional noise reduction along edges away from the corners. We also are investigating the use of iterative forms of this technique in simultaneously identifying and restoring images and image sequences.

8. REFERENCES

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