

# AN ADAPTIVE CHANNEL ESTIMATOR FOR CDMA SYSTEMS IN MULTIPATH FADING CHANNELS

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## ABSTRACT

CDMA systems in multipath fading channels need to estimate channel parameters for coherent detection of the transmitted signals. In this paper we present a simple but effective channel estimation algorithm that can be incorporated into most types of multiuser receivers to obtain good detection performance. This technique uses a set of correlation filters to independently estimate each of the channel parameters. One advantage our method has over subspace-based algorithms for channel estimation is that it can estimate the channel parameters without phase or amplitude ambiguity. Simulation results demonstrating that our channel estimator is capable of tracking reasonably fast fading channels are also presented in the paper.

## 1. INTRODUCTION

Multipath fading effects in CDMA communication channels often cause severe detection problems at the receiving end. These non-ideal conditions necessitate channel estimation in practical wireless communication systems. Distortions due to channel imperfections are in addition to and independent of the interference from the other users in the network. However, a multiuser receiver cannot correctly cancel interference caused by other users without knowing the channel channel characteristics. Since the parameters of channels are in general time-varying and must be tracked over time, adaptive algorithms are required for satisfactory mitigation of multipath fading effects. This paper presents a simple but effective adaptive channel estimation scheme that mitigates intersymbol interference (ISI) in CDMA systems. Unlike the bank of matched filters (RAKE receivers) that are used in IS-95 systems [1], this technique uses a set of correlation filters to independently estimate each of the channel parameters. Unlike the subspace based approaches in [2] and [3], our channel estimation results in no phase ambiguity and amplitude ambiguity.

## 2. DATA FORMULATION

We consider a  $K$ -user CDMA system whose processing gain is  $N$ . Let us define  $d_j[i]$  as the data symbol of the  $j$ th user at symbol index  $i$  and we assume that the symbols are independent and equally likely to be  $-1$  or  $+1$ . We also define an  $N$ -dimensional vector  $\mathbf{c}_j[i]$  as the spreading code associated with the  $j$ th user at symbol index  $i$ . For fixed code CDMA,  $\mathbf{c}_j[i]$  repeats itself in every symbol period. For long code CDMA,  $\mathbf{c}_j[i]$  varies. Our channel estimator can deal with both fixed code or long code CDMA. In this paper, we consider fixed code CDMA, and drop the symbol index

for  $\mathbf{c}_j[i]$  in the following representations. Finally, we assume that each channel is linear but time-varying. Forney showed in [4] that a finite impulse response (FIR) filter can represent the discrete-time channel model that combines the effects of the transmitter filtering, the physical channel, the receiver filtering, and symbol sampling. Hoeher [5] applied the Monte Carlo-based analog channel model in [6] to develop a discrete-time representation [4, 7] and proposed a slowly time-varying multipath fading channel model. Let an  $L$ -element vector given by

$$\mathbf{h}_j[i] = [h_{j,i}(0), h_{j,i}(1), \dots, h_{j,i}(L-1)]^T, \quad (1)$$

represent the discrete-time response of a multipath fading channel for the  $j$ th user at the symbol time index  $i$ . For simplicity of presentation, we assume that sampling is done at the chip rate. In (1), each element of  $\mathbf{h}_j[i]$  is usually modeled as a complex Gaussian process. The phase of a complex Gaussian process is uniformly distributed. The amplitude of a complex Gaussian process is *Rice-distributed* when there is a line of sight between the transmitter and the receiver. Otherwise, the amplitude of the process is *Rayleigh-distributed* [7]. In the experiments described later in the paper, we do not assume any line of sight and hence the channel response  $\mathbf{h}_j[i]$  is modeled as Rayleigh distributed in its amplitude and uniformly distributed in its phase [5].

Let us define  $\mathbf{c}_j^{(m)}$  as the spreading code of the pilot channel delayed by  $m$  chips, *i.e.*, if  $\mathbf{c}_j = [c_j(0) \ c_j(1) \ \dots \ c_j(N-1)]^T$ , then

$$\mathbf{c}_j^{(m)} = \left[ \underbrace{0 \ 0 \ \dots \ 0}_m \ c_j(0) \ c_j(1) \ c_j(N-1-m) \right]^T. \quad (2)$$

Mathematically, the signal at the CDMA receiver input can be expressed in the form

$$\mathbf{r}[i] = \sum_{m=0}^{L-1} \sum_{j=1}^K h_{j,i}(m) d_j[i] \mathbf{c}_j^{(m)} + \eta[i], \quad (3)$$

where  $\eta[i]$  denotes an  $N$ -dimensional vector of additive noise samples. We assume that this noise sequence has zero mean value and has a covariance matrix given by  $\sigma^2 \mathbf{I}_N$ , where  $\mathbf{I}_N$  represents an identity matrix containing  $N \times N$  elements. We further assume that the noise sequence is uncorrelated with the transmitted data sequence and the channel parameters.

### 3. THE ADAPTIVE CHANNEL ESTIMATOR

For the purpose of developing the channel estimation method, let us start by considering a CDMA system that has a pilot channel. The derivations in this section have been extended to the case when pilot signals are not available using decision feedback approaches in [8], and described briefly later in the paper. Without loss of generality, let us assume that the pilot user is indexed by one. RAKE receivers similar to those employed in the IS-95 standard estimate the channel parameters as a moving average of the product of the received signal, the pilot signal, and the matched filter with unit impulse response. That is, the  $m$ th coefficient of the channel is estimated at the  $i$ th instant as

$$\hat{h}_{1,i}(m) = \frac{1}{P+1} \sum_{k=i-P}^i \mathbf{r}^T[k] d_1[k] \mathbf{c}_1^{(m)}. \quad (4)$$

This approach assumes that the spreading sequence is normalized such that  $\|\mathbf{c}_1^{(m)}\|^2 = 1$ . An alternate approach for tracking the coefficients is to use a recursive estimate of the channel parameters given by

$$\hat{h}_{1,i}(m) = \alpha \hat{h}_{1,i-1}(m) + (1 - \alpha) \left( \mathbf{r}^T[i] d_1[i] \mathbf{c}_1^{(m)} \right), \quad (5)$$

where the forgetting factor  $\alpha$  is a small positive value close to but less than one. Assuming that the delayed versions of  $\mathbf{c}_1$  are uncorrelated with each other, and that the pilot symbols are uncorrelated with the signals transmitted by the other users in the network, both (4) and (5) are capable of eliminating the interference from the other users' signals and estimating the channel parameters without bias. However, the estimates may not be sufficiently accurate when the delayed versions of  $\mathbf{c}_1$  have significant correlations among them.

We now propose a modification to the above approach that will eliminate the need to preserve zero correlation among all delayed versions of the spreading sequence. In our approach we simply replace  $\mathbf{c}_1^{(m)}$  in (4) by a correlation vector  $\mathbf{v}_m$  that satisfies the constraints

$$\mathbf{v}_m^T \mathbf{c}_1^{(m)} = 1 \quad (6)$$

and

$$\mathbf{v}_m^T \mathbf{c}_1^{(l)} = 0, \quad (7)$$

for  $l, m = 0, 1, \dots, L-1$  and  $m \neq l$ . The correlation vector  $\mathbf{v}_m$  can be calculated using an appropriate Gram-Schmidt orthogonalization procedure: Remove any component of  $\mathbf{c}_1^{(m)}$  that can be estimated using  $\{\mathbf{c}_1^{(l)}; l \neq m\}$ , and then normalize the residual vector such that (6) is satisfied.

For instance, if the channel length were three, the first correlation vector  $\mathbf{v}_0$  can be found by first evaluating  $\mathbf{t1}_0$ , the component of  $\mathbf{c}_1^{(2)}$  that is uncorrelated with  $\mathbf{c}_1^{(1)}$  as

$$\mathbf{t1}_0 = \mathbf{c}_1^{(2)} - ((\mathbf{c}_1^{(2)})^T \mathbf{c}_1^{(1)}) \mathbf{c}_1^{(1)} / ((\mathbf{c}_1^{(1)})^T \mathbf{c}_1^{(1)}), \quad (8)$$

then evaluating  $\mathbf{t2}_0$ , the component of  $\mathbf{c}_1^{(0)}$  that is uncorrelated with  $\mathbf{t1}_0$  and  $\mathbf{c}_1^{(1)}$  as

$$\begin{aligned} \mathbf{t2}_0 = \mathbf{c}_1^{(0)} & - ((\mathbf{c}_1^{(0)})^T \mathbf{c}_1^{(1)}) \mathbf{c}_1^{(1)} / ((\mathbf{c}_1^{(1)})^T \mathbf{c}_1^{(1)}) \\ & - ((\mathbf{c}_1^{(0)})^T \mathbf{t1}_0) \mathbf{t1}_0 / ((\mathbf{t1}_0)^T \mathbf{t1}_0), \end{aligned} \quad (9)$$

and finally normalizing  $\mathbf{t2}_0$  with  $\mathbf{c}_1^{(0)} \mathbf{t2}_0$  as

$$\mathbf{v}_0 = \mathbf{t2}_0 / (\mathbf{c}_1^{(0)} \mathbf{t2}_0). \quad (10)$$

The procedure for finding  $\mathbf{v}_1$  and  $\mathbf{v}_2$  is similar. This procedure can be easily extended to the cases where the number of coefficients in the channel model is different from three.

Using the set of correlation vectors  $\mathbf{v}_m$ , a new channel estimation method may be derived as

$$\hat{h}_{1,i}(m) = \frac{1}{P+1} \sum_{k=i-P}^i \mathbf{r}^T[k] d_1[k] \mathbf{v}_m. \quad (11)$$

A recursive estimate of the channel parameters may also be developed as

$$\hat{h}_{1,i}(m) = \alpha \hat{h}_{1,i-1}(m) + (1 - \alpha) (\mathbf{r}^T[i] d_1[i] \mathbf{v}_m). \quad (12)$$

To see why such a scheme would work, let us assume for the time being that the channel parameters are time-invariant. Therefore,  $\mathbf{h}_j[i]$  is a constant vector. Taking the statistical expectation of the product of the correlation filter output  $\mathbf{v}_m^T \mathbf{r}[i]$  and the pilot symbol  $d_1[i]$ ,

$$\begin{aligned} E \{ \mathbf{v}_m^T \mathbf{r}[i] d_1[i] \} &= \mathbf{v}_m^T \mathbf{c}_1^{(m)} \hat{h}_{1,i}(m) \\ &= \hat{h}_{1,i}(m). \end{aligned} \quad (13)$$

The above derivation made use of the signal model for  $\mathbf{r}[i]$  and the fact that the data sequences generated by various users are uncorrelated with each other. Since the procedures in (11) and (12) compute the averages effectively over a short number of recent samples, they are able to track slowly varying (slow relative to the window size) changes in the channel characteristics.

### 4. A SIMULATION EXAMPLE

In this section, we present an example that compares the abilities of the channel estimation methods described in (5) and (12). We consider the forward link of a CDMA channel in which eight signals were transmitted with equal power. One of the signals was the pilot signal. All the signals were transmitted via the same multipath fading channel. The finite impulse response (FIR) of the multipath fading channel used the statistical discrete-time model in [5]. We assumed that the CDMA chip rate was 1.23 MHz, and the carrier frequency was 900 MHz. We set the length of the FIR filter to be 3, *i.e.*, the channel vector was  $\mathbf{h}_1[i] = [h_{1,i}(0) \ h_{1,i}(1) \ h_{1,i}(2)]^T$ . The maximum Doppler frequency was set to be 100 Hz, which corresponded to a relative speed of 120 km per hour between the base and mobile stations. A set of 15-bit *Gold codes* were used as the spreading codes for the eight signals. All the systems used a forward error control code defined by a four-state convolutional code with generator matrix  $\mathbf{g} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$  [7]. The correlation values between the delayed versions of  $\mathbf{c}_1$  were  $(\mathbf{c}_1^{(0)})^T \mathbf{c}_1^{(1)} = 0.533$ ,  $(\mathbf{c}_1^{(0)})^T \mathbf{c}_1^{(2)} = 0.067$ , and  $(\mathbf{c}_1^{(1)})^T \mathbf{c}_1^{(2)} = 0.467$ , respectively. The forgetting factor  $\alpha$  was chosen to be 0.95. The signal-to-noise ratio was set to be 10 dB<sup>1</sup>. The channel estimator started the estimation with the initialized vector  $\mathbf{h}_1[i] = [1 \ 0 \ 0]^T$ .

<sup>1</sup>In practice, the noise level may remain unchanged for a period of time, but the received signal power will vary along with the changes of channel attenuation. Therefore, we calculated the average signal-to-noise ratios (SNRs) over the duration of the experiment in the simulation.

Figure 1 displays the evolution of the three estimated channel parameters (solid line) using the RAKE receiver-type estimation method described in (5). Also displayed is the evolution of the three actual channel parameters (dash line). The left column of the figure shows the evolution of the amplitude, and the right column of the figure shows the evolution of the absolute value of phase<sup>2</sup>. The results in the figure show that the estimated channel parameters do not follow the actual channel parameters in many cases. The reason, as explained earlier, is that the correlations among the delayed versions of  $c_1$  were quite large.

The same CDMA system and the same channel were used for evaluating our method described in (12). Figure 2 displays the evolution of the three estimated channel parameters (solid line) along with the evolution of the three actual channel parameters (dash line), both in amplitude and absolute value of phase. The results in this figure show that the estimated channel parameters follow the actual channel parameters quite well. Such an observation was expected since in our newly proposed method the correlation filter  $\mathbf{v}_m$  had been selected so as to cancel the interference from the other delayed versions of  $c_1$ .

## 5. CONCLUDING REMARKS

This paper presented a simple but effective adaptive channel estimation scheme for CDMA systems. Our new method eliminates the interference among the replica signals of the user of interest while estimating each channel parameters independently. Consequently, our method provides better channel estimation than the current approach used in IS-95 systems [1]. Compared with the subspace based approaches [2][3], our channel estimation is also better because it does not exhibit any phase ambiguity and amplitude ambiguity. Experimental performance evaluation of our method and comparison of the method with an existing scheme showed that our method is capable of tracking the channel parameters well in environments in which the relative motion between the mobile and the base station is fast and also the number of users in the network is reasonably high compared with the spreading gain. These results make us believe that our approach is a viable and better alternative to competing approaches to channel estimation available in the literature.

Our channel estimation method can be easily incorporated into most multiuser receivers for CDMA systems. It can also be easily extended to the situation when there is no pilot signal available through a decision feedback mechanism. For example, the FEC decoder decision  $\hat{d}_1[i - D]$  can be used in place of  $d_1[i]$ . Thus, a recursive estimate of the channel parameters is given by

$$\hat{h}_{1,i}(m) = \alpha \hat{h}_{1,i-1}(m) + (1-\alpha) (\mathbf{v}_m^T \mathbf{r}[i - D]) \hat{d}_1[i - D], \quad (14)$$

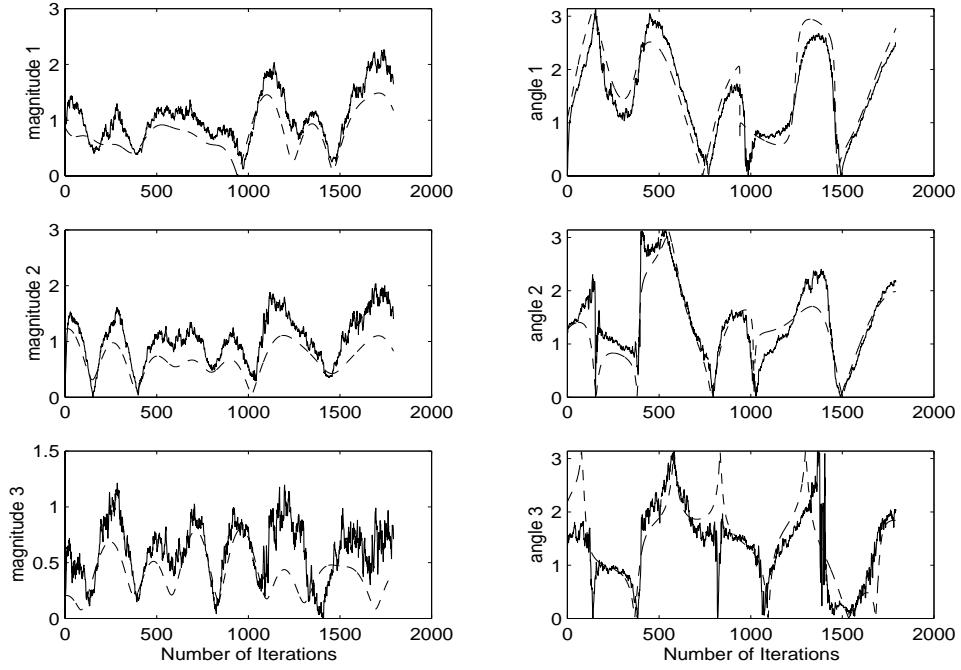
where we assume that the estimation delay  $D$  is much smaller than the rate at which the channel characteristics are changing. However, a decision feedback algorithm would only work assuming the decisions that are made by the receiver are correct most of time, and is thus most useful when the signal-to-noise ratio in the

<sup>2</sup>Showing the absolute value of phase instead of the original value avoids the confusion caused by periodicity of phase. For example, the actual value of phase at one time could be  $\pi$ , but the estimated value could be  $-\pi$ . Even though these two values are equivalent in phase, they would look quite different in a figure that shows the whole range of phase from  $-\pi$  to  $\pi$ . This ambiguity could also have been avoided by unwrapping the phase, which might result in a larger dynamic range for the phase values than shown in the figure.

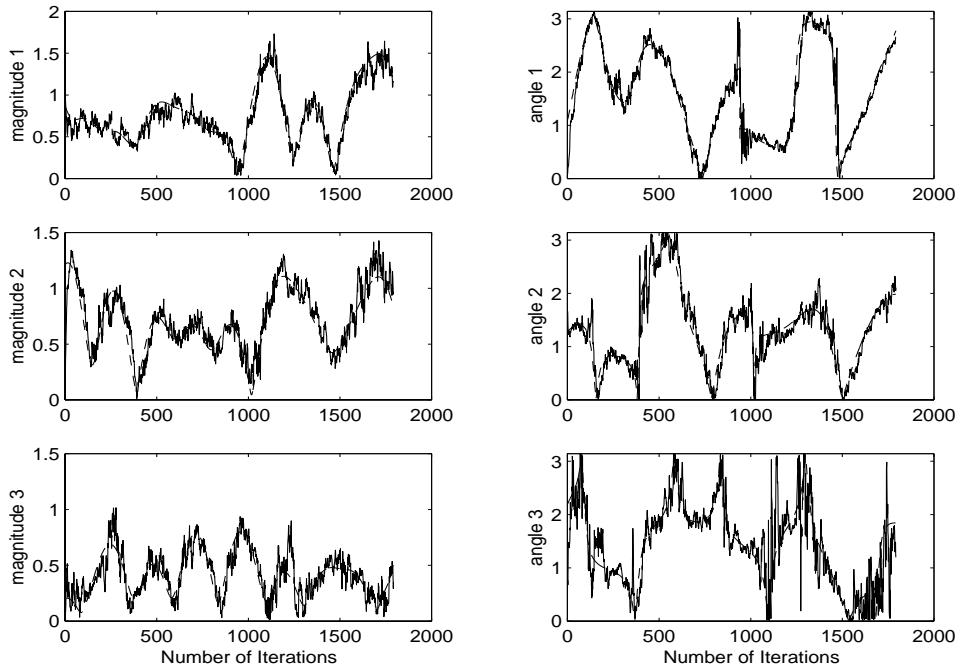
communication channel is relatively high. Performance evaluation of incorporating our channel estimation method into a blind and adaptive projection receiver is given in [8].

## 6. REFERENCES

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**Fig. 1.** Evolution of the channel parameters with  $K = 8$  users and  $f_{dmax} = 100$  Hz. Dash line: actual parameters. Solid line: estimated parameters. The channel estimation method was described in (5).



**Fig. 2.** Evolution of the channel parameters with  $K = 8$  users and  $f_{dmax} = 100$  Hz. Dash line: actual parameters. Solid line: estimated parameters. The channel estimation method was described in (12).