

A NEW APPROACH TO POWER ADJUSTMENT FOR SPATIAL COVARIANCE BASED DOWNLINK BEAMFORMING

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ABSTRACT

When using downlink beamforming at the base station of a cellular system, care must be taken to assure that all users satisfy the given SINR requirements. Generally, this leads to a computationally prohibitive joint optimization problem, therefore decoupled techniques based on eigenvalue decomposition of the spatial covariance have been proposed. However, these techniques require additional power adjustment of the steering vectors in order to satisfy the given SINR requirements. Conventional power control schemes are not suited for this purpose because the SINRs primarily depend on the shape of the radiation patterns. In this paper we propose a new power adjustment algorithm, which adjusts the beamforming weights, such that all users have the same SINR. Additionally, it is possible to limit the maximum radiated user power.

1. INTRODUCTION

Due to the increasing demand in wireless Internet applications, future wireless systems will have to cope with highly asymmetric data traffic. A scenario where a short uplink message entails large amount of downlink multimedia data is likely to occur frequently, so proper resource allocation schemes as well as bandwidth-efficient transmission techniques are needed.

The spectral efficiency of the uplink can be improved by using advanced receiver algorithms together with antenna arrays at the base station (BS). For the downlink, however, a different approach is required. This is mainly due to the limited energy storage capability and size of the mobile station (MS), which makes it extremely difficult to use antenna arrays together with intensive signal processing here. One way to get around this problem is to carry out downlink signal processing at the BS prior to transmission, which means that the properties of the propagation medium must be estimated *before* the transmitted signal encounters the channel. This leads to a situation where high capacity demands must be accomplished with only limited knowledge of the radio channel. Thus, the downlink can be seen as the bottleneck in the design of cellular systems.

A promising technique to significantly improve the downlink capacity is to use beamforming based on linear half-wavelength antenna arrays at the BS. Downlink beamforming has been the subject of intense research throughout the past decade [1]–[8]. The principle is illustrated in Fig. 1. By adaptively weighting the i^{th} user signal with an antenna weighting vector $\mathbf{w}_i = [w_i^{(1)} \dots w_i^{(M)}]^T$, $1 \leq i \leq K$, where M is the number of antenna elements, a transmit radiation pattern can be adaptively steered towards the desired user, thereby avoiding cross-talk between inter-

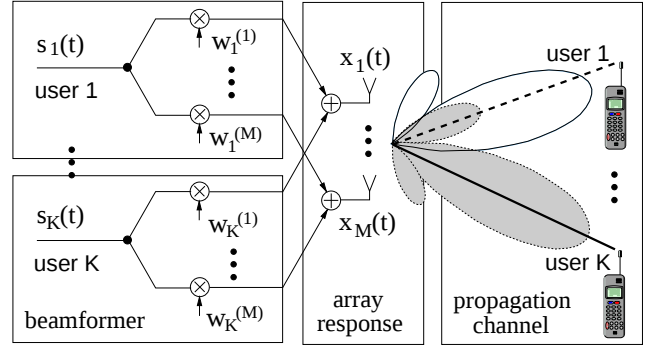


Fig. 1. Schematic illustration of downlink beamforming

cell users as well as inter-cell interference received by neighboring cells. This, in turn, can be traded for improved data rate, increased user capacity or better Quality-of-Service (QoS).

In case that $K > M$, cross-talk cannot be suppressed completely due to the side lobes of the radiation pattern (see Fig. 1). Thus, the Signal-to-Interference-plus-Noise-Ratio (SINR) of the i^{th} user is a function of *all* antenna vectors $\mathbf{w}_1 \dots \mathbf{w}_K$. As a consequence, all antenna vectors must be computed jointly, which generally leads to a highly nonlinear optimization problem, being computationally prohibitive for real time applications.

Computationally attractive suboptimum techniques have been proposed [3]–[8], which break up the original joint optimization problem into two smaller subproblems:

1. compute the shape of the radiation pattern for each user independently,
2. adjust the user powers such that all users have the same SINR.

While the beamforming step can be efficiently computed by eigenvalue decomposition, the power control step cannot be accomplished by conventional techniques. This is mainly due to the fact that the cross-talk caused by a certain user primarily depends on the shape of the radiation pattern rather than on the transmitted user power. This means that after computing the antenna weights, all users may have completely different SINRs (see Fig. 2). These differences cannot be balanced by classical power control since the situation changes completely each time when a new set of beamforming weights is computed. Hence, each new beamforming vector must be corrected by an additional power adjustment scheme, which balances the transmitted user powers according to the SINR

requirements.

In this paper, a new algorithm is presented for the case that all users have equal SINR requirements and the radiated power is limited by P_{max} . Following a brief review of the signal model and possible beamforming solutions, we first study the noiseless case, which is closely related to the optimum SINR optimization problem. Then an algorithm is presented which computes the optimum user powers $P_1 \dots P_K$ for a given set of radiation patterns. The complete proofs are beyond the scope of this paper, instead some sketches are given in the appendix. Some notational conventions are: vectors in boldface lower case and matrices in boldface upper case. The complex conjugate and the complex conjugate transpose are given by $\overline{(\cdot)}$ and $(\cdot)^H$, respectively. Furthermore, $\mathcal{E}\{\cdot\}$ denotes the expectation operator and $*$ the convolution operator.

2. RADIO CHANNEL MODEL

From Fig. 1 it can be seen that the array output vector is given by

$$\mathbf{x}(t) = \sum_{i=1}^K \mathbf{w}_i \cdot s_i(t), \quad (1)$$

where $s_i(t)$ are the modulated and spreaded user signals. The radio channel associated with the i^{th} user can be modeled in good approximation as a linear system with time-variant impulse response

$$\mathbf{h}_i(t; \tau) = \sum_{l=1}^{L_i} \alpha_{i,l}(t) \mathbf{a}(\theta_{i,l}) \delta(\tau - \tau_{i,l}),$$

where L_i denotes the number of transmission paths, $\tau_{i,l}$ the path delay, $\alpha_{i,l}$ is the complex path attenuation which is subject to fading and path loss and $\mathbf{a}(\theta_{i,l})$ is the array response, which is parametrized by $\theta_{i,l}$ (the Angle-of-Departure, AoD, measured relative to array broadside). Considering a calibrated, $\lambda/2$ -spaced uniform linear array (ULA), we have

$$\mathbf{a}(\theta_{i,l}) = [1, e^{-j\pi \sin \theta_{i,l}}, e^{-j2\pi \sin \theta_{i,l}}, \dots, e^{-j(M-1)\pi \sin \theta_{i,l}}]^T.$$

This model is valid as long as the signals are narrow-band with respect to the array observation time, i.e., the transmission time of the electromagnetic wave across the array is much less than the inverse signal bandwidth (narrowband assumption).

The signal received at the i^{th} mobile is given by

$$\begin{aligned} r_i(t) &= \mathbf{x}(t) * \mathbf{h}_i(t; \tau) + n(t) \\ &= \sum_{k=1}^K \mathbf{w}_k^H \sum_{l=1}^{L_i} \alpha_{i,l}(t) \mathbf{a}(\theta_{i,l}) s_k(t - \tau_{i,l}) + n(t), \end{aligned}$$

where $n(t)$ denotes additive white Gaussian noise. The desired component $r_i^{(d)}(t)$ of the signal $r_i(t)$ is obtained for $k = i$. The remaining terms are due to cross-talk and noise and are denoted by $r_i^{(c)}(t)$. The average SINR of the i^{th} mobile is given by

$$\text{SINR}_i = \frac{\mathcal{E}\{|r_i^{(d)}(t)|^2\}}{\mathcal{E}\{|r_i^{(c)}(t)|^2\}} = \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum_{\substack{k=1 \\ k \neq i}}^K \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2}, \quad (2)$$

where $\sigma^2 = \mathcal{E}\{n(t)\overline{n(t)}\}$ denotes the noise variance and

$$\mathbf{R}_i = \sum_{l=1}^{L_i} \mathcal{E}\{|\alpha_{i,l}(t)|^2\} \mathbf{a}(\theta_{i,l}) \mathbf{a}^H(\theta_{i,l})$$

is the channel covariance matrix. This result has been obtained by assuming that the averaging period is chosen sufficiently large such that the transmission paths are uncorrelated. On the other hand the observation time must be sufficiently small such that $\theta_{i,l}$ and $\tau_{i,l}$ can be regarded as stationary. Without loss of generality we have normalized $\mathcal{E}\{s_k(t)\overline{s_k(t)}\} = 1$.

The weighting vectors can be split up into

$$\mathbf{w}_i = \sqrt{P_i} \mathbf{u}_i, \quad 1 \leq i \leq K, \quad (3)$$

where P_i is proportional to the signal power radiated towards the i^{th} user. The vector $\mathbf{u}_i$ is the orientation of the antenna weighting vector, which determines the shape of the radiation pattern. Without loss of generality, we assume

$$\mathbf{u}_i^H \mathbf{R}_i \mathbf{u}_i = 1, \quad \forall i. \quad (4)$$

For simplicity we will refer to P_i and $\mathbf{u}_i$ as the ‘signal power’ and the ‘orientation’ of the radiation pattern, respectively. Defining $\mathbf{p} = [P_1, \dots, P_K]$, the SINR of the i^{th} mobile as a function of $\mathbf{p}$ and σ^2 is given by

$$\text{SINR}_i^{(l)}(\mathbf{p}, \sigma^2) = \frac{P_i}{\sum_{\substack{k=1 \\ k \neq i}}^K P_k \cdot (\mathbf{u}_k^{(l)})^H \mathbf{R}_i \mathbf{u}_k^{(l)} + \sigma^2}, \quad \forall i, \quad (5)$$

where the specific choice of beamforming algorithm is denoted by the superscript (l) , $l \in \mathbb{N}$. Possible solutions are:

1. Generalized Sidelobe Canceler (GSC) [6], [7]

$$\mathbf{u}_i^{(1)} = \arg \min_{\mathbf{u}_i} \mathbf{u}_i^H \left(\sum_{k=1}^K \mathbf{R}_k \right) \mathbf{u}_i \quad \text{s.t.} \quad \mathbf{u}_i^H \mathbf{R}_i \mathbf{u}_i = 1, \quad \forall i \quad (6)$$

2. Maximum Directivity Beamformer (MDB) [8]

$$\mathbf{u}_i^{(2)} = \arg \min_{\mathbf{u}_i} \mathbf{u}_i^H \mathbf{u}_i \quad \text{s.t.} \quad \mathbf{u}_i^H \mathbf{R}_i \mathbf{u}_i = 1, \quad \forall i. \quad (7)$$

3. In [6] an alternative approach is proposed, which can be shown [5] to be equivalent to (7):

$$\mathbf{u}_i^{(3)} = \arg \max_{\mathbf{u}_i} \mathbf{u}_i^H \mathbf{R}_i \mathbf{u}_i \quad \text{s.t.} \quad \|\mathbf{u}_i\|_2 = 1, \quad \forall i. \quad (8)$$

3. OPTIMUM SIR SOLUTION

We start by considering the noiseless case and define $\text{SIR}_i^{(l)}(\mathbf{p}) = \text{SINR}_i^{(l)}(\mathbf{p}, \sigma^2 = 0)$. In the following we assume that the beam pattern orientations $\mathbf{u}_1^{(l)}, \dots, \mathbf{u}_K^{(l)}$ are already available as solutions of (6), (7) or other beamforming algorithms.

Fig. 2 shows the radiation patterns of the GSC without power control ($P_1 \dots P_K = 1/\sqrt{K}$). A single-path channel with equal path attenuations has been applied. The positions of the distinct user AoDs are marked by vertical lines and the total power radiated by the array is denoted by P_t . It can be seen, that even under these simplifying assumptions the resulting SIRs are quite different. This means, that the users with oversized SIR could reduce their power in order to maximize the minimum SIR, which would maximize the overall system capacity. Mathematically, this can be expressed as:

$$\text{SIR}_{opt}^{(l)} := \max_{\mathbf{p}} \min_{1 \leq i \leq K} \text{SIR}_i^{(l)}(\mathbf{p}). \quad (9)$$

Our goal is to find a vector $\mathbf{p}^{(l)}$ such that all SIR are equal, i.e.

$$\text{SIR}_1^{(l)}(\mathbf{p}^{(l)}) = \dots = \text{SIR}_K^{(l)}(\mathbf{p}^{(l)}) = \text{SIR}_{opt}^{(l)}. \quad (10)$$

Defining the cross-power matrix

$$\Psi^{(l)} = \begin{bmatrix} 1 & (\mathbf{u}_2^{(l)})^H \mathbf{R}_1 \mathbf{u}_2^{(l)} & \dots & (\mathbf{u}_K^{(l)})^H \mathbf{R}_1 \mathbf{u}_K^{(l)} \\ (\mathbf{u}_1^{(l)})^H \mathbf{R}_2 \mathbf{u}_1^{(l)} & 1 & & \\ \vdots & & \ddots & \\ (\mathbf{u}_1^{(l)})^H \mathbf{R}_K \mathbf{u}_1^{(l)} & \dots & & 1 \end{bmatrix}$$

Eq. (10) can be rewritten as a set of linear equations

$$\Psi^{(l)} \cdot \mathbf{p}^{(l)} = \left(1 + \frac{1}{\text{SIR}_{opt}^{(l)}}\right) \mathbf{p}^{(l)}, \quad (11)$$

where $\mathbf{p}^{(l)}$ is an eigenvector associated with the eigenvalue $\lambda^{(l)} = (1 + 1/\text{SIR}_{opt}^{(l)})$ of the matrix $\Psi^{(l)}$.

Theorem 1 Let $\lambda_{max}^{(l)}$ denote the maximum eigenvalue of the matrix $\Psi^{(l)}$, then

$$\text{SIR}_{opt}^{(l)} = \frac{1}{\lambda_{max}^{(l)} - 1} \quad (12)$$

is the maximum SIR which can be supported for equal SIR requirement and given $\mathbf{u}_1^{(l)}, \dots, \mathbf{u}_K^{(l)}$. The power vector $\mathbf{p}^{(l)}$ satisfying (10) is given by the eigenvector associated with $\lambda_{max}^{(l)}$.

To illustrate this, Fig. 3 shows the GSC after SIR equalization. Although all SIR are equal, this solution is not appropriate for practical implementation, since the transmitted power can become arbitrarily small. This can be avoided by considering noise, which will be done in the following section.

4. SINR EQUALIZATION

Having determined the optimum solution $\text{SIR}_{opt}^{(l)}$ for the noiseless case, we will now investigate the case, where all mobiles require the same SINR γ_{min} . For notational simplicity, the optimum power vector for given $\mathbf{u}_1^{(l)} \dots \mathbf{u}_K^{(l)}$ will again be denoted by $\mathbf{p}^{(l)}$. Assuming $\sigma^2 \neq 0$, it can be shown that there is a set of power vectors $\mathbf{p} \in \mathbb{R}_+^K$, which satisfy

$$\gamma_{min} \leq \min_{1 \leq i \leq K} \text{SINR}_i^{(l)}(\mathbf{p}, \sigma^2) < \text{SIR}_{opt}^{(l)}. \quad (13)$$

Theorem 2 Assume that $\gamma_{min} < \text{SIR}_{opt}^{(l)}$, then among all possible solutions of (13), there exists one solution $\mathbf{p}^{(l)}$, which minimizes the radiated array power. This solution is characterized by $\text{SINR}_i^{(l)}(\mathbf{p}^{(l)}, \sigma^2) = \gamma_{min}$, $1 \leq i \leq K$. It can be obtained by solving a set of linear equations:

$$\mathbf{p}^{(l)} = \sigma^2 \left[\left(1 + \frac{1}{\gamma_{min}}\right) \cdot \mathbf{I} - \Psi^{(l)} \right]^{-1} \cdot \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}. \quad (14)$$

A similar scheme has been proposed in [6], however it could not be assured that the solution of (14) is positive, i.e. $\mathbf{p}^{(l)} \in \mathbb{R}_+^K$. From Theorem 1 and Theorem 2, it can be seen, that the existence of a valid solution only depends on the maximum eigenvalue $\lambda_{max}^{(l)}$ of the matrix $\Psi^{(l)}$, which must satisfy $\lambda_{max}^{(l)} < (\gamma_{min})^{-1} + 1$.

Clearly, the goal should be to minimize $\lambda_{max}^{(l)}$ in order to support large γ_{min} .

Next, there is an additional requirement that the radiated power of an individual user must not exceed P_{max} . If $\text{SIR}_{opt}^{(l)}$ approaches γ_{min} , then the user power tends to infinity. This case must be avoided by introducing an additional constraint

$$\max_{1 \leq k \leq K} P_k(\gamma_{min}) < P_{max}. \quad (15)$$

Theorem 3 A necessary (but not sufficient) condition that (15) is fulfilled, is given by

$$\gamma_{min} \leq \left(\frac{\sigma^2}{P_{max}} + \frac{1}{\text{SIR}_{opt}^{(l)}} \right)^{-1} =: \tilde{\gamma}. \quad (16)$$

In other words: if (16) is not fulfilled, then (15) is also not fulfilled. In this case, γ_{min} cannot be provided by the given beamforming scenario characterized by $\Psi^{(l)}$. Otherwise, the power vector $\mathbf{p}^{(l)}$ can be computed by (14). Numerical simulations (see Fig. 4) show that even in the worst case ($\gamma_{min} = \tilde{\gamma}$), the maximum user power only slightly exceeds P_{max} . Thus, for practical applications, the resulting error is controllable.

5. SUMMARY AND CONCLUSIONS

This paper has addressed the problem of open-loop power adjustment for decoupled downlink beamforming. Beamforming transmission differs from omnidirectional transmission in that the SINR cannot be controlled by conventional power control. This is because the SINR is primarily determined by the shape of each radiation pattern, rather than by the individual user powers. Thus, after computing the steering vectors, the user powers must be adjusted such that the given SINR requirements are satisfied. In case of equal SINR requirements, this can be accomplished in a computationally efficient way:

1. estimate the channel covariance matrices $\mathbf{R}_1 \dots \mathbf{R}_K$,
2. compute the beam pattern orientations $\mathbf{u}_1^{(l)} \dots \mathbf{u}_K^{(l)}$ as solutions of (6), (7) or other beamforming schemes,
3. compute $\text{SIR}_{opt}^{(l)}$ by (12),
4. check if condition (16) is fulfilled, otherwise γ_{min} cannot be supported for the given user scenario,
5. compute the user powers $P_1^{(l)} \dots, P_K^{(l)}$ by (14),
6. compute the final antenna weights $\mathbf{w}_i^{(l)} = \sqrt{P_i^{(l)}} \cdot \mathbf{u}_i^{(l)}$.

The optimization has been performed for a single-cell scenario. Yet, inter-cell interference is minimized to a certain extend because the algorithm minimizes $P_1 \dots P_K$ for given γ_{min} and $\mathbf{u}_1^{(l)} \dots \mathbf{u}_K^{(l)}$.

Future work will have to analyze multiple cell systems more thoroughly. It might not be feasible to estimate the user distribution of neighboring cells due to the limited bandwidth of the network signaling channel. However, the assumption of spatially colored *a priori* distributions seems to be an interesting alternative and could be considered within the proposed downlink beamforming framework.

Finally, the case of unequal SINR requirements should be addressed. Extensive simulations are still necessary in order to fully exploit the potential capacity gain offered by downlink beamforming.

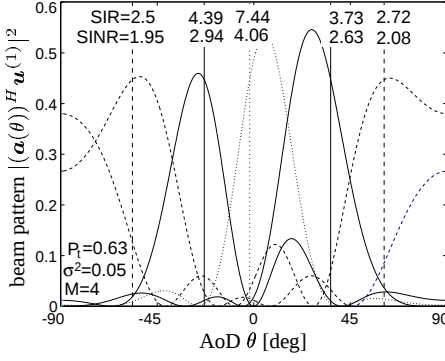


Fig. 2. GSC without power adjustment

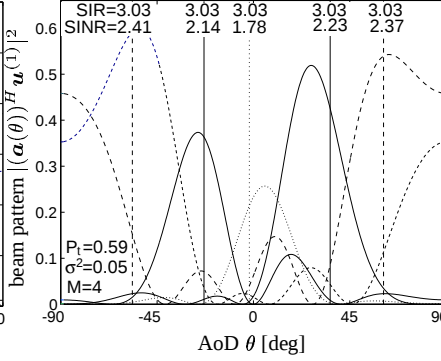


Fig. 3. GSC after SIR equalization

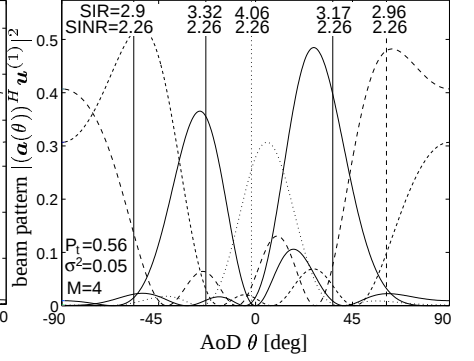


Fig. 4. GSC after SINR equalization

6. APPENDIX

Proof of Theorem 1

The proof is beyond the scope of this paper and will be published in [9].

Proof of Theorem 2

Let $\mathbf{p}^{opt}$ denote the optimum SIR solution which satisfies $\text{SIR}_i^{(l)}(\mathbf{p}^{opt}) = \text{SIR}_{opt}^{(l)}, \forall i$. Considering an arbitrary scalar $\lambda > 0$, we have

$$\begin{aligned} \text{SINR}_i^{(l)}(\lambda \cdot \mathbf{p}^{opt}, \sigma^2) &= \frac{P_i^{opt}}{\sum_{k=1, k \neq i}^K P_k^{opt} (\mathbf{u}_k^{(l)})^H \mathbf{R}_i \mathbf{u}_k^{(l)} + \sigma^2 / \lambda} \\ &= \text{SINR}_i^{(l)}(\mathbf{p}^{opt}, \sigma^2 / \lambda) \end{aligned}$$

The function $\min_{1 \leq i \leq K} \text{SINR}_i^{(l)}(\lambda \cdot \mathbf{p}^{opt}, \sigma^2)$ is monotonically increasing in λ and bounded by the supremum $\text{SIR}_{opt}^{(l)}$. From this follows

$$\min_{1 \leq i \leq K} \text{SINR}_i^{(l)}(\mathbf{p}, \sigma^2) < \text{SIR}_{opt}^{(l)}$$

for arbitrary $\mathbf{p}$ and σ^2 . Thus, a valid solution can only exist if $\gamma_{min} < \text{SIR}_{opt}^{(l)}$. Next, it is shown that in this case there exists exactly one solution, which is given by (14).

The matrix $\mathbf{C}^{(l)} = ((1 + \gamma_{min}^{-1})\mathbf{I} - \Psi^{(l)})$ is regular, thus $(\mathbf{C}^{(l)})^{-1}$ exists. Using Taylor series expansion, we have

$$\begin{aligned} \mathbf{p}^{(l)} &= (\mathbf{C}^{(l)})^{-1} \sigma^2 \cdot [1, \dots, 1]^T \\ &= \frac{\sigma^2}{1 + \frac{1}{\gamma_{min}}} \sum_{r=0}^{\infty} \frac{1}{(1 + \frac{1}{\gamma_{min}})^r} (\Psi^{(l)})^r \cdot [1, \dots, 1]^T. \end{aligned} \quad (17)$$

It can be shown that (17) is convergent only if $\gamma_{min} < \text{SIR}_{opt}^{(l)}$. The components of $\Psi^{(l)}$ are positive, thus the components of $\mathbf{p}^{(l)}$ must also be positive. Since there is only one solution, this solution minimizes the total radiated power.

Proof of Theorem 3

Let γ_{min} be the desired SINR. With (5) we can write

$$\frac{P_i}{\gamma_{min}} - \frac{P_i}{\text{SIR}_{opt}} + I(i) = \sigma^2,$$

where

$$I(i) = \frac{P_i}{\text{SIR}_{opt}} - \sum_{k=1, k \neq i}^K \mathbf{u}_k^H \mathbf{R}_i \mathbf{u}_k.$$

It can be shown that there must exist at least one user with index i_0 , which satisfies $I(i_0) \leq 0$. Thus we have

$$\max_{1 \leq k \leq K} P_k \geq \sigma^2 \cdot \left(\frac{1}{\gamma_{min}} - \frac{1}{\text{SIR}_{opt}} \right)^{-1}.$$

7. REFERENCES

- [1] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using semidefinite optimization," in *Proc. of 37th Annual Allerton Conference*, Sept. 1999, pp. 987–996.
- [2] F. Rashid-Farrokh, L. Tassiulas, and K. J. Liu, "Transmit beamforming and power control for cellular wireless systems," *IEEE Trans. on Comm.*, vol. 46, no. 10, pp. 1313–1323, Oct. 1998.
- [3] W. Utschick and J. A. Nossek, "Downlink beamforming for FDD mobile radio systems based on spatial covariances," in *Proc. European Wireless 99, Munich, Germany*, Oct. 1999, pp. 65–68.
- [4] D. Gerlach and A. Paulraj, "Base station transmitting antenna arrays for multipath environments," *Signal Processing*, vol. 54, pp. 59–73, 1996.
- [5] H. Boche and M. Schubert, "Performance analysis of downlink beamforming algorithms," in *Proc. MICRO.tec, Hanover, Germany*, Sept. 2000, pp. V2/161–166.
- [6] Ch. Farsakh and J. A. Nossek, "Spatial covariance based downlink beamforming in an SDMA mobile radio system," *IEEE Trans. on Comm.*, vol. 46, no. 11, pp. 1497–1506, Nov. 1998.
- [7] P. Zetterberg and B. Ottersten, "The spectrum efficiency of a base station antenna array system for spatially selective transmission," in *Proc. VTC99 Fall*, 1995.
- [8] M. Schubert and H. Boche, "Downlink beamforming for TD/CDMA multipath channels," in *Proc. ICASSP 2000, Istanbul, Turkey*, June 2000, pp. V.2993–V.2996.
- [9] H. Boche and M. Schubert, "Theoretical and experimental comparison of optimization criteria for downlink beamforming," *preprint Heinrich-Hertz Institute*, 2001.