CHIRPED-OFDM FOR TRANSMISSIONS OVER TIME-VARYING CHANNELS WITH LINEAR DELAY/DOPPLER SPREADING

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ABSTRACT

In this work we show that the optimal digital communication strategy for transmissions over time-varying channels with spread function maximally concentrated along a line of the delay-Doppler domain consists in multiplexing the input symbol block with an IFFT, as in OFDM, and modulating the IFFT output with a chirp signal whose sweep rate is matched to the channel. We show how to allocate the transmit power optimally across the chirped subcarriers, derive the limits of applicability of the proposed chirped-OFDM scheme and compute the resulting BER curves.

1. MOTIVATION

Orthogonal frequency division multiplexing (OFDM) is by now a well established technology and OFDM combined with proper precoding of the information symbols can be shown to achieve optimal performance for communications over linear time- invariant (LTI) channels. OFDM can be extended to linear time-varying (LTV) channels only as far as the OFDM symbol duration is kept much smaller than the channel coherence time T_c, i.e., roughly speaking, the time interval over which the channel is approximately constant. Although this does not represent a serious limitation in most current applications, the trend towards broader bandwidth services for users with possibly higher mobility pushes the research towards the extension of OFDM to time-varying channels.

Standard OFDM can be extended to slowly time-varying systems only at the expense of efficiency. In fact, let us consider an OFDM system where $1/T$ is the input data rate, N is the number of symbols multiplexed in each OFDM block, and LT is the duration of the cyclic prefix, with $LT \geq \Delta_s$, where Δ_s is the delay spread. To prevent severe distortion in the transit through a time-varying channel, NT must be a fraction of T_c, let us say $NT = \alpha T_c$, with $\alpha < 1$. This imposes an upper bound on the OFDM efficiency ϵ, defined as the ratio between the duration of the input N-symbols block and the duration of the final OFDM symbol, equal to $\alpha T_c/(\alpha T_c + \Delta_s)$.

In this work we show that if the time-varying channel is characterized by a spread function $S(\nu, \tau)$ maximally concentrated along a line in the delay-Doppler plane, we may extend the duration of the OFDM symbol well beyond the channel coherence time, without suffering *any* distortion, provided that we send a chirped-OFDM signal, i.e. an OFDM signal modulated by a chirp whose sweep rate is equal to the slope of the line where $S(\nu, \tau)$ is maximally concentrated. Important special cases of channels with linear delay-Doppler distributions, besides LTI channels, are multiplicative channels and two-ray multipath channels, often encountered in HF links, satellite communications and extensively used for performance evaluations, see e.g. [7], [5]. Indeed, standard OFDM can be intepreted as a special case of our chirped-OFDM, corresponding to channels whose spread function is concentrated along a line with slope equal to zero. The capability of accommodating linear distributions of $S(\nu, \tau)$ with any slope is what gives to our chirped-OFDM the ability of avoiding distortions even using symbols durations much greater than the channel coherence time. In this paper we derive the new limits for the maximum symbol duration allowing distortionless transmissions and show the BER curves of the proposed scheme.

2. EIGENFUNCTIONS OF LTV CHANNELS WITH LINEAR DELAY-DOPPLER SPREADING

A linear time-varying (LTV) channel is fully characterized by its impulse response $h(t, \tau)$ which allows us to write the input/output (I/O) relationship

$$y(t) = \int_{-\infty}^{\infty} h(t, \tau) x(t - \tau) d\tau. \quad (1)$$

Equivalently, the channel can be described by its delay-Doppler spread function $S(\nu, \tau)$, defined as [2]

$$S(\nu, \tau) := \int_{-\infty}^{\infty} h(t, \tau) e^{-j2\pi\nu t} dt. \quad (2)$$

We say that a channel has a linear delay-Doppler spread if $S(\nu, \tau)$ is concentrated along a straight line.

In several applications, and primarily in digital communications, it is important to know the channel eigenfunctions or, more precisely, talking about time-varying channels, the channel left and right singular functions (see, e.g. [3]). Specifically, if the system impulse response is square-integrable, i.e. [1]

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |h(t, \tau)|^2 dt d\tau < \infty, \quad (3)$$

then there exists a countable set of *singular values* λ_i and two classes of orthonormal functions $v_i(t)$ and $u_i(t)$, named *right and left singular functions*, such that the following system of integral equations holds true [3]

$$\lambda_i u_i(t) = \int_{-\infty}^{\infty} h(t, t - \tau) v_i(\tau) d\tau, \quad (4)$$

$$\lambda_i v_i(\tau) = \int_{-\infty}^{\infty} h^*(t, t - \tau) u_i(t) dt. \quad (5)$$

[1] In principle, (3) is not satisfied for many ideal models. However (3) is certainly satisfied for observations (transmissions) within finite time and frequency intervals, and this encompasses all practical situations.

Unfortunately, there is no analytic expression for the singular functions of a general time-varying channel. However, there are approximate formulas valid for slowly-varying operators [6] and for underspread channels, i.e. channels whose spread function is maximally concentrated within a rectangle in the delay-Doppler domain whose area $BT \ll 1$ [4], [1][2]. Now we show that if the channel spread function is maximally concentrated along a straight line, the channel singular functions are known exactly, whichever is the extension of the spread function support.

Theorem: The left and right singular functions of a channel whose spread function is concentrated along a line in the delay-Doppler plane, i.e.

$$S(\nu, \tau) = g(\tau)\delta(\nu - f_0 - \mu\tau), \tag{6}$$

are *chirp* signals with sweep rate μ, i.e.

$$\begin{aligned} v_i(t) &= e^{j\pi\mu t^2} e^{j2\pi f_i t} & (7)\\ u_i(t) &= e^{j\pi\mu t^2} e^{j2\pi f_i t} e^{j2\pi f_0 t} = v_i(t) e^{j2\pi f_0 t}. & (8) \end{aligned}$$

Proof: The impulse response corresponding to (6) is

$$h(t, \tau) = g(\tau) e^{j2\pi\mu\tau t} e^{j2\pi f_0 t} \tag{9}$$

and, substituting (9) and (7) in (4) we get

$$\lambda_i u_i(t) = e^{j2\pi f_0 t} e^{j2\pi f_i t} e^{j\pi\mu t^2} G_\mu(f_i) = G_\mu(f_i) e^{j2\pi f_0 t} v_i(t),$$

so that (4) is proved with $\lambda_i = G_\mu(f_i)$, where $G_\mu(f)$ is the Fourier transform (FT) of $g_\mu(t) := g(t)e^{j\pi\mu t^2}$. We can also check immediately that (5) is also satisfied and that the two classes of functions $v_i(t)$ and $u_i(t)$ are orthogonal [QED].

Important particular cases of channels with linear delay-Doppler spreading are LTI channels, corresponding to $\mu = 0$, multiplicative channels, where $\mu = \infty$, and two-ray multipath channels. In fact, two-ray channels have a spread function

$$S(\nu, \tau) = h_0\delta(\tau - \tau_0)\delta(\nu - f_0) + h_1\delta(\tau - \tau_0)\delta(\nu - f_1) \tag{10}$$

and, since there is always a straight line passing through two points, we have $\mu = (f_1 - f_0)/(\tau_1 - \tau_0)$.

3. CHIRPED OFDM

Recalling the pioneering work of Gallager on time-varying channels [3], we know that the optimal strategy for transmitting a sequence of symbols $s[k]$, $k = \ldots, -1, 0, 1, \ldots$ through an LTV channel consists in sending the signal

$$x(t) = \sum_{k=-\infty}^{\infty} \Phi_k s[k] v_k(t) \tag{11}$$

where $v_k(t)$ is the k-th right singular function associated to the k-th singular value and Φ_k are coefficients used to allocate power across the transmitted symbols, see e.g. [3], [1]. Using (4), the received signal is

$$y(t) = \sum_{k=-\infty}^{\infty} \Phi_k \lambda_k s[k] u_k(t) + w(t), \tag{12}$$

[2]The underspread property indeed holds true for most communication channels.

where $w(t)$ is additive noise. Hence, the transmitted symbols can be estimated by simply taking the scalar products of $y(t)$ with the left singular functions, i.e.

$$\hat{s}[m] = \frac{1}{\lambda_m \Phi_m} \int_{-\infty}^{\infty} y(t) u_m^*(t) dt = s[m] + w[m], \tag{13}$$

where $w[m] := \int_{-\infty}^{\infty} w(t) u_m^*(t) dt / \lambda_m \Phi_m$. If the additive noise is white and Gaussian, the random variables $w[m]$ are iid Gaussian random variables. In this way, the initial LTV channel has been converted into a set of parallel independent subchannels and the symbol-by-symbol detector is also the maximum likelihood detector. If we specialize (11) to channels with spread function as in (6), using (7), we obtain

$$x(t) = e^{j\pi\mu t^2} \sum_k \Phi_k s[k] e^{j2\pi f_k t}.$$

This expression shows that the optimal transmission strategy for channels with linear delay-Doppler spread, consists in multiplexing the symbols as in OFDM and then modulating the OFDM signal with a chirp whose sweep rate is the slope of the line where $S(\nu, \tau)$ is maximally concentrated. Equation (11) presupposes the parallel transmission of an infinite set of data, which is not practical. Now, we show how to specialize (14) to the finite block case. Given the i-th block of N symbols $(s_i[0], \ldots, s_i[N-1])$, with $s_i[n] := s[iN + n]$, we multiplex them in order to form the continuous time waveform

$$x_i(t) = e^{j\pi\mu(t - i(N+L)T)^2} \sum_{n=0}^{N-1} \Phi_i(n) s_i[n] e^{j2\pi n(t - i(N+L)T)/NT}, \tag{14}$$

with $t \in [iNT - LT, (i+1)NT]$ so as to include a prefix of length LT in each block. This multiplexing strategy is what we term *chirped-OFDM*. To grasp better physical insight into the channel output, we consider the spread function expressed as a series of delta functions, $S(\nu, \tau) = \sum_q h_q \delta(\tau - \tau_q)\delta(\nu - f_0 - \mu\tau_q)$, without making any assumption about the number of paths. We only assume, as in OFDM, that our guard interval is longer than the channel duration, i.e. $LT \geq \max_q(\tau_q)$. The channel output corresponding to (14), after discarding the initial guard interval and setting $\theta := t - i(N+L)T$, is

$$y_i(\theta) = e^{j\pi(\mu\theta^2 + 2f_0(\theta + (N+L)iT))} \sum_{n=0}^{N-1} \Phi_i(n) s_i[n] \lambda_i(n) e^{j2\pi n\theta/NT},$$

where

$$\lambda_i(n) := \sum_q h_q e^{j\pi\mu\tau_q(\tau_q + 2i(N+L)T)} e^{-j2\pi n\tau_q/NT}. \tag{15}$$

The recovery of the symbols $s_i[n]$ proceeds then as in OFDM, except for an initial dechirping operation. Specifically, the decoding steps are:

i) dechirp $y_i(\theta), \Rightarrow z_i(\theta) := y_i(theta) e^{-j\pi(\mu\theta^2 + 2f_0(\theta + (N+L)iT))}$;
ii) sample $z_i(theta)$ at rate $1/T$ and discard the guard interval;
iii) compute the N-point FFT $Z_i[k]$ of $z_i[n] := z_i(nT)$;
iv) equalize and detect the symbols. Using zero-forcing equalization, for example, $\hat{s}_i[k] := Z_i[k]/\Phi_i(k)\lambda_i(k)$.

The last operation is critical if $\lambda_i(k)$ is zero (small). This situation may be circumvented by resorting to MMSE decoding or avoiding transmissions over the most faded subcarriers, as shown in Section 4. It is important to remark that, in the presence of additive Gaussian noise, the dechirping operation does not alter the statistics of the noise. The initial dechirping operation in continuous time, followed by sampling, is analoguous to the scheme implemented in the radar altimeter onboard the satellite ERS-1 currently orbiting around the Earth, and it is useful because it reduces the bandwidth and then allows us to sample at the minimum rate.

4. OPTIMAL POWER ALLOCATION

In this section we show how to choose the amplitudes $\Phi_i(n)$ in (14) according to different optimization criteria, when the channel is known at the tranmitter side. Dealing with time-varying channel, optimal coding requires in general the prediction of the channel evolution, at least within the time interval assigned to the next block to be transmitted. Assuming a multipath model, we only have to estimate the channel parameters, using for example the method of [1] and update the estimate with a period depending on the time interval over which we presume that the channel parameters are constant (this interval may be much longer than the channel coherence time). Assuming the channel perfectly known at the transmitter side, in [1] we showed that the optimal precoding matrix has always the following structure $\boldsymbol{F}(n) = \boldsymbol{V}(n)\boldsymbol{\Phi}(n)$, where the columns of $\boldsymbol{V}(n)$ are the right singular vectors of the channel matrix describing the transit of the n-th block, whereas $\boldsymbol{\Phi}(n)$ is a diagonal matrix whose entries $\Phi_i(n)$ are a function of the channel singular values only, according to a law which depends on the optimality criterion. Here we report some of the results, derived under the assumption of white Gaussian noise, for different criteria.

1. Minimum mean square error between transmitted and received symbols, subject to a given average transmit power (MMSE/AP):

$$\Phi_i(n)^2 = \max\left(\frac{K_1}{|\lambda_i(n)|} - \frac{\sigma_v^2}{|\lambda_i(n)|^2}, 0\right). \tag{16}$$

2. Maximal mutual information between, subject to a given average transmit power (MIR/AP):

$$\Phi_i(n)^2 = \max\left(K_2 - \frac{\sigma_v^2}{|\lambda_i(n)|^2}, 0\right). \tag{17}$$

The constants K_1 and K_2 in (16) and (17) are chosen in order to enforce the prescribed average transmit power and σ_v^2 is the noise variance.

Interestingly, both solutions (16) and (17) prevent the transmissions through the most faded subchannels. The important property for channels with linear delay-Doppler spreading is that we do not need to compute any SVD to evaluate the optimal precoder because the columns of $\boldsymbol{V}(n)$ are chirp signals with sweep rate μ and the singular values $\lambda_i(n)$ are given by (15).

5. LIMITS OF APPLICABILITY

In general, we cannot expect the channel spread function to be totally concentrated along a line, except a few case, e.g. two-ray channels. Therefore, it is important to establish the limits of applicability of our chirped-OFDM. Given a spread function $S(\nu, \tau)$, first of all we find the parameters f_m and μ_m of the line $\nu = f_m + \mu_m\tau$ where $S(\nu, \tau)$ is maximally concentrated. Considering the normalized spread function $\bar{S}(\nu, \tau) := S(\nu, \tau) / \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |S(\nu, \tau)| d\nu d\tau$, the parameters f_m and μ_m are solution of

$$(f_m, \mu_m) := arg \min_{f,\mu} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\bar{S}(\nu, \tau)|(\nu - f - \mu\tau)^2 d\nu d\tau.$$

Introducing the moments $m_{k,l} := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\bar{S}(\nu, \tau)|\nu^k\tau^l d\nu d\tau$, we obtain $f_m = (m_{1,0}m_{0,2} - m_{0,1}m_{1,1})/(m_{0,2} - m_{0,1}^2)$ and $\mu_m = (m_{1,1} - m_{0,1}m_{1,0})/(m_{0,2} - m_{0,1}^2)$. Then we measure the spread of $S(\nu, \tau)$ around the maximum concentration line $\nu = f_m + \mu_m\tau$ as

$$B^2 := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\bar{S}(\nu, \tau)|(\nu - f_m - \mu_m\tau)^2 d\nu d\tau. \tag{18}$$

We state that the spread function distribution is approximately linear, within a time interval of duration NT, if $B < 1/NT$, i.e. if the spread around the maximum concentration line is smaller than the bandwidth of the FFT filter operating on symbols of duration NT. Equivalently, the new upper bound for the duration of the chirped-OFDM symbol guaranteeing (almost) distortionless transmission is $NT < 1/B$, with B given by (18). This limit may be well beyond the standard OFDM limit which, because of its inability of accommodating linear distributions different from the line $\nu = 0$, corresponds to a spread B_0 given by (18) with $f_m = 0$ and $\mu_m = 0$ and thus it is certainly greater than B.

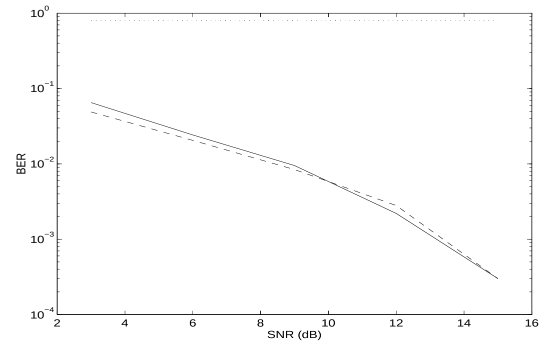

Fig. 1. BER vs. SNR for i) chirped-OFDM (solid line) over LTV channel; ii) standard OFDM (dotted line) over LTV channel; iii) standard OFDM over equivalent LTI channel (dashed line).

6. PERFORMANCE

In this section we show the bit error rate (BER) obtained using our chirped-OFDM technique.

Ex. 1 - Chirped-OFDM vs. OFDM: In Fig. 1 we show the BER vs. SNR (dB) for a two-ray channel with parameters $\boldsymbol{f}_d = [0.1, 0.5]/T$, $\boldsymbol{h} = [1.3, 2]$, $\boldsymbol{\tau} = [1.2, 3.6]T$. The symbols are QPSK and the number of symbols per block is $N = 128$. Fig. 1 shows the BER for our chirped OFDM (solid line) and for a standard OFDM (dotted line). We also report (dashed line) the BER for a standard OFDM system operating on the equivalent LTI channel (dashed line), i.e. the channel having the same parameters as the LTV channel, except the Doppler shifts which are set to 0. Comparing solid and dashed lines, we observe that the chirping

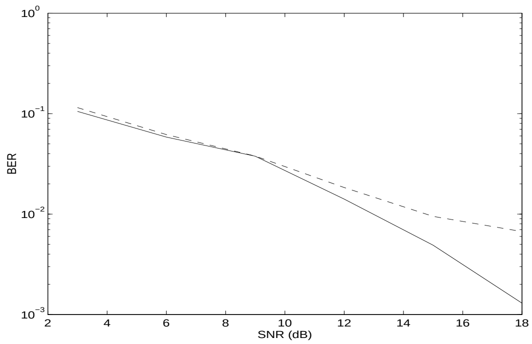

Fig. 2. BER vs. SNR with MMSE/AP power loading (solid line) and without loading (dashed line).

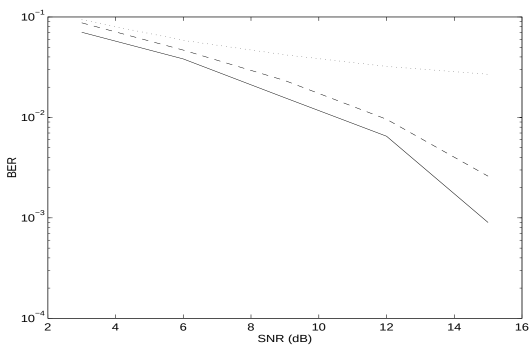

Fig. 3. BER vs. SNR of chirped-OFDM over a three-ray channel, with different distances from the maximum concentration line: $\Delta f = 0$ (solid line), $\Delta f = .1/NT$ (dashed line), and $\Delta f = 1/NT$ (dotted line).

operation allows the chirped-OFDM system to perform as if the channel is time-invariant, whereas standard OFDM on the same LTV channel performs very poorly (dotted line).

Ex. 2 - Loading: In Fig. 2 we report the BER obtained using our chirped OFDM with optimal power loading, according to the MMSE/AP criterion (solid line), and without loading (dashed line). The simulations are averaged over 100 independent channel realizations, where each realization has the same parameters as in Fig. 1 except the amplitudes which are complex zero mean Gaussian random variables. Fig. 2 shows how loading improves the performance, especially at high SNR.

Ex. 3 - Nonlinear delay-Doppler spread: In Fig. 3 we show the BER vs. SNR of a chirped-OFDM over a three ray channel, whose rays are not aligned. Specifically, the different curves in Fig. 3 refer to increasing distances Δf between one ray and the maximum concentration line. We can see that when the distance becomes comparable with $1/NT$, the chirped-OFDM is no longer valid. To overcome this furhter limit, we need to use nonlinear modulation waveforms, as suggested in [1]. However, this generalization is more difficult to implement because it is no longer possible to single out a common (nonlinear) frequency modulation signal and there is not an underlying OFDM structure.

Ex. 4 - Channel estimate: The main price to be paid for using the chirped-OFDM strategy is essentially the need for channel estimation and prediction. For multipath channels we can estimate the

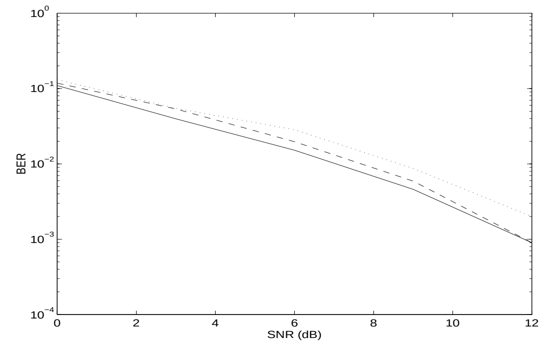

Fig. 4. BER vs. SNR of chirped-OFDM with known channel parameters (solid line) and channels estimated using chirp signals of length 512 (dashed line) and 256 (dotted line).

channel parameters using, for example, the methods suggested in [1], based on the periodic transmission of sounding chirp signals. A numerical example comparing the case of known channel (solid line) and channel estimated transmitting chirp signals of length 512 (dashed line) or 256 (dotted line) is reported in Fig. 4.

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