Exact Performances Analysis of a Selective Coefficient Adaptive Algorithm in Acoustic Echo Cancellation

I.Kammoun[(1)(2)], *M.Jaidane*[(1)]
(1) L.S.Télécoms, ENIT, Campus Universitaire, le Belvédère, Tunis, Tunisie
meriem.jaidane@enit.rnu.tn, nej_jaidane@exite.com
(2) Departement TSI, ENST, 46, rue Barrault-75634 Paris Cedex 13, France
ikammoun@excite.com

Abstract

In the hands-free communications, the identification of long impulse response in acoustic echo cancellation requires very important load calculation. One way to reduce the complexity of the classical Normalized Least Mean Square (NLMS) adaptive algorithm, is to use the Mmax NLMS algorithm [1]. It is shown that this algorithm is a very promising one, that maintains a closest performance to the full update NMLS filter in spite of the updating of a small number of coefficients. However, due to its complexity, the mean square analysis uses unrealistic hypothesis. It was then not possible to consider practical context such as high input correlation or high step size.

In this paper, we present an exact performances analysis inspired from [2], when the input signal remains in a finite alphabet set. With this realistic hypothesis, dedicated to the digital context, we can describe accurately the Mmax NLMS'behavior without any unrealistic assumption. In particular, we evaluate the exact value of critical and optimal step size and we provide the exact Mean Square Error (MSE) for all step size and input correlation. The influence of high order statistics can be enhanced.

Keywords :
Mmax NLMS algorithm - Finite alphabet - convergence performances - acoustic echo cancellation.

1 The Mmax NLMS performances for long impulse responses identification

In the hands-free communications, acoustic echo cancellation is usually realized by adaptive filtering. The adaptive finite-impulse-response (FIR) filter may require thousands of coefficients to accurately model the echo return path. The huge calculation load is beyond the capabilities of current digital signal processing (DSP) chips. To reduce the processing requirements, different algorithms were proposed (see for example[3], [4]). Such algorithms include the Mmax NLMS [1] analyzed in this paper. The Mmax NLMS updates a portion of his coefficients at each sample time; those selected coefficients are the ones with large magnitude gradient components on the error surface. The Mmax NLMS algorithm update equation is given by:

$$H_{k+1}(i) = \begin{cases} H_k(i) + \frac{\mu}{X_k^T.X_k}.e_k.x_{(k-i+1)} & \text{if } i \text{ corresponds} \\ & \text{to one of the first } M \text{ maxima} \\ & \text{of } \left|x_{(k-i+1)}\right|,\ i=1,...,L. \\ H_k(i) & \text{otherwise} \end{cases} \quad (1)$$

e_k is defined by $e_k = y_k - X_k^T.H_k$ and y_k is defined by $y_k = X_k^T.F + b_k$. Where $H_k = [H_k(1), H_k(2), ..., H_k(L)]^T$ is the coefficient vector of the adaptive filter at time k, $X_k = [x_k, x_{k-1}, ..., x_{k-L+1}]^T$ is the input signal vector, y_k is the desired response signal, e_k is the error signal, F is the L-length optimal weight vector and b_k is a zero mean independent disturbance signal.

First, to illustrate the importance of the Mmax NLMS, this algorithm has been simulated in the mobile-radio and videoconference contexts for a highly correlated input signal generated by passing a zero mean white Gaussian signal with unity variance through the filter $H(z) = \frac{1}{1-1.58z^{-1}+0.81z^{-2}}$. A white noise of 0.0001 variance is added to the desired signal. In the first one (L=300), for $\mu = 0.8$ we obtain good results with a small number of updates at each iteration (M=30). However, with very long impulse response (L=1500) encountered in the second context, a small number gives unsatisfactory results for all step size. In order to improve these results, we apply the Mmax NLMS algorithm on a decimate/interpolate structure[5].

We notice from figure 1 that, for the long impulse response considered, the proposed algorithm result

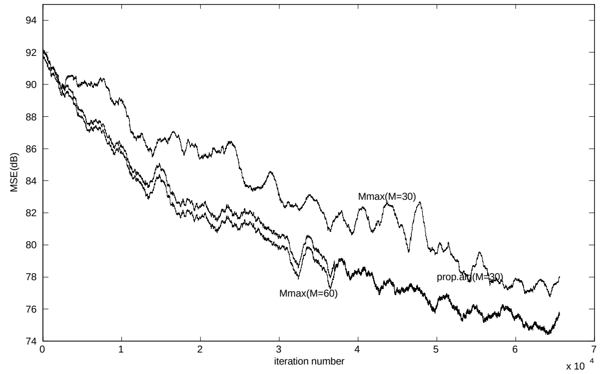

Figure 1 – MSE variation for the Mmax and dec/int Mmax

(M=30) is better than the Mmax NLMS one (M=30) and that it is very close to the Mmax NLMS result (M=60). In [1], the theoretical analysis of the mean square error (MSE) convergence and steady-state performance is made for i.i.d signal and is provided for the extreme case of one update/iteration (M=1). Moreover, it uses the common independence hypothesis between successive coefficient vectors. They prove in this context [1] that to ensure the Mmax NLMS algorithm convergence, the step size μ should be bounded by :

$$0 < \mu < \frac{2L\sigma_x^4}{\eta + (L-1).\sigma_x^4} \qquad (2)$$

where $\eta = E(x_k^4)$ and $\sigma_x^2 = E(x_k^2)$. When the MSE convergence is guaranteed, the steady-state excess MSE $\varepsilon_{ex}(\infty)$ is:

$$\varepsilon_{ex},_{M=1(\infty)} = \frac{\mu.\varepsilon_{\min}}{2 - \frac{\mu.(\eta+(L-1).\sigma_x^4)}{L.\sigma_x^4}} \qquad (3)$$

where $\varepsilon_{\min} = E(b_k^2)$. These results were obtained for the simple case M=1. Moreover, they use unrealistic calculatory hypothesis, so, they don't permit an exact study of the convergence performances especially for correlated inputs as speech and small step size. To overcome these problems, we propose, as in [2] to use a discrete Markov chain in order to model the input data. This approach is dedicated to the digital context.

2 Exact analysis of the Mmax NLMS algorithm

In the acoustic echo cancellation, the input sequence x_k remains in a finite alphabet set $\{a_1, a_2, ..., a_d\}$. Consequently, the observation vector X_k remains also in a finite alphabet $\{W_1, W_2, ..., W_N\}$ with cardinality $N = d^L$. Since the input sequence is stationary, it can be modeled by a discrete time chain $\{\theta(k) : k \in Z^+\}$ with finite state $\{1, 2, ..., N\}$ [2] such that $X_k = W_{\theta(k)}$. The discrete time Markov chain is characterized by its probability transition matrix $P = [P_{ij}]$.

2.1 The finite alphabet approach

The equation (1) can be written in this way :

$$H_{k+1} = H_k + \mu.f(X_k).e_k \qquad (4)$$

where $f(X_k) = (0, ..., \frac{x_{k-i_1+1}}{X_k^T.X_k}, ..., \frac{x_{k-i_M+1}}{X_k^T.X_k}, ..., 0)$; $(x_{k-i_j+1})_{1 \leqslant j \leqslant M}$ are the first M maxima of $|x_{k-i+1}|$, $i = 1, 2, ..., N$. The behavior of the algorithm can be described by the evolution of deviation vector $V_k = H_k - F$. The recursion of V_k is given by:

$$V_{k+1} = (I - \mu.f(X_k).X_k^T).V_k + \mu.f(X_k).b_k \qquad (5)$$

The performance analysis are made through the evolution of $E(V_k)$ and $E(V_k V_k^T)$. The main idea is, since there is N possibilities of $W_{\theta(k)}$, to split the vector $E(V_k)$ and the matrix $E(V_k V_k^T)$ in N components defined respectively by $q_j(k) = E(V_k.1_{(\theta(k)=j)})$ and $Q_j(k) = E(V_k V_k^T.1_{(\theta(k)=j)})$. Where $1_{(.)}$ is the Dirac indicator. Since the matrix $(I - \mu.f(X_k).X_k^T)$ remains in a finite alphabet $\{A_1, A_2, ..., A_N\}$, b_k is zero mean independent of the input sequence and according to (5), we obtain $q_j(k+1) = \sum_{i=1}^{N} E(A_i.V_k.1_{\theta(k+1)=j}.1_{\theta(k)=i})$. Since A_i are constant matrices, the difficulties to analyze the algorithm are avoided, and we can deduce the recursive formulae between $q_j(k+1)$ and $q_j(k)$ without any independence assumption by:

$$q_j(k+1) = \sum_{i=1}^{N} A_i.P_{ij}.q_j(k) \qquad (6)$$

In order to write the last recursion in linear form, we introduce the useful notations $\tilde{q}(k) = \begin{bmatrix} q_1(k)) \\ \vdots \\ q_N(k)) \end{bmatrix} \in R^{NL}$. We have then

$$\tilde{q}(k+1) = \Gamma.\tilde{q}(k) \qquad (7)$$

where $\Gamma = (P^T \otimes I_L).diag(A_i)$. In similar way, we prove that :

$$Q_j(k+1) = \sum_{i=1}^{N} P_{ij}.A_i.Q_i(k).A_i^T + z_j(k) \qquad (8)$$

where

$$z_j(k) = \mu^2.E(b_k^2).\sum_{i=1}^{N} E(f(W_i).f(W_i)^T).P_{ij}.P(1_{\theta(k)=i}).$$

By introducing these notations $\tilde{Q}(k) = \begin{bmatrix} vec(Q_1(k)) \\ \vdots \\ vec(Q_N(k)) \end{bmatrix} \in R^{NL^2}$ and $\tilde{Z}(k) = \begin{bmatrix} vec(z_1(k)) \\ \vdots \\ vec(z_N(k)) \end{bmatrix}$, we obtain :

$$\tilde{Q}(k+1) = \Lambda.\tilde{Q}(k) + \tilde{Z}(k) \tag{9}$$

where $\Lambda = (P^T \otimes I_{L^2}).diag(A_i \otimes A_i)$. The equations (7) and (9) contain all relevant informations about the Mmax NLMS algorithm performances. Since matrix Γ and Λ are constants and depend only on the step size and statistical properties of the input signal, algorithm performances depend only on eigenvalues of these matrix.

2.2 Necessary condition for convergence

We prove in this paragraph that a necessary condition for the convergence of the Mmax NLMS algorithm is that the finite alphabet $\{f(W_1), f(W_2), ..., f(W_N)\}$ generates the space $\Re^L$. In fact, if this condition is not assumed, there exists a vector $Y \neq 0 \in \Re^L$ which verifies $f(W_i)^T.Y = 0, \forall\, i \in \{1,2,...,N\}$. We have then $Y^T.A_i = Y^T, \forall\, i \in \{1,2,...,N\}$. Since P is a transition matrix, we have $\sum_{j=1}^{N} P_{ij} = 1$, and consequently the vector $u = [1,...,1]$ is an eigenvector of P. We deduce that $(u^T \otimes Y^T).\Gamma = (u^T \otimes Y^T)$. Due to the fact that 1 is an eigenvalue of Γ, the algorithm diverges for all step size. To illustrate the necessity of the condition aforesaid, let's consider a particular identification scheme with $L = 5$ ($F = [0.1, 0.3, 0.95, 0.5, 0.8]^T$)and $M = 1$(number of coefficients updated at each iteration). The input is periodic and verifies $x_k = \sin(\frac{k\pi}{4} + \frac{\pi}{3})$. Then x_k belongs to the finite alphabet $\{\pm\sin(\frac{\pi}{12}), \pm\sin(\frac{\pi}{6}), \pm\sin(\frac{\pi}{3}), \pm\sin(\frac{5\pi}{12})\}$. In this case, X_k belongs to the alphabet $\{\pm W_1, \pm W_2, \pm W_3, \pm W_4\}$ and $f(X_k)$ belongs to $\{\pm f(W_1), \pm f(W_2), \pm f(W_3), \pm f(W_4)\}$ which don't generate $\Re^5$. Figure 2 represents the evolution of the deviation vector components for a step size equal to 0.5. As expected, the Mmax NLMS diverges.

If the stability condition is respected, V_k will converge to the origin since the channel has the same length as the adaptive filter.

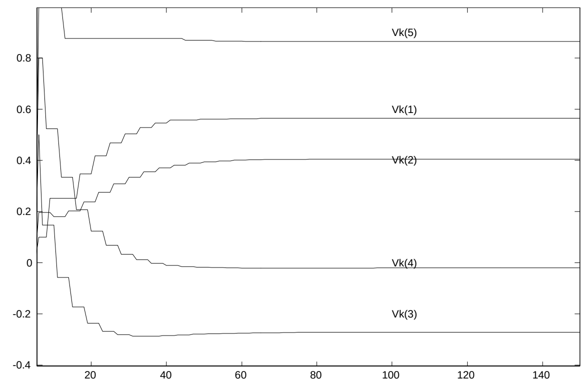

Figure 2 – The divergence of the deviation vector

2.3 Exact determination of the algorithm performances

- The critical step size is defined as the step from which the algorithm diverges. It can be defined for the mean and the mean square convergence. For the mean convergence, it is found when Γ has an eigenvalue higher than 1. So $\mu_{mc}^c = \arg(\lambda_{\max}^{\Gamma}(\mu) = 1)$. In similar way, the critical step size for the mean square convergence is defined by $\mu_{msc}^c = \arg(\lambda_{\max}^{\Lambda}(\mu) = 1)$.

- The optimal step size is defined as the step that gives the maximal speed convergence. It can be found for the mean and the mean square convergence : $\mu_{mc}^{opt} = \arg(\min(\lambda_{\max}^{\Gamma}(\mu)))$ and $\mu_{msc}^{opt} = \arg(\min(\lambda_{\max}^{\Lambda}(\mu)))$.

- After algorithm convergence, the mean square error (MSE) is defined by:

$$MSE = E(e_k^2) = E((V_k^T X_k + b_k))^2 \tag{10}$$

It is easy to prove that $vec(E(V_k^T X_k)^2) = \sum_{i=1}^{N} W_i^T \otimes W_i^T vec(Q_i(k))$.

If we note $\tilde{Q} = \lim_{x\to\infty} \tilde{Q}(k)$ and $\tilde{Z} = \lim_{x\to\infty} \tilde{Z}(k)$, then according to (12), we have $\tilde{Q} = (I - \Lambda)^{-1}.\tilde{Z}$. We have then :

$$EQM = pb + \sum_{i=1}^{N} W_i^T \otimes W_i^T.vec(Q_i) \tag{11}$$

We note that the performances analysis is performed from the matrix Γ and Λ. So, it may lead to a significant calculation when the filter order or the alphabet number is high. To illustrate the exactness of the proposed approach and the inaccuracy of the classical approach which uses the common independence hypothesis, let's consider a particular identification scheme with $L = 2$ and where x_k belongs to the finite alphabet set $\{1, -3\}$ with a transition matrix $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Referring to the equation (11), we can determine the MSE calculated by the new approach. To determine the MSE we run a Monte-Carlo simulations over 50 realizations. Figure 3 represents the evolution of the MSE versus the step size.

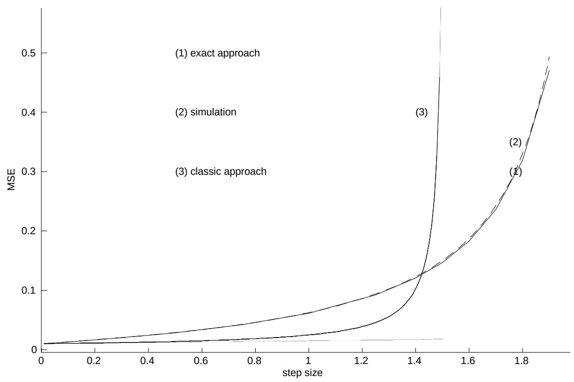

Figure 3 – MSE variation with the step size

The main contribution of the new approach is its exactness. This is illustrated by the complete agreement between simulations and theoretical results. Figure 3 shows also that the result of the classic approach is correct for the small step sizes and that it differs from the exact result for large ones.

This exact analysis allows to study the influence of the input statistics on the algorithm performances through the input transition matrix. To show for example that the critical step size depends on the correlation of the input, let us consider this particular example : the filter length is $L = 2$ and the input sequence x_k belongs to the alphabet $\{\pm 1, \pm 3\}$ with a particular transition matrix which has the following form $\begin{bmatrix} 1-\alpha & \frac{\alpha}{3} & \frac{\alpha}{3} & \frac{\alpha}{3} \\ \frac{\alpha}{3} & 1-\alpha & \frac{\alpha}{3} & \frac{\alpha}{3} \\ \alpha & \alpha & 1-3\alpha & \alpha \\ \alpha & \alpha & \alpha & 1-3\alpha \end{bmatrix}$. The correlation depends on the factor α. The variations of μ^c_{msc} versus the correlation is depicted in Figure 4.

Figure 4 shows, in this precise case of alphabet, that the step size of adaptation does not have to exceed the value 2 to ensure the convergence of the algorithm whatever is the correlation. Indeed, this value can be reached

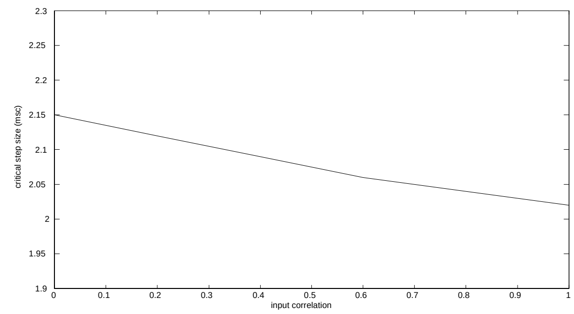

Figure 4 – μ^c_{msc} variation with the input correlation

for high correlations.

3 Conclusion

The Mmax NLMS is a way to identify long impulse responses encountered in acoustic echo cancelation. In this paper, performances of the Mmax NLMS algorithm are analyzed in the real context of the digital transmission where the input signal belongs to a finite alphabet. This exact analysis was easily done without any unrealistic hypothesis. Instead of the classical approaches, we calculate the exact values of critical and optimum step size and provide at the algorithm convergence, the exact Mean Square Error for all step size and input correlation. Moreover, it is then possible to study the influence of the input statistics on the algorithm convergence.

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