TRANSMITTER CHANNEL TRACKING FOR OPTIMAL POWER ALLOCATION

Francesc Rey, Meritxell Lamarca, Gregori Vázquez

Department of Signal Theory and Communications, Polytechnic University of Catalonia
UPC Campus Nord - Mòdul D5, c/Jordi Girona 1-3, 08034 Barcelona (Spain)
e-mail:{frey,xell,gregori}@gps.tsc.upc.es

ABSTRACT

The design of accurate equalization schemes, optimizing the transmission strategy, can be achieved if channel status information is available not only at the receiver but also at the transmitter side. Accordingly, we propose a feasible scheme to track the channel response by the transmitter based on channel prediction, becoming a suitable solution in time-varying channels. Moreover, to follow faithfully the channel variability, the receiver can estimate the channel prediction error and next update the transmitter predictor through a return link. This paper, provides an accurate study of the predictor design, and analyzes the way to minimize the amount of information exchanged through the feedback channel concerning the differential entropy of channel evolution. Furthermore, the prediction error is quantified focusing on the rate-distortion function, and it is shown that a low throughput is enough for tracking the channel coefficients even in the presence of fast time-varying channels.

1. INTRODUCTION

Knowledge of the channel status information (CSI) at the transmitter is of paramount importance for the design of reliable high speed communication systems. When the transmitter has information on the propagation channel, it is possible to design pre-equalization schemes and power allocation strategies, which optimize the use of the available resources. If transmitter precoders are combined with transmitter diversity and the channel variability dynamics, the resulting schemes become more favorable. Time-varying channels have traditionally been a handicap in communications systems, nevertheless those channel fluctuations can be exploited as an additional diversity source when the transmitter knows the channel dynamics. Furthermore, combining precoding with transmitter diversity, the total transmitted power can be optimally distributed through the antennas according to the channel profiles.

In this paper a pragmatic solution to convey channel response to the transmitter is proposed. Since channel estimation can only be carried out at the receiver, the transmitter has to obtain those coefficients through a feedback channel. However, in the presence of time-varying channels this previous solution becomes inefficient, and the CSI must be frequently updated. A more interesting design can be achieved if the transmitter is able to predict the channel as a function of the past behaviour. Hence, we propose to tackle the channel variability with a scheme containing identical linear predictors at the transmitter and the receiver, and a feedback link to assist the transmitter predictor with the prediction error, which can only be measured at the receiver. The proposed solution addresses the severe limitations of the feedback channel capacity by means of a scheme which reminds the well known DPCM transmitter, and allows tracking slow and fast varying channels fitting the predictor to the channel dynamics.

This paper is organized as follows. Next section introduces the signal and channel model, and briefly summarizes the optimal equalization scheme when the CSI is available at the transmitter [1]. Section 3.1 addresses an accurate study of linear predictors based on the Kalman filter, as a function of the normalized Doppler frequency, while section 3.2 focuses on prediction error quantification analysis according to a rate-distortion criterion, to determine the required return link throughput. Section 4 illustrates some simulation results, and finally section 5 concludes the paper.

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2. BACKGROUND

2.1. Signal and Channel Model

Let us consider a discrete-time sequence transmitted through a linear time-variant (LTV) frequency selective fading channel. The equivalent baseband received waveform can be modeled as:

$$r(t) = \sum_{k=-\infty}^{\infty} h\left(t; t - kT_s\right) x(k) + w(t), \tag{1}$$

where $x(k)$ is the discrete-time sequence corresponding to an arbitrary complex alphabet; T_s is the transmitted symbol period; $h\left(t, \tau\right)$ is the continuous-time LTV impulse response; and finally $w(t)$ models the AWGN term, with zero mean and variance σ_w^2.

Focusing on the time dispersive channel, and sampling the received signal waveform at one sample per symbol T_s, the equivalent discrete-time channel model becomes:

$$h\left(n, \tau_k\right) \triangleq h\left(nT_s, kT_s\right) = \textstyle\sum_{k=0}^{L_c-1} h\left(nT_s, kT_s\right) \delta\left(\tau - kT_s\right) \tag{2}$$

referring L_c as the number of relevant paths, τ_k as the different multipath delays and n as the discrete time evolution.

Next, a linear prediction model, to estimate the channel coefficients and tracking its time evolution, is accurately introduced. Considering stochastic assumptions over the LTV channel for each multipath delay, the channel evolution is represented as a low-pass circular complex Gaussian process, and the time variation rate is directly related to the Doppler shift. Defining the channel response vector as $\mathbf{h}(n) \triangleq \left[\begin{array}{ccc} h(n,0) & ,\ldots, & h(n,L_c-1) \end{array} \right]^T$, the LTV channel evolution can be modeled as:

$$\mathbf{h}(n) = \sum_{l=1}^{N} \mathbf{F}_l \mathbf{h}(n-l) + \mathbf{v}(n) \qquad (3)$$

where $\mathbf{v}(n)$ is a complex Gaussian process modeling the prediction error, with zero mean and correlation $\mathbf{R_{vv}} = \sigma_v^2 \mathbf{I}$, and $\mathbf{F}_i \quad i = 1,\ldots,N$ matrices contain the prediction coefficients.

When no information is available on the channel fluctuations, a simple first-order prediction assuming next channel state equals the previous one (i.e. $\mathbf{F}_1 = \mathbf{I}; N = 1$) can be considered for slow to moderate LTV channels. Nevertheless, a more accurate approach in predicting the time evolution and capturing the channel dynamics consists in modeling the LTV channel as a Nth-order autoregresive (AR) process [2]. In the latter case, knowledge of channel time correlation is required in Yule-Walker equations to fit prediction coefficients to the channel features. However, the aim of this paper is not to obtain those AR coefficients, and thus the Doppler shift for all scatters will be assumed to be known. A useful method to derive the $\mathbf{F}_i$ sequence matrices can be found in [3].

2.2. Global Equalization

In this section we will summarize the joint design of the transmitter and receiver assuming perfect knowledge of the channel response at both sides [1]. Figure 1 illustrates a joint transmitter and receiver equalization scheme which achieves optimal performance when both, transmitter and receiver, have an accurate knowledge of the CSI. Hence, a feedback link is essential to inform the pre-equalizer or encoder filter $F(n)$ about the CSI. Subsequently, next section will discuss the way to pass on the CSI to the transmitter minimizing the amount of data to be transmitted over the feedback link.

Transmitting in a burst mode, collecting M consecutive symbols as $\mathbf{x}(n) \triangleq \left[\begin{array}{ccc} x(nM) & ,\ldots, & x(nM+M-1) \end{array} \right]^T$, and adding the appropriate guard time in order to avoid inter-block interference, the received signal vector becomes: $\mathbf{r}(n) \triangleq \left[\begin{array}{ccc} r(nM') & ,\ldots, & r(nM'+M'-1) \end{array} \right]^T$ and $M' = M + L_c - 1$. Therefore, the input-output equivalent relationship can be denoted in a matrix form as follows:

$$\mathbf{r}(n) = \mathbf{H}(n)\mathbf{x}(n) + \mathbf{w}(n) \qquad (4)$$

where $\mathbf{H}(n)$ is the convolution matrix containing the channel coefficients, and $\mathbf{w}(n)$ is the sampled noise vector $\mathbf{w}(n) \triangleq \left[\begin{array}{ccc} w(nM') & ,\ldots, & w(nM'+M'-1) \end{array} \right]^T$. We will assume that the channel remains constant within a burst, and so channel coefficients need to be estimated only once per block.

When linear transformations $\mathbf{F}(n)$ and $\mathbf{G}(n)$ are applied at the transmitter and receiver, estimated symbols can be expressed as:

$$\hat{\mathbf{s}}(\mathbf{n}) = \mathbf{G}(n)\mathbf{H}(n)\mathbf{F}(n)\mathbf{s}(n) + \mathbf{G}(n)\mathbf{w}(n) \qquad (5)$$

Optimal values of $\mathbf{F}(n)$ and $\mathbf{G}(n)$ can be derived under the hypothesis of i.i.d symbols, constrained transmitted power and perfect CSI knowledge at the transmitter side. By means of the singular value decomposition (SVD) of the channel matrix, $\mathbf{H}(n) = \mathbf{U}(n)\mathbf{\Lambda}(n)\mathbf{V}^H(n)$, those optimal expressions are:

$$\begin{array}{rcl} \mathbf{F}_{opt} & = & \mathbf{V}(n)\mathbf{\Phi}(n) \\ \mathbf{G}_{opt} & = & \mathbf{\Gamma}(n)\mathbf{\Lambda}^{-1}(n)\mathbf{U}^H(n) \end{array} \qquad (6)$$

where $\mathbf{\Phi}(n)$ and $\mathbf{\Gamma}(n)$ are diagonal matrices allowing different design rules. Moreover, according to (6) the time dispersive channel expression (4) can be transformed into a set of M parallel independent flat-fading channels. Different criteria can be found to design $\mathbf{\Phi}(n)$ and $\mathbf{\Gamma}(n)$ matrices: minimum mean square error with zero-forcing constraint [1], minimum mean square error with average power constraint [1],[4], and maximum information rate with average power constraint [5].

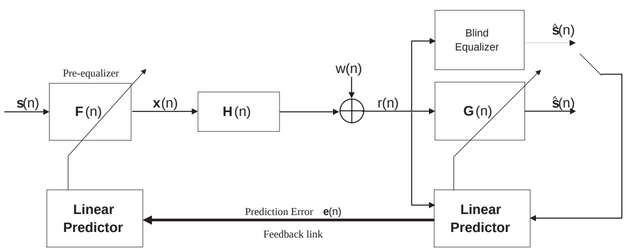

Figure 1: Global Equalization Scheme

3. TRANSMITTER CHANNEL TRACKING

As shown previously, to design optimal $\mathbf{F}(n)$ and $\mathbf{G}(n)$ matrices, it is of paramount importance not only to identify the channel response at the receiver, but also to inform about its status to the transmitter. Moreover, in LTV channels, because of its time evolution, information at both sides must be continuously updated. In this section a scheme is proposed to update the CSI at the transmitter while minimizing the amount of data to be transmitted through the feedback channel.

Previous challenges, as shown in figure 1, are resolved using two identical predictors at the transmitted and receiver sides, and just transmitting the error prediction over the feedback channel. We propose to assist the transmitter with the prediction error instead of transmitting the channel coefficients themselves. As will be shown in section 3.2, the differential entropy of the error prediction is much lower than the differential entropy of the channel coefficients, and therefore the whole number of required bits is accordingly reduced. Next, Kalman filter is introduced, as minimum variance linear predictor [6], to track the channel evolution. Afterwards, the way to to transfer the CSI to the transmitter is analyzed. Finally, it will be shown how to quantize the prediction error and employ it to compensate the transmitter linear prediction.

3.1. Linear prediction by Kalman Filter

Rewriting (3) in matrix notation, and defining $\mathcal{H}(n) \triangleq \left[\begin{array}{cccc} \mathbf{h}(n) & \mathbf{h}(n-1) & \cdots & \mathbf{h}(n-N+1) \end{array}\right]^T$, time evolution of vector $\mathcal{H}(n)$ can be written as:

$$\mathcal{H}(n) = \left[\begin{array}{cccc} \mathbf{F}_1 & \mathbf{F}_2 & \cdots & \mathbf{F}_N \\ I & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & I & \mathbf{0} \end{array}\right] \mathcal{H}(n-1) + \left[\begin{array}{c} \mathbf{v}(n) \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{array}\right] \quad (7)$$

This expression can be regarded as the Kalman state equation, having $\mathcal{H}(n)$ as the Kalman state vector, while the measurement equation can be obtained from the channel output (4) as:

$$\mathbf{r}(n) = \left[\begin{array}{cccc} \mathbf{X}(n) & \mathbf{0}^T & \cdots & \mathbf{0}^T \end{array}\right] \mathcal{H}(n) + \mathbf{w}(n). \quad (8)$$

The measurement matrix $\mathbf{X}(n)$ is a Sylvester matrix whose columns are scrolled versions of vector $\mathbf{x}(n)$. Notice that the transmitted symbols $x(n)$ are needed to build the measurement matrix, so the information data $s(n)$ must be known at the receiver. In tracking, this information is available at the receiver because $\mathbf{F}(n)$ and $\mathbf{G}(n)$ equalize the channel. However, during acquisition $\hat{s}(n)$ is not reliable, and an auxiliary deterministic blind equalizer (e.g. [7]) or a training sequence must be introduced.

Equations (7) and (8) define the Kalman state space representation. Thus, the minimum variance estimator for the channel vector $\mathbf{h}(n)$, conditioned to the channel output observation $\mathbf{r}(n)$ is given by [6]:

$$\hat{\mathbf{h}}(n) = \hat{\mathbf{h}}(n/n-1) + \underbrace{\mathbf{K}(n)\left(r(n) - \mathbf{X}(n)\hat{\mathbf{h}}(n/n-1)\right)}_{\mathbf{e}(n)} \quad (9)$$

where $\hat{\mathbf{h}}(n/n-1)$ is the channel prediction according to the Kalman state transition matrix (7), and $\mathbf{K}(n)$ is the Kalman gain matrix.

Notice in (9) that the deviation of the predicted measurement from the actual measurement $\mathbf{r}(n)$, i.e. the innovation, is used to define the prediction error, which compensates the differences between the real channel evolution and the predicted model. This prediction error can be only measured at the receiver, because the observation vector is required. Moreover, note that $\mathbf{e}(n)$ is the minimum required information to track the LTV channel by the transmitter. Therefore, this prediction error information, whose dynamics range is much lower than the channel coefficient dynamics, should be transferred to the transmitter across the feedback channel.

3.2. Prediction Error Quantization

The optimal way to quantize and encode the prediction error vector in a finite number of bits according to information theory principles is next analyzed. First, it is essential to identify the statistics of the aforementioned vector defined in (9). As a consequence of the central limit theorem, real and imaginary elements in $\mathbf{e}(n)$ are approximately Gaussian random processes. Besides, after some manipulations, and considering the steady-state Kalman filter, it can be shown that the mean and variance of the error vector are:

$$\begin{aligned} E\{\mathbf{e}(n)\} &= \mathbf{0} \quad (10) \\ E\left\{\mathbf{e}(n)\mathbf{e}^H(n)\right\} &= \left(\mathbf{F}_1\mathbf{F}_1^H - \mathbf{I}\right)\bar{\mathbf{\Sigma}} + \mathbf{R}_{\mathbf{vv}} \end{aligned}$$

where $\bar{\mathbf{\Sigma}}$ denotes the steady-state covariance matrix of the channel estimate.

Channel coefficients variance $\sigma_{\mathbf{h}}^2$ and the prediction error variance $\sigma_{\mathbf{e}}^2$ always verify $\sigma_{\mathbf{h}}^2 \geq \sigma_{\mathbf{e}}^2$, and most of the times $\sigma_{\mathbf{h}}^2 \gg \sigma_{\mathbf{e}}^2$. Next, we will deduce that lower variance means a minor number of bits to transfer the same information. The $\mathbf{e}(n)$ vector elements require an infinite number of bits to be encoded without loosing information. Therefore, when a finite number of bits are employed, a quantization distortion is always unavoidable. We propose to minimize such difference making use of the rate-distortion function, which gives the minimum number of bits required to quantize a source with distortion less or equal to D. For a zero mean Gaussian source, with variance σ^2, the rate-distortion function is given by [8]:

$$R(D) = \left\{ \begin{array}{cc} \frac{1}{2}\log\frac{\sigma^2}{D} & 0 \leq D \leq \sigma^2 \\ 0 & D > \sigma^2 \end{array} \right. \quad (11)$$

where D is defined as the square-error distortion: $D = (x-\hat{x})^2$. Note that the rate-distortion function is a theoretical bound in the sense that it can only be achieved by increasing the encoding-decoding complexity.

In our application, the prediction error is a complex vector of length L_c, hence the total number of real Gaussian variables to encode is $2L_c$. Vector quantization is the optimal solution to attain the performance given by the rate-distortion function (11). Nevertheless, due to the excessive complexity of vector quantization, a suboptimum solution, quantizing each vector element separately, will be used. As $\mathbf{e}(n)$ elements are approximately Gaussian random processes, an optimum non-uniform quantifier for Gaussian sources designed by Max [9] will be applied.

4. SIMULATION RESULTS

Computer simulation were carried out to illustrate the performance of the proposed algorithm. A block transmission mode, QPSK modulation gathering $M = 32$ symbol per block, a symbol rate of $r = 0.72$Mbauds (1.44Mb/s), and a random multipath Rayleigh channel with $L_c = 5$ uncorrelated scatters (US assumption), has been considered. Two LTV channels were simulated, a *slow LTV channel* with a normalized Doppler frequency $\bar{f}_m = 0.016$ (carrier frequency $f_c = 9$GHz, mobile speed $v = 40kph.$), and a *fast LTV channel* with $\bar{f}_m = 0.05$ ($f_c = 9$GHz, mobile speed $v = 120kph.$). The normalized Doppler frequency is defined as:

$$\bar{f}_m = \frac{f_c v}{c}\frac{M + L_c - 1}{r}. \quad (12)$$

Figure 2 plots the LTV prediction error σ_e^2 versus the normalized Doppler shift frequency. Linear prediction without Doppler information $\mathbf{F} = \mathbf{I}$ (dashed line), and linear AR(1)-AR(2) models, assuming perfect knowledge on Doppler for all scatters, were considered for different quantization bits. Several conclusions can be drawn from figure 2. First, it can be seen that identity matrix (i.e. no Doppler information) and AR(1) predictors have similar performance, and only a slight difference appears in fast time-varying channels. Moreover, a second important result is the threshold for the prediction order. As it can be concluded, a 1st order predictor provides a low prediction error in low Doppler scenarios, while this error grows as Doppler increases requiring the introduction of 2nd order

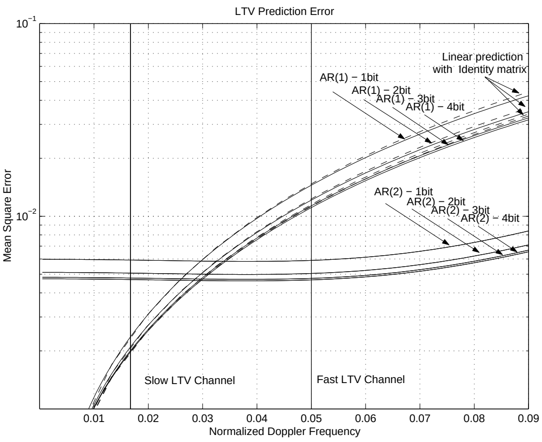

Figure 2: Channel Prediction Mean-Square Error. Identity/AR(1)/AR(2). Bits:1,2,3,4

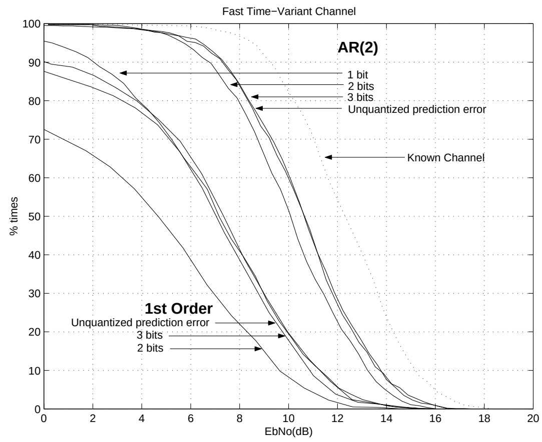

Figure 3: EbNo output distribution. EbNo input 14dB. Diversity B=2. Fast LTV Channel. Identity/AR(2). Bits:1,2,3,Unquantized

predictors. Finally, illustrates the dependency of the prediction error with the number of quantization bits. Notice that $Q = 3$ bits is a reasonable solution as verified in next simulation.

Figure 3 compares the equalization performance of the proposed algorithm when no information is available on the channel variability (assuming next channel state equals the previous one), and modeling the LTV channel as an AR(2) process (assuming AR coefficients are known). A MMSE criterion with average power constraint was considered to design $\mathbf{F}_{opt}$ and $\mathbf{G}_{opt}$ matrices, and a MISO scheme with diversity order $B = 2$ was reproduced. Simulations were carried out for different number of quantization bits, and a fast LTV channel, as the worst choice, was considered. It is not surprising that the AR(2) model outperforms the first order solution. As illustrated in figure 2, AR(2) predictor provides a lower prediction error, and consequently residual intersymbol interference is reduced. Furthermore, with $Q = 3$ quantization bits, the solution is in both cases close to the unquantized solution. Finally, we point out that although AR(2) outperforms 1st order, prediction coefficients are required, and hence Doppler shifts for all paths have to be estimated. Consequently, the 1st order predictor still remains interesting when straightforward designs are required.

5. CONCLUSIONS

This paper has introduced a suitable scheme to assist the transmitter with the CSI in optimal transmitter-receiver equalization designs. The proposed solution is based on identical linear predictors at both sides, and a feedback channel to aid the transmitter with the prediction error computed at the receiver. Thus, it is possible to track fast fading channels with a low rate feedback link. The paper evaluates the performance of different channel predictors and derives results as a function of the channel variability according to the normalized Doppler frequency. Simulations have shown 1st order predictors are good in slow LTV channels, while 2nd order predictors are required in fast LTV channels at the expense of higher complexity. Moreover, considering the severe limitations of the feedback channel capacity, the paper addresses the return link required throughput, analyzing the prediction error quantization.

6. REFERENCES

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