CHANNEL-INDEPENDENT NON-DATA AIDED SYNCHRONIZATION OF GENERALIZED MULTIUSER OFDM

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ABSTRACT

We develop and analyze timing and carrier frequency offset synchronization algorithms for generalized asynchronous and quasi-synchronous orthogonal frequency division multiple access systems using null subcarriers and subcarrier hopping. We derive an approximate analytic expression for the variance of the frequency offset estimators as a function of the number of active users and the SNR and show that the performance of our algorithms is asymptotically independent of the channel zero locations for quasi- synchronous systems. Finally, we validate our theoretical expressions with simulations.

1. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) has been recently proposed for cable TV [8], broadband radio access networks (BRAN) [2], and multiuser communications through satellite links [13]. One of the most critical aspects of OFDMA is synchronization. It is well known, that single-user or broadcast systems based on Orthogonal Frequency Division Multiplexing (OFDM) are highly sensitive to time and/or frequency offsets which cause intersymbol interference (ISI) [5], [7]. The problem becomes more challenging for OFDMA, especially in the uplink channel, where the users are all potentially asynchronous. Several works have addressed the synchronization of OFDM systems [5], [10], [6], but only recently Van de Beek *et al.* have considered the synchronization of multi-user OFDM (a.k.a. OFDMA) systems [11], by generalizing the method of [10]. The algorithm proposed in [10] was shown to attain maximum likelihood optimality under certain assumptions, namely Gaussianity of the OFDMA samples and whiteness of the received signal spectrum. The variance of the frequency offset estimator in [10] is indeed pretty close to the theoretical limit for ideal channels, but it exhibits a floor in the presence of frequency selective channels. Furthermore, the whiteness assumption is not as well justified in OFDMA as it is for OFDM, because the spectrum utilized by an OFDMA system depends on the number of active users.

The main *objective* of this paper is to develop non-data aided synchronizers for both the downlink and the uplink channels with performance that (unlike existing alternatives [10]) is *independent of the channel zero locations*. The resulting algorithms exploit the null guard intervals present when the system is not fully loaded and rely on subcarrier-hopping to render the performance of different users uniform. Null (or virtual) subcarriers were also used for synchronization of single-user OFDM systems in [4, 9]. However, unlike [4, 9], the estimators herein are suitable for multiuser systems, they have low implementation complexity, and remain consistent regardless of the underlying channels. We describe our method by building on the generalized (G) OFDMA scheme of [3, 12]. However, applicability of the proposed synchronizers is not restricted to linearly precoded block transmissions. We also carry out theoretical performance analysis of our frequency offset estimator and validate our results with simulations (the detailed derivations are omitted due to lack of space but are available from [1]).

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2. OFDMA MODULATION STRATEGIES

We choose different strategies for assigning symbols to subcarriers in the uplink and downlink channels.

2.1. Asynchronous Transceivers

In the *downlink* channel, where all user signals are synchronous by construction, we assign a block of symbols from each user to a set of frequencies (subcarriers) which are as far apart as possible, so as to maximize the frequency diversity gain. As a consequence, the spectra of different users are interleaved. Specifically, we multiplex the data to be transmitted in blocks. Each block has length $N+L$, where $N = JM$ denotes the total number of (information bearing and null) subcarriers available to the system; J is the number of symbols transmitted per user's block; M is the maximum number of active users which can be accommodated by the available bandwidth; and L is the length of the cyclic prefix (guard interval).

In accordance to realistic scenarios, we suppose that the system is not fully loaded; i.e., the number of active users $M_A < M$. The p-th block transmitted from the base station (BS) to each mobile unit (MU) has entries $x(p;n), n \in [-L, N-1]$, given by:

$$x(p;n) = \sum_{m=0}^{M_A-1} \sum_{l=0}^{J-1} u_m(p;l) e^{j\frac{2\pi}{N}(m+lM+i_p)n}, \qquad (1)$$

where $u_m(p;l)$ is the l-th (possibly precoded) symbol of the m-th user within the p-th block, and i_p is a frequency hopping index. In general, we assign $J > 1$ subcarriers per user to gain diversity and guarantee symbol recovery regardless of the channel zeros thanks to redundant precoding [3, 12]; however, the synchronization method works also for $J = 1$. Throughout this paper, the first argument (e.g., p) will be the block index, while the second one (e.g., l) will denote the indices within the block. Subscripts will index users.

The J subcarriers allocated to the m-th user in the p-th block are: $(m+i_p, m+M+i_p, \ldots, m+(J-1)M+i_p)/NT$, where $1/T$ is the symbol rate, and they shift from block to block. Let $c_k(t)$, τ_k and f_k denote the equivalent baseband channel and the residual time and frequency offsets[1] between the BS and the k-th MU.

The received sequence from the k-th user in the absence of noise is [1]

$$y_k(q;i) = \sum_{p=-\infty}^{\infty} \sum_{n=-L}^{N-1} \sum_{m=0}^{M_A-1} \sum_{l=0}^{J-1} u_m(p;l) e^{j2\pi\nu_k[q(N+L)+i-\theta_k]} \cdot e^{j\frac{2\pi}{N}[(m+lM+i_p)n]} c_k[((q-p)(N+L)+(i-n)-\theta_k)T] \quad (2)$$

with $i = 0, \ldots, N-1$, $\nu_k := f_k T$ and $\theta_k := \tau_k/T$.

For the *uplink*, simplicity of the synchronization process motivates maximum separation (as opposed to interleaving) of different users' spectra. For this reason, we assign each user's block of symbols to a set of adjacent subcarriers, but we separate the spectra of different users' signals as much as possible. Specifically, the p-th block with entries $n \in [-L, N-1]$ transmitted from the m-th MU is mapped to an OFDMA block as follows:

$$x_m(p;n) = \sum_{l=0}^{J-1} u_m(p;l) e^{j\frac{2\pi}{N}(l+mJ_A+i_p)n}, \quad (3)$$

where $u_m(p;l)$ is the l-th (possibly precoded) symbol of the m-th user's p-th block; i_p is as before the subcarrier hopping index; the length L of the cyclic prefix is assumed to be at least equal to the greatest channel order; $J_A = J + J_N$, where $J_N := \lfloor J(M/M_A - 1) \rfloor$. According to (3), the set of subcarriers allocated to the m-th user is $(mJ_A + i_p, mJ_A + 1 + i_p, \ldots, mJ_A + J - 1 + i_p)/NT$ and it shifts from block to block. Therefore, there is a frequency guard interval separating two adjacent users equal to J_N/NT.

The aggregate signal received at the BS from all M_A active users is:

$$y(q;i) = \sum_{p=-\infty}^{\infty} \sum_{n=-L}^{N-1} \sum_{m=0}^{M_A-1} \sum_{l=0}^{J-1} u_m(p;l) e^{j2\pi\nu_m[q(N+L)+i-\theta_m]} \cdot e^{j\frac{2\pi}{N}[(l+mJ_A+i_p)n]} c_m[((q-p)(N+L)+i-n-\theta_m)T], \quad (4)$$

where $c_m(t)$ is the impulse response of the channel between the m-th MU and the BS.

2.2. Quasi-Synchronous Transceivers

In quasi-synchronous systems, time offsets are kept small because MUs are approximately aligned with the BS's pilot signal. One may thus incorporate the small time offsets in the unknown channel responses. This entails a certain efficiency loss because guard intervals become longer than necessary, but it simplifies considerably the synchronization process.

The noise-free samples at the k-th MU's receive-filter output are now [1]

$$y_k(q;i) = e^{j2\pi\nu_k(q(N+L)+i)} \sum_{m=0}^{M_A-1} \sum_{l=0}^{J-1} \tilde{u}_{m,k}(q;l) e^{j\frac{2\pi}{N}(m+lM+i_q)i} \quad (5)$$

where $\tilde{u}_{m,k}(q;l) := u_m(q;l)C_k(m+lM+i_q)$ and $C_k(m) := \sum_{l=0}^{L} c_k(l) e^{-j\frac{2\pi}{N}ml}$, with $m \in [0, N-1]$, is the k-th channel transfer function.

Similarly for the uplink, the received noise-free samples at the BS after cyclic prefix removal are given by:

$$y(q;i) = \sum_{m=0}^{M_A-1} e^{j2\pi\nu_m(q(N+L)+i)} \sum_{l=0}^{J-1} \tilde{u}_m(q;l) e^{j\frac{2\pi}{N}[(l+mJ_A+i_q)i]}, \quad (6)$$

where $\tilde{u}_m(q;l) := u_m(q;l)C_m(l+mJ_A+i_q)$.

3. SYNCHRONIZATION IN OFDMA

The basic idea behind our synchronization algorithm exploits the redundancy present in the received signal when the system is not fully loaded ($M_A < M$). Specifically, with perfect time and frequency synchronization, the q-th received $N \times 1$ vector $\boldsymbol{y}(q)$ with entries as in (2),(4),(5) or (6) can be expressed as $\boldsymbol{y}(q) = \boldsymbol{A}\boldsymbol{u}(q)$, where $\boldsymbol{u}(q)$ is the $M_A J \times 1$ vector obtained by stacking the $J \times 1$ symbol vectors $\boldsymbol{u}_m(q) := [u_m(q;0), \ldots, u_m(q;J-1)]^T$, for $m = 0, \ldots, M_A - 1$, corresponding to the M_A active users, and $\boldsymbol{A}$ is an $N \times JM_A$ tall mixing matrix. If we stack a sufficiently large number of vectors $\boldsymbol{y}(q)$, with $q \in [0, N_b - 1]$, so that the $M_A J \times N_b$ matrix $\boldsymbol{U} := [\boldsymbol{u}(0), \ldots, \boldsymbol{u}(N_b - 1)]$ has full row rank, the corresponding $MJ \times N_b$ matrix $\boldsymbol{Y} := [\boldsymbol{y}(0), \ldots, \boldsymbol{y}(N_b - 1)]$ will have rank$(\boldsymbol{Y}) \leq M_A J$, and thus it will be rank deficient. This implies that $\boldsymbol{Y}$ will have a nullspace of dimensionality $\nu \geq (M - M_A)J$. Thus, if we project the received signal onto the nullspace of $\boldsymbol{Y}$ we should obtain a null vector under perfect synchronization. Therefore, the norm of the projected vector is a quantitative measure of our lack of synchronization and we may search for the time and frequency offsets which minimize this norm.

The main problems with the implementation of this algorithm are: i)we do not know a-priori the nullity ν of $\boldsymbol{Y}$ and it might not be easy to determine it from a finite set of data in the presence of noise; ii)we are not able to guarantee a channel-independent performance; and iii)the nullspace has to be estimated from the received data which requires a computationally complex SVD. These challenges go against our goal of a low-complexity method with channel-independent performance. Interestingly, if we specialize the aforementioned idea to OFDMA, we are able to get rid of all previous limitations. In fact, for OFDMA systems that are not fully loaded, the nullspace is spanned by the orthogonal sub-carriers that are not utilized for information transmission (virtual sub-carriers), and are known a-priori. The projector onto the nullspace of $\boldsymbol{Y}$ is thus simply the FFT of the received sequence, evaluated at the frequencies of the virtual sub-carriers.

Relying on the virtual (null) subcarriers, our approach is reminiscent of the methods in [4, 9] for single-user OFDM. However, unlike [4], the estimators herein are suitable for multiuser systems, they have low implementation complexity, and most important they exhibit performance independent of the underlying channels.

[1] We define the residual offsets as the differences between the time and carrier offsets $\bar{\tau}_k$ and $\bar{f}_k$ and our guesses $\hat{\tau}_k$ and $\hat{f}_k$: $\tau_k := \bar{\tau}_k - \hat{\tau}_k$ and $f_k := \bar{f}_k - \hat{f}_k$.

3.1. Cost Functions for Synchronization

Motivated by the previous arguments, we introduce first the cost function for the downstream received at the k-th MU

$$\mathcal{J}(\nu_k;\theta_k) := \frac{1}{N_b}\sum_{q=0}^{N_b-1}\sum_{m'=M_A}^{M-1}\sum_{l'=0}^{J-1}\left|Y_k(q;m'+l'M+i_q)\right|^2, \quad (7)$$

where

$$Y_k(q;m'+l'M+i_q) := \frac{1}{N}\sum_{i=0}^{N-1} y_k(q;i)e^{-j2\frac{\pi}{N}(m'+l'M+i_q)i}, \quad (8)$$

with $y_k(q;i)$, given by (2). Based on (7) and (8), our estimates for timing and frequency offsets of the k-th user are found as:

$$(\hat{\nu}_k, \hat{\theta}_k) = \arg\min_{\nu,\theta} \mathcal{J}(\nu_k;\theta_k). \quad (9)$$

Similarly, introducing the parameters $\boldsymbol{\nu} := (\nu_0, \ldots, \nu_{M_A-1})$ and $\boldsymbol{\theta} := (\theta_0, \ldots, \theta_{M_A-1})$, we define the cost function for the uplink as:

$$\mathcal{J}(\boldsymbol{\nu};\boldsymbol{\theta}) := \sum_{q=0}^{N_b-1}\sum_{m'=0}^{M_A-1}\sum_{l'=J}^{J_A-1}\left|Y(q;l'+m'J_A+i_q)\right|^2, \quad (10)$$

where

$$Y(q;m'+l'M+i_q) := \sum_{i=0}^{N-1} y(q;i)e^{-j\frac{2\pi}{N}[(l'+m'J_A+i_q)i]}, \quad (11)$$

with $y(q;i)$ given by (6).

Similar to (9), our estimates for the *uplink* are found as:

$$(\hat{\boldsymbol{\nu}}, \hat{\boldsymbol{\theta}}) = \arg\min_{\boldsymbol{\nu},\boldsymbol{\theta}} \mathcal{J}(\boldsymbol{\nu},\boldsymbol{\theta}). \quad (12)$$

In *quasi-synchronous* systems, the time offsets are considered as part of the channel impulse responses, so that the only unknowns are the frequency offsets. We assume that: i) the symbols $u_m(q;i)$ are uncorrelated, with zero mean and variance σ_u^2; ii) the noise is white, with zero mean and variance σ_v^2, and uncorrelated from $u_m(q;i)$; and iii) the hopping indices i_q, with $q = 0, \ldots, N-1$, span the range $[0, N-1]$.

Under i)-iii), it is possible to express the *downlink* cost function (5) in closed form, for $N_b \to \infty$, as (see [1] for details):

$$J(\nu_k) = (\sigma_u^2 C_k^2/N)\sum_{m=1}^{M-1}\sum_{l=-J+1}^{J-1}(J-|l|)g_N^2(m+lM-\nu_k N)$$
$$\cdot \min(m, M-m, M_A, M-M_A) + \sigma_v^2 J(M-M_A)/N \quad (13)$$

where $C_k^2 := (1/N)\sum_{q=0}^{N-1}|C_k(q)|^2$ and $g_N(x) := \sin(\pi x)/\sin(\pi x/N)$.

Likewise, for *uplink* quasi-synchronous transmissions we find that (10) tends asymptotically, as $N_B \to \infty$, to [1]

$$\mathcal{J}(\boldsymbol{\nu}) := (\sigma_u^2/N^2)\sum_{m=0}^{M_A-1} C_m^2 \sum_{m'=0}^{M_A-1}\sum_{l'=J}^{J_A-1}\sum_{l=0}^{J-1}$$
$$g_N^2[l-l'+(m-m')J_A+\nu_m N] + \sigma_v^2 M_A(J_A-J)/N. \quad (14)$$

From (13) and (14) we can draw two important remarks.

Remark 1: Thanks to subcarrier hopping, the cost functions depend on the channel impulse responses only through the coefficient C_m^2, so that the synchronization parameter estimators minimizing the cost functions (13) or (14) have performance *independent of the channel zero locations*.

Remark 2: As the number of blocks tends to infinity, the additive white noise simply adds a pedestal to the cost functions, so that the timing and frequency offset estimators based on (9) and (12) are also *consistent*.

To gain further insight, it is useful to analyze $\mathcal{J}(\boldsymbol{\nu})$ for small residual offsets. Taking the first order Taylor series expansion of (14) around the origin, we obtain

$$\mathcal{J}(\boldsymbol{\nu}) \approx \sigma_u^2 \sum_{m=0}^{M_A-1}\alpha_m C_m^2 \nu_m^2 + \frac{\sigma_v^2}{N}M_A(J_A-J), \quad (15)$$

where the coefficients α_m are independent of ν_m. Eq. (15) reveals that $\mathcal{J}(\boldsymbol{\nu})$ is basically a paraboloid in the neighborhood of the origin. Thus, if properly initialized, a simple iterative algorithm searching for one frequency offset at a time, yields all the unknown offsets ν_m sequentially. The major challenge in implementing such an algorithm is that we need a feedback channel between BS and MU's to exchange synchronization information, as in [11]. Nevertheless, if the guard intervals are sufficiently large, we can attempt the synchronization even without the feedback channel (see also Example 2 in Section 4).

3.2. Synchronization Algorithm

In the following, we describe a block-iterative procedure to estimate the frequency offset f_0 of, say, user 0, in the downlink. We start with an initial guess $\hat{f}_0(0)$ at time 0 and at the kth iteration we multiply the received sequence by $\exp(-j2\pi\hat{f}_0(k)i)$. We then estimate the cost function (7) and its gradient with respect to $\hat{f}_0(k)$, by taking averages over a finite number of blocks N_b (taken as an integer multiple of N). Depending on the value assumed by the cost function and its gradient, we decide to exit from the loop if both fall below suitable thresholds; otherwise, we update $\hat{f}_0(k)$, and then $\hat{\nu}_0(k)$, according to the steepest descent method:

$$\hat{\nu}_0(k+1) = \hat{\nu}_0(k) - \mu\frac{\partial\mathcal{J}(\hat{\nu}_0(k))}{\partial\hat{\nu}_0(k)}, \quad (16)$$

where μ is the step size. The range of μ guaranteeing convergence in (16) can be obtained after taking the second order series expansion of $d\mathcal{J}(\nu_0(k))/d\nu_0(k)$ in the neighborhood of $\nu_0(k) = 0$. We then have $d\mathcal{J}(\nu_0(k))/d\nu_0(k) \approx \lambda\nu_0(k)$ with [1]

$$\lambda = -\frac{4\pi^2\sigma_u^2 C_2}{N^2}\sum_{i=-(N-1)}^{N-1}(N-|i|)i^2 g_{M_A}(i/N)$$
$$\cdot \quad g_{M-M_A}(i/N)g_J^2(i/J). \quad (17)$$

From (16), we find that $\hat{\nu}_0(k+1) = \hat{\nu}_0(k)(1-\mu\lambda)$, and hence

$$\nu_0(k+1) = \nu_0(0)(1-\mu\lambda)^{k+1}, \quad (18)$$

so that our iterative frequency offset estimator (16) converges if $|1-\mu\lambda| < 1$, or equivalently, if $0 < \mu < 2/\lambda$.

In the uplink channel, thanks to the particular structure of the signal in (3), we filter the received signal so as to estimate the frequency offsets pertaining to different users separately. Of course, the filtering is not perfect, so that multiuser interference inevitably appears. Nevertheless, at least for small residual offsets, when estimating say the k-th offset, the contributions from the other users act as additive noise [c.f.(15)].

4. PERFORMANCE ANALYSIS

In this section we report the variance of the frequency offset estimator for the quasi-synchronous downlink channel. It is approximately valid at high SNR, but for any finite number of observation data (see [1] for the derivations). The results have been derived for the downlink channel, but they can be considered as a lower bound for the uplink channel as well. Using a small perturbation analysis, we prove that the variance of the frequency offset estimate is [1]

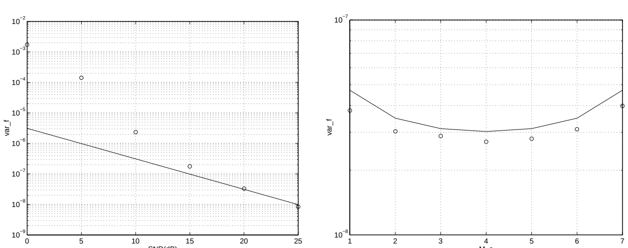
$$\sigma_\nu^2 = \frac{N}{2\pi^2 SNR} \frac{\sum_{n,p,q=0}^{M-1} (n-p)(q-p)A(n,p,q)}{\sum_{n,p=0}^{M-1} (n-p)^2 B(n,p)}, \quad (19)$$

where $A(n,p,q) := g_{M_A}((n-q)/M)\, g_{M_A}((n-p)/M)\, g_{M_A}((p-q)/M)$, and $B(n,p) := g_{M_A}^2((n-p)/M)$.

We now present some simulations that validate (19) and assess the goodness of the estimators.

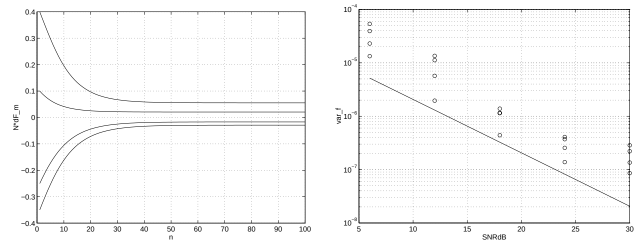
Example 1(`Downlink with time offsets`): In Fig. 1 we show the frequency estimation variance in the downlink. The system is asynchronous and we estimate time and frequency offsets in cascade (time offset is estimated first). We observe very good agreement between theory and simulation at high SNR, where there are practically no errors in the time offset estimation.

Example 2(`Quasi-synchronous uplink`): Fig. 2a shows that all frequency offsets converge to very small values in about 20 iterations, whereas Fig. 2b depicts the estimation variance vs. SNR, for each user. The solid line is the single-user benchmark. As expected, the estimated variance corresponds to its single-user counterpart with excess noise. At high SNR, there is floor due to multiuser interference, which can be lowered by increasing the guards, if possible, or by sending frequency information back to the MUs to enable updating of their time and frequency references.



(a) (b)

Fig. 1. Theory (solid line) and simulation (∘): (a) σ_ν^2 vs. SNR, $M = 8$, $M_A = 3$, $L = 2$, $J = 8$; (b) σ_ν^2 vs. M_A, $M = 8$, $L = 2$, $J = 8$, $SNR = 20$ dB.



(a) (b)

Fig. 2. Theory (solid line) and simulations (∘):(a) frequency offset of each user vs. iteration index; (b) variance of all frequency estimators vs. SNR; $M = 8$, $M_A = 4$, $L = 2$, $J = 4$; $J_A = 16$, $SNR = 20$ dB.

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