EFFECT OF IMPERFECT CHANNEL ESTIMATION ON SYNCHRONOUS MULTI-RATE DS/CDMA SYSTEMS WITH HIGH SPREADING FACTORS

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ABSTRACT

This paper analyzes the influence of channel estimation errors on the performance of linear multi-user receivers. Assuming randomized codes and asymptotically high spreading factors and noise power, we show that the performance of the decorrelating and the minimum mean squared error (MMSE) receivers tend to the same limit in terms of signal to noise ratio and bit error rate (but not in terms of mean squared error). Using these results and assuming Gaussian-distributed channel estimators, we derive two simple approximations to the bit error rate and compare them with the actual values via simulation.

1. INTRODUCTION AND SIGNAL MODEL

The increasing interest in the implementation of DS/CDMA mobile communication systems has motivated the study of the performance of these systems under realistic conditions. In particular, the effect of imperfect parameter estimation on the performance of linear multi-user receivers has been considered in several recent papers. For instance, the influence of imperfect timing along with phase and frequency errors on linear multi-user detectors was discussed in [1] and [2], whereas the influence of a mismatched channel impulse estimate was investigated in [3] and [4]. Particularly, in [4] the authors assumed randomized spreading sequences and studied the performance of linear multi-user receivers when the number of users of the system increases without bound. Here, we take a similar approach to the problem: assuming randomly generated spreading sequences, the performance of linear multi-user receivers is analyzed in terms of bit error probability under the approximation of high spreading factors and noise power.

Let us consider a synchronous multi-rate DS/CDMA system with simultaneous multi-code and variable spreading factor transmission. Note that this is the basic configuration of the wideband CDMA (WCDMA) system proposed for the Frequency Division Duplex (FDD) mode of UTRA (UMTS Terrestrial Radio Access). The signal model proposed here could correspond either to an uplink communication in a single user scenario where the user of interest transmits with several codes or to a downlink communication where different users (with different bit rates) are being served. The following parameters characterize the multi-rate structure of the signal:

N_c	Period of the spreading code sequences (in number of chips)
Q	Number of codes transmitted in parallel
SF_q	Spreading factor associated with the qth code
$N_s(q)$	Number of symbols transmitted on the qth code within N_c chips.

We assume that at the basestation the signal is synchronously sampled at the chip rate (modulation with no excess bandwidth) and that symbol detection is made in observation windows of MN_c chips. Stacking MN_c samples of the received signal into a column vector $\mathbf{x} \in \mathbb{C}^{MN_c \times 1}$ we can describe the received signal as

$$\mathbf{x} = \mathbf{Gs} + \mathbf{n} \tag{1}$$

with $\mathbf{G}$ a matrix of received signatures, $\mathbf{s}$ a vector containing the complex-valued transmitted symbols and $\mathbf{n}$ the noise component. The column vector $\mathbf{h} \in \mathbb{C}^{L\times 1}$ contains the channel impulse response, assumed constant on the observation window and of length $L < N_c$.

Let us now concentrate on the matrix of received signatures $\mathbf{G}$. This matrix is formed stacking side by side the signature matrices corresponding to each of the code sequences $\mathbf{G} = \left[\mathbf{G}_1 \cdots \mathbf{G}_Q\right]$. These $\mathbf{G}_q$, $q = 1 \ldots Q$, can in turn be expressed as

$$\mathbf{G}_q = \mathcal{C}_q\left(\mathbf{I}_{M_s(q)} \otimes \mathbf{h}\right)$$

$$\mathcal{C}_q = \begin{bmatrix} \mathcal{D}_q(2)\,\mathcal{C}_q(1) & \cdots & \mathbf{0} \\ \mathbf{0} \quad \mathcal{C}_q(2) & \ddots & \vdots \\ \vdots & \ddots \quad \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathcal{C}_q(2)\,\mathcal{C}_q(1) \end{bmatrix}_{M\times M+1 \text{ blocks}},$$

with

$$M_s(q) = \underbrace{\left\lceil \frac{L-1}{SF_q} \right\rceil}_{\text{Past symbols}} + MN_s(q)$$

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the effective number of symbols received in an observation interval and $\mathcal{C}_q(1)$, $\mathcal{C}_q(2)$ the upper and lower parts of the next convolution matrix:

$$\begin{bmatrix}\mathcal{C}_q(1)\\ \mathcal{C}_q(2)\end{bmatrix} = \left[\mathcal{C}_{q,1}\,\mathcal{C}_{q,2}\cdots\mathcal{C}_{q,N_s(q)}\right] \in \mathbb{C}^{2N_c\times N_s(q)L}$$

$$\mathcal{C}_{q,i} = \begin{bmatrix} c_{q,r}(1) & 0 & \mathbf{0}_{L-1\times 1}\\ \vdots & \ddots & c_{q,r}(1)\\ c_{q,r}(N_c) & \ddots & \vdots\\ 0 & \ddots & c_{q,r}(N_c)\\ \vdots & & \cdots\mathbf{0}_{N_c-L+1\times 1}\end{bmatrix} \in \mathbb{C}^{2N_c\times L},$$

where $\mathbf{I}_i$ and $\mathbf{0}_{j\times k}$ stand for the $i\times i$ identity matrix and the $j\times k$ all-zero entries matrix respectively. Matrix $\mathcal{D}_q(2)$ is obtained as the $\left\lceil\frac{L-1}{SF_q}\right\rceil$ columns on the right of $\mathcal{C}_q(2)$, and contains the contribution from symbols transmitted prior to the observation interval. The complex-valued modified code sequences $c_{q,r}(n)$ are defined from the original codes $c_q(n)$ setting to zero all the samples outside the rth symbol interval,

$$c_{q,r}(n) = \begin{cases} c_q(n) & (r-1)SF_q < n \le rSF_q\\ 0 & \text{otherwise}\end{cases}.$$

Finally, returning to (1), $\mathbf{s}\in\mathbb{C}^{M_s\times 1}$ is a complex-valued vector with length $M_s = \sum_{q=1}^{Q} M_s(q)$ containing all the symbols that contribute to the signal within the observation interval

$$\mathbf{s} = \left[\mathbf{s}_1^T\,\mathbf{s}_2^T\cdots\mathbf{s}_Q^T\right]^T.$$

Each vector $\mathbf{s}_q\in\mathbb{C}^{M_s(q)\times 1}$ contains the symbol stream transported by the qth spreading sequence and received during the observation interval.

We consider the use of multi-user receivers to perform a joint detection of the symbols transported by the Q spreading codes. In particular, we will study the best linear unbiased estimator (BLUE) of $\mathbf{s}_q$ under the assumption of circularly symmetric white noise –referred to as decorrelating receiver–,

$$\hat{\mathbf{s}}^{DEC} = \left[\hat{\mathbf{G}}^H\hat{\mathbf{G}}\right]^{-1}\hat{\mathbf{G}}^H\mathbf{x},$$

and the linear Bayesian estimator under the assumption of circularly symmetric i.i.d. symbols and white noise –referred to as MMSE receiver–,

$$\hat{\mathbf{s}}^{MMSE} = \left[\hat{\mathbf{G}}^H\hat{\mathbf{G}} + \hat{\sigma}^2\mathbf{I}\right]^{-1}\hat{\mathbf{G}}^H\mathbf{x},$$

where $\hat{\mathbf{h}}$ and $\hat{\sigma}^2$ are estimations of the channel impulse response $\mathbf{h}$ and noise power σ^2 respectively and $\hat{\mathbf{G}}$ equal to $\mathbf{G}$ replacing $\mathbf{h}$ with $\hat{\mathbf{h}}$.

2. EFFECT OF IMPERFECT CHANNEL ESTIMATION

From the set of symbols mapped to the qth code sequence, we are only interested in those for which our observation $\mathbf{x}$ constitutes a sufficient statistic. The vector containing this selection of symbols will be denoted by $\mathbf{t}$, so that for a particular code q, we will have

$$\hat{\mathbf{t}}_q = \left[\hat{\mathbf{s}}_q\left(\left\lceil\frac{L-1}{SF_q}\right\rceil+1\right)\cdots\hat{\mathbf{s}}_q\left(M_s(q)-\left\lceil\frac{L-1}{SF_q}\right\rceil\right)\right]^T$$

with $\hat{\mathbf{s}}_q(i)$ representing the ith element of $\hat{\mathbf{s}}_q$. The same definition holds for all the estimators presented in the last section, as well as for the vector of actual values $\mathbf{s}_q$.

Consider now the following statistical assumption:

(**As1**) The spreading sequences are circularly symmetric independent identically distributed (i.i.d.) random variables with zero mean, variance $E\left[c_p(n)c_q^*(m)\right] = \alpha_q\delta_{p-q}\delta_{m-n}$, finite higher order moments and independent of the noise vector $\mathbf{n}$.

(**As2**) The noise samples are circularly symmetric i.i.d. with zero mean and covariance $E\left[\mathbf{n}\mathbf{n}^H\right] = \sigma^2\mathbf{I}_{MN_c}$.

As shown in the Appendix, under (**As1**, **As2**) and assuming that the spreading factors SF_q and the noise power σ^2 increase without bound while their quotient remains constant, the random variables $\hat{\mathbf{t}}_q^{DEC}$ and $\hat{\mathbf{t}}_q^{MMSE}$ tend in law to a circularly symmetric jointly Gaussian-distributed random vector with mean

$$\mathbf{m}_q^{DEC} = \frac{\hat{\mathbf{h}}^H\mathbf{h}}{\left\|\hat{\mathbf{h}}\right\|^2}\mathbf{t}_q,\quad \mathbf{m}_q^{MMSE} = \frac{1}{1+\hat{\gamma}_s^{-1}(q)}\mathbf{m}_q^{DEC}$$

and covariance

$$\mathbf{C}_{p,q}^{DEC} = \frac{\|\mathbf{h}\|^2\gamma_s^{-1}(q)}{\left\|\hat{\mathbf{h}}\right\|^2}\mathbf{I}_{M_s(q)}\delta_{p-q},\quad \mathbf{C}_{p,q}^{MMSE} = \frac{\mathbf{C}_{p,q}^{DEC}}{\left[1+\hat{\gamma}_s^{-1}(q)\right]^2},$$

being

$$\gamma_s(q) = \frac{\alpha_q SF_q\|\mathbf{h}\|^2}{\sigma^2}$$

the symbol energy to noise spectral density associated with the qth spreading sequence.

Since the symbol estimates are asymptotically independent, one can evaluate the performance in terms of probability of error for a generic ith symbol transported over the qth code sequence. Under the asymptotic conditions established, the output signal to noise ratio is the same for the two detectors and can be expressed as

$$SNR_q^{DEC} = SNR_q^{MMSE} = \mathrm{tr}\left[\mathbf{P}_{\mathbf{h}}\mathbf{P}_{\hat{\mathbf{h}}}\right]\gamma_s(q), \qquad (2)$$

where, for a generic column vector $\mathbf{a}$, $\mathbf{P}_{\mathbf{a}} = \frac{\mathbf{a}\mathbf{a}^H}{\|\mathbf{a}\|^2}$.

3. EFFECT ON THE SYMBOL ERROR RATE

Let us first concentrate on the asymptotic expression for the output of the two receivers,

$$\hat{\mathbf{t}}_q^{DEC} = \frac{\hat{\mathbf{h}}^H\mathbf{h}}{\left\|\hat{\mathbf{h}}\right\|^2}\mathbf{t}_q + \mathbf{n}_{DEC}$$

$$\hat{\mathbf{t}}_q^{MMSE} = \frac{1}{1+\hat{\gamma}_s^{-1}(q)}\frac{\hat{\mathbf{h}}^H\mathbf{h}}{\left\|\hat{\mathbf{h}}\right\|^2}\mathbf{t}_q + \mathbf{n}_{MMSE},$$

with $\mathbf{n}_{DEC}$ and $\mathbf{n}_{MMSE}$ the output noise contributions. The effect of an imperfect channel estimation is a reduction of the signal to noise ratio, characterized by the term

$\mathrm{tr}\left[\mathbf{P_h P_{\hat{h}}}\right]$ in (2) as well as a phase error at the output of the equalizer. The bit error rate can be expressed as,

$$BER = E\left[P_b|_{\hat{\mathbf{h}}}\right]$$

with $P_b|_{\hat{\mathbf{h}}}$ the bit error probability conditioned on the channel estimator and where the expectation is carried out with respect to $\hat{\mathbf{h}}$. Assuming Gray coding and QPSK modulation, one can express the conditional bit error probability as [5]:

$$\begin{aligned} Pb|_{\hat{\mathbf{h}}} = &\frac{1}{2}Q\left(\cos\left(\frac{\pi}{4}+\phi\right)\sqrt{2\zeta\gamma_s(q)}\right) + \\ &+\frac{1}{2}Q\left(\cos\left(\frac{\pi}{4}-\phi\right)\sqrt{2\zeta\gamma_s(q)}\right), \end{aligned} \quad (3)$$

with

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\lambda^2/2}d\lambda, \quad \zeta = \mathrm{tr}\left[\mathbf{P_h P_{\hat{h}}}\right]$$
$$\phi = \arctan\left[\mathrm{Im}\left(\hat{\mathbf{h}}^H\mathbf{h}\right)/\,\mathrm{Re}\left(\hat{\mathbf{h}}^H\mathbf{h}\right)\right].$$

Note that, under the asymptotic conditions described, this probability is equivalent for the two detectors under consideration. Making use of several trigonometric identities, (3) can be expressed as

$$\begin{aligned} Pb|_{\hat{\mathbf{h}}} = &\frac{1}{2}Q\left(\sqrt{\gamma_s(q)}\left[\mathrm{Re}(\eta)-\mathrm{Im}(\eta)\right]\right) + \\ &+\frac{1}{2}Q\left(\sqrt{\gamma_s(q)}\left[\mathrm{Re}(\eta)+\mathrm{Im}(\eta)\right]\right), \end{aligned} \quad (4)$$

with $\eta = \frac{\hat{\mathbf{h}}^H\mathbf{h}}{\|\hat{\mathbf{h}}\|\|\mathbf{h}\|}$.

In order to give more intuitive approximations for the bit error probability, consider now the first order Taylor series development of the decorrelator output with respect to the channel estimation around the true channel impulse response (note that under the asymptotic conditions stated, the MMSE receiver will yield the same performance),

$$\begin{aligned} \hat{\mathbf{s}}^{DEC} - \mathbf{s} \simeq &\left[\mathbf{G}^H\mathbf{G}\right]^{-1}\mathbf{G}^H\mathbf{n} - \left[\mathbf{G}^H\mathbf{G}\right]^{-1}\mathbf{G}^H\left(\hat{\mathbf{G}}-\mathbf{G}\right)\mathbf{s} \\ &-\left[\mathbf{G}^H\mathbf{G}\right]^{-1}\mathbf{G}^H\left(\hat{\mathbf{G}}-\mathbf{G}\right)\left[\mathbf{G}^H\mathbf{G}\right]^{-1}\mathbf{G}^H\mathbf{n} \\ &+\left[\mathbf{G}^H\mathbf{G}\right]^{-1}\left(\hat{\mathbf{G}}-\mathbf{G}\right)^H\mathbf{P}_{\mathbf{G}}^{\perp}\mathbf{n}, \end{aligned} \quad (5)$$

with $\mathbf{P}_{\mathbf{G}}^{\perp} = \mathbf{I}_{MN_c} - \mathbf{G}\left[\mathbf{G}^H\mathbf{G}\right]^{-1}\mathbf{G}^H$. The first term corresponds to the noise at output of the decorrelator assuming perfect channel estimation, whereas the rest can be associated with an additional noise due to channel mismatching. Let us now consider a Gaussian-distributed circularly symmetric $\hat{\mathbf{h}}$ with mean $\mathbf{h}$ and variance $E\left[\left(\hat{\mathbf{h}}-\mathbf{h}\right)\left(\hat{\mathbf{h}}-\mathbf{h}\right)^H\right] = \mathbf{C_h}$ independent of both the noise vector $\mathbf{n}$ and the transmitted symbols $\mathbf{s}$. It can be shown that, under (**As1**, **As2**) and assuming that noise power and spreading factors increase without bound at the same rate, the covariance of the total noise contribution in (5) tends in probability to

$$\frac{1}{\gamma_s(q)}\left(1+\frac{\mathrm{tr}\left(\mathbf{C_h}\right)}{\|\mathbf{h}\|^2}\right) + \frac{\mathbf{h}^H\mathbf{C_h}\mathbf{h}}{\|\mathbf{h}\|^4},$$

so that the bit error rate (BER) of the two systems can be approximated as

$$BER \simeq Q\left(\left[\gamma_s^{-1}(q)\left(1+\frac{\mathrm{tr}\left(\mathbf{C_h}\right)}{\|\mathbf{h}\|^2}\right)+\frac{\mathbf{h}^H\mathbf{C_h}\mathbf{h}}{\|\mathbf{h}\|^4}\right]^{-1/2}\right). \quad (6)$$

We can identify two terms influenced by imperfect channel estimation. At high signal to noise ratios, $\frac{\mathbf{h}^H\mathbf{C_h}\mathbf{h}}{\|\mathbf{h}\|^4}$ is the dominating term and only the part of the covariance matrix generated by the channel subspace contributes as effective interference. This term is responsible for an irreducible error floor at the output of the linear receivers. At low signal to noise ratios, the term $\frac{\mathrm{tr}(\mathbf{C_h})}{\|\mathbf{h}\|^2}$ becomes dominant and the whole covariance matrix span is responsible for the performance degeneration. This is in strong connection with the performance of semiblind ML channel estimators in mutirate synchronous CDMA discussed in [6].

4. GOODNESS OF FIT OF THE APPROXIMATIONS

Figures 1 and 2 represent the performance of the decorrelator with an observation window of $M = 4$ spreading periods ($N_c = 256$) for the cases of $Q = 1$ and $Q = 2$ codes respectively. The channel length was set to $L = 5$ and the spreading factor to $SF_q = 64$, so that each code represented $N_s(q) = 4$ virtual users per spreading period. The results obtained via Monte-Carlo simulations with randomly generated codes (asterisks) are plot together with the bit error rate approximations derived in (4) and (6) –dash-dotted and solid line respectively–. The channel estimator was assumed unbiased and Gaussian-distributed with diagonal normalized covariance $\frac{\mathbf{C_h}}{\|\mathbf{h}\|^2} = \alpha_h^2\mathbf{I}_L$, where α_h^2 ranged from 0.01 to 0.5. Results for the MMSE receiver are omitted here for clarity (in any case, its performance is very close to that of the decorrelating receiver).

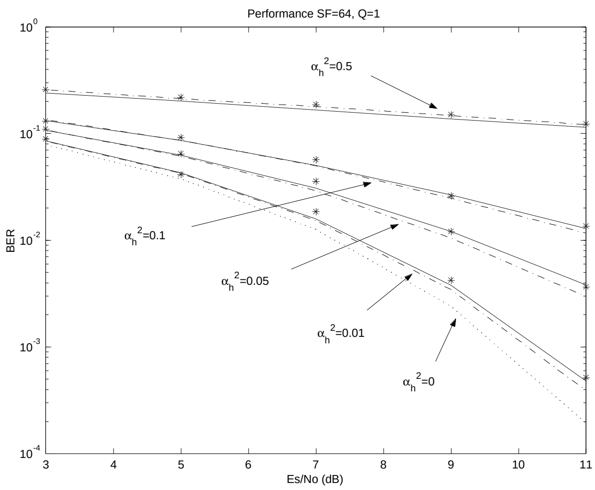

Figure 1: Asymptotic performance of the multiuser detectors with $SF = 64$, $Q = 1$. Dotted line: performance with perfect channel estimation. Dash-dotted line: BER in (4). Solid line: BER in (6). Asterisks: simulated performance.

In spite of the fact that the BER expressions are derived under the assumption of infinite SF_q, they turn out to be quite tight for relatively moderate spreading factors. Note that, in WCDMA, a low bit rate user can transmit with a SF_q as high as 256.

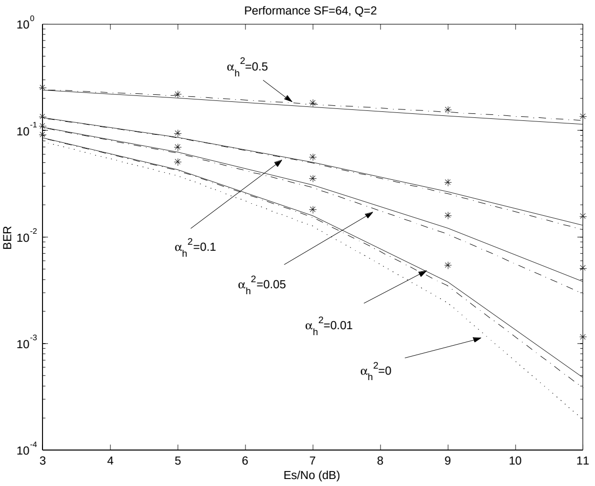

Figure 2: Asymptotic performance of the multiuser detectors with $SF = 64$. Line conventions are as in Fig. 1.

5. CONCLUSIONS

This paper has presented an asymptotic analysis of the influence of an imperfect channel estimation on the performance of linear multiuser receivers when both spreading factor and noise power increase without bound. Under these conditions, the performance of the MMSE and decorrelating receivers has been shown to be equivalent in terms of signal to noise ratio and symbol error probability. Finally, the asymptotic bit error rate and an approximation based on first order perturbations have been derived and their accuracy has been compared via simulation.

6. APPENDIX. ASYMPTOTIC DISTRIBUTION OF THE OUTPUT OF THE RECEIVERS

Recalling the structure of matrix $\mathbf{G}$ and applying the Weak Law of Large Numbers, the i,jth element of matrix $\hat{\mathbf{G}}_p^H \mathbf{G}_q$ for $\left\lceil \frac{L-1}{SF_q} \right\rceil < i,j \leq M_s(q) - \left\lceil \frac{L-1}{SF_q} \right\rceil$ can be expressed, under **(As1)**, as

$$\frac{1}{\sigma^2}\left[\hat{\mathbf{G}}_p^H \mathbf{G}_q\right]_{i,j} = \frac{\alpha_q SF_q}{\sigma^2} \hat{\mathbf{h}}^H \mathbf{h}\ \delta_{p-q}\delta_{i-j} + O_p\left(\kappa^{-1/2}\right),$$

with κ denoting the order of magnitude of either SF_q or σ^2, and $O_p(.)$ the in-probability version of the corresponding deterministic notation. The ith output of the decorrelating receiver can be asymptotically expressed as

$$\left[\hat{\mathbf{s}}_q^{DEC}\right]_i = \frac{\hat{\mathbf{h}}^H \mathbf{h}}{\left\|\hat{\mathbf{h}}\right\|^2}\left[\mathbf{s}_q\right]_i + \frac{\hat{\mathbf{h}}^H \mathcal{C}_{q,i}^H \mathbf{n}}{\alpha_q SF_q \left\|\hat{\mathbf{h}}\right\|^2} + O_p\left(\kappa^{-1/2}\right). \quad (7)$$

Convergence in law to a Gaussian distribution follows from the central limit theorem for sums of i.i.d. random variables. The mean of the asymptotic distribution of $\hat{\mathbf{t}}_q^{DEC}$ is readily identified from (7), and its covariance takes the expression:

$$\frac{E\left[\hat{\mathbf{h}}^H \mathcal{C}_{q,i}^H \mathbf{n}\mathbf{n}^H \mathcal{C}_{p,j}\hat{\mathbf{h}}\right]}{\alpha_q^2 SF_q^2 \left\|\hat{\mathbf{h}}\right\|^4} = \frac{1}{\gamma_s(q)} \frac{\left\|\mathbf{h}\right\|^2}{\left\|\hat{\mathbf{h}}\right\|^2}\ \delta_{p-q}\delta_{i-j} + O_p\left(\kappa^{-1/2}\right).$$

Since it is assumed that all the moments of $c_q(n)$ are finite, the limiting value of the covariance coincides with the covariance of the limiting distribution [7].

Let us now turn to the ith output of the MMSE receiver, which can asymptotically be expressed as

$$\left[\hat{\mathbf{s}}_q^{MMSE}\right]_i = \frac{1}{1+\hat{\gamma}_s^{-1}(q)} \frac{\hat{\mathbf{h}}^H \mathbf{h}}{\left\|\hat{\mathbf{h}}\right\|^2}\left[\mathbf{s}_q\right]_i + \\ + \frac{1}{1+\hat{\gamma}_s^{-1}(q)} \frac{\hat{\mathbf{h}}^H \mathcal{C}_{q,i}^H \mathbf{n}}{\alpha_q SF_q \left\|\hat{\mathbf{h}}\right\|^2} + O_p\left(\kappa^{-1/2}\right).$$

Convergence in law to a Gaussian distribution follows from the central limit theorem for sums of i.i.d. random variables. Once again, the mean of the asymptotic distribution of $\hat{\mathbf{t}}^{MMSE}$ is readily identified from (7), and its covariance can be calculated as

$$\frac{\gamma_s^{-1}(q)}{\left[1+\hat{\gamma}_s^{-1}(q)\right]^2} \frac{\left\|\mathbf{h}\right\|^2}{\left\|\hat{\mathbf{h}}\right\|^2}\ \delta_{p-q}\delta_{i-j} + O_p\left(\kappa^{-1/2}\right).$$

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