A BIORTHOGONAL TRANSFORM WITH OVERLAPPING AND NON-OVERLAPPING BASIS FUNCTIONS FOR IMAGE CODING

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ABSTRACT

This paper presents a new framework for a biorthogonal lapped transform that consists of long and short basis functions called the VLLBT. It is shown that when the biorthogonal long basis functions of the VLLBT are given, the optimal short basis functions in the energy compaction sense are derived by solving an eigenvalue problem without iterative searching techniques. We also provide design and image coding examples of the VLLBT. The resulting VLLBT attains high coding gain comparing to other lapped transforms. Moreover, experimental results show that the proposed VLLBT is superior to other conventional transforms in terms of PSNR at high compression ratio. Furthermore, it significantly reduces the annoying blocking artifacts.

1. INTRODUCTION

Transform coding is one of the most efficient methods for data compression of images. However, it is extensively known that at high compression ratios, reconstruction from compressed data results in low-quality images. More specifically, one of the most noticeable artifacts that these images exhibit is the *blocking effect*. This effect manifests itself as an artificial discontinuity between adjacent blocks and is a direct result of the independent processing of the blocks. It has been reported that the blocking effects can be reduced with the use of lapped transforms (LTs) [1] instead of the use of block-independent transforms such as the DCT. For blocks with length M, the lapped orthogonal transform (LOT), which is one of the most fundamental LTs, has M basis functions of length $2M$, so that the functions overlap across block boundaries. These overlapping basis functions result in reducing the blocking effects. In addition, the LOT has been extended to the GenLOT whose basis functions have length LM [2, 3]. Although lapped transforms reduce the blocking effects, due to their long basis functions, the quantization error is spread out over adjacent blocks. To avoid the spread of high frequency noise, recently, the LOT with variable length functions (VLLOT) has been proposed [4, 5]. The VLLOT consists of overlapping (long) basis functions, which can reduce the blocking artifacts, and block-independent (short) basis functions, which can restrict the ringing artifacts around edges. The VLLOT is based on GenLOT's lattice structure, where the initial building block is usually assumed to be the DCT. Thus, the short basis functions of the VLLOT are chosen from the DCT basis functions, directly.

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In the lapped transform or subband transform case, however, it has been reported that biorthogonal lapped transforms show higher coding gain than orthogonal ones [6]. In this paper, therefore, we provide a novel lapped biorthogonal transform with the optimal short basis functions, which is called the *VLLBT*, and its design method based on an eigenvalue problem.

1.1. Notation and Preliminaries

Bold-faced characters are used to denote vectors and matrices. The following conventions are adopted in terms of notation: $\mathcal{R}^N$ denotes the N-dimensional Euclidean space. $\boldsymbol{I}_N$, $\boldsymbol{J}_N$, and $\mathbf{0}_N$ stand for the identity, the reversal, and the null matrices of size $N \times N$, respectively. $\boldsymbol{A}^T$ stands for the transposition of the matrix $\boldsymbol{A}$. $\langle \boldsymbol{f}, \boldsymbol{g} \rangle$ stands for the inner product of the vectors $\boldsymbol{f}$ and $\boldsymbol{g}$. $\|\boldsymbol{f}\|$ stands for the Euclidean norm of the vector $\boldsymbol{f}$. $\boldsymbol{e}_n$ denotes the n-th “canonical" vector.

2. LAPPED ORTHOGONAL TRANSFORM – REVIEW

The lapped orthogonal transform (LOT) is a powerful tool for image compression [1] since it can reduce the blocking effect, which is a serious drawback in transform coding of images. Let us first review the LOT.

Let $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_0 \\ \boldsymbol{A}_1 \end{bmatrix}$ be a $2M \times M$ matrix where $\boldsymbol{A}_0$ and $\boldsymbol{A}_1$ contain the first M rows and the last M rows of $\boldsymbol{A}$, respectively. When the matrix $\boldsymbol{A}$ satisfies

$$\boldsymbol{A}_0^T\boldsymbol{A}_0 + \boldsymbol{A}_1^T\boldsymbol{A}_1 = \boldsymbol{A}_0\boldsymbol{A}_0^T + \boldsymbol{A}_1\boldsymbol{A}_1^T = \boldsymbol{I}_M, \quad (1)$$

$$\boldsymbol{A}_1^T\boldsymbol{A}_0 = \boldsymbol{A}_1\boldsymbol{A}_0^T = \mathbf{0}_M, \quad (2)$$

$\boldsymbol{A}$ is called the lapped orthogonal transform (LOT) [1]. Each column vector of $\boldsymbol{A}$ corresponds to the LOT basis function, whose length is $2M$. Equation (1) forces orthogonality of the basis functions, whereas (2) forces orthogonality of the overlapping parts of the basis functions of adjacent blocks.

Let $\boldsymbol{D}$ be an orthogonal matrix of size $M \times M$, and let $\boldsymbol{U}_1$ and $\boldsymbol{V}_1$ be orthogonal matrices of size $M/2 \times M/2$. Then, with an orthogonal projection matrix $\boldsymbol{P}$, the LOT matrix $\boldsymbol{A}$ can be rewritten [6] as

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_0 \\ \boldsymbol{A}_1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{D}\boldsymbol{P} \\ \boldsymbol{D}(\boldsymbol{I} - \boldsymbol{P}) \end{bmatrix} \underbrace{\begin{bmatrix} \boldsymbol{U}_1 & \mathbf{0}_{M/2} \\ \mathbf{0}_{M/2} & \boldsymbol{V}_1 \end{bmatrix}}_{\boldsymbol{Z}}. \quad (3)$$

This form gives the factorized LOT. It can easily checked that the LOT conditions (1) and (2) are imposed on $\boldsymbol{A}$ given as in (3). Malvar proposed a fast LOT where $\boldsymbol{D}$ corresponds to the DCT matrix, and $\boldsymbol{P}$ is the Haar butterfly [1]. In this fast LOT, the free parameter is the orthogonal matrix $\boldsymbol{Z}$.

The lapped orthogonal transform with variable length (VLLOT) [4, 5] has developed in order to avoid the spread of high-frequency distortion into neighboring blocks. The most basic VLLOT consists of K long and $(M-K)$ short basis functions of length $2M$ and M, respectively. The transform matrix of the VLLOT is given as

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_0 \\ \boldsymbol{A}_1 \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{A}}_0 & \bar{\boldsymbol{A}}_0 \\ \hat{\boldsymbol{A}}_1 & \boldsymbol{0} \end{bmatrix}, \tag{4}$$

where both $\hat{\boldsymbol{A}}_0$ and $\hat{\boldsymbol{A}}_1$ are size of $M \times K$, and $\bar{\boldsymbol{A}}_0$ is size of $M \times (M-K)$. The columns of $[\hat{\boldsymbol{A}}_0^T \ \hat{\boldsymbol{A}}_1^T]^T$ and $\hat{\boldsymbol{A}}_1$ correspond to the long and the short basis functions, respectively. In this type of the VLLOT, according to the necessary conditions for an existing linear phase perfect reconstruction (LPPR) filter bank [4,6], K must be even. The VLLOT matrix can be also written by the factorized form as given as in (3). In this case, the projection matrix $\boldsymbol{P}$ is written by

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{I}_{K/2} & \boldsymbol{0} & -\boldsymbol{I}_{K/2} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{(M-K)/2} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{I}_{K/2} & \boldsymbol{0} & \boldsymbol{I}_{K/2} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \tag{5}$$

and the orthogonal matrices of the last stage $\boldsymbol{Z}$ are given by

$$\boldsymbol{U}_1 = \begin{bmatrix} \hat{\boldsymbol{U}}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{(M-K)/2} \end{bmatrix}, \quad \boldsymbol{V}_1 = \begin{bmatrix} \hat{\boldsymbol{V}}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{(M-K)/2} \end{bmatrix}, \tag{6}$$

where $\hat{\boldsymbol{U}}_1$ and $\hat{\boldsymbol{V}}_1$ are $K/2 \times K/2$ orthogonal matrices. Some fast algorithms for the VLLOT has been well studied [4,5]. On the other hand, Tanaka and Yamashita found the optimal short functions in the energy compaction sense, and proposed adaptive transforms with overlapping and non-overlapping basis functions [7].

3. THE VLLBT: BIORTHOGONALIZATION OF THE VLLOT

Consider the use of biorthogonal lapped transforms. In this case, $\boldsymbol{A}$ given as in (4) is used for the forward transformation. The inverse transformation is defined as a matrix $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_0 \\ \boldsymbol{B}_1 \end{bmatrix}$. The condition for perfect reconstruction is obtained by rewriting the orthogonal constraints (1) and (2). Thus, the transform matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ requires

$$\boldsymbol{A}_0^T \boldsymbol{B}_0 + \boldsymbol{A}_1^T \boldsymbol{B}_1 = \boldsymbol{B}_0 \boldsymbol{A}_0^T + \boldsymbol{B}_1 \boldsymbol{A}_1^T = \boldsymbol{I}_M, \tag{7}$$

$$\boldsymbol{A}_1^T \boldsymbol{B}_0 = \boldsymbol{A}_0^T \boldsymbol{B}_1 = \boldsymbol{0}_M, \boldsymbol{B}_1 \boldsymbol{A}_0^T = \boldsymbol{B}_0 \boldsymbol{A}_1^T = \boldsymbol{0}_M, \tag{8}$$

Equation (7) implies biorthogonality of long basis functions, and (8) describes biorthogonality of tails. Let us define a biorthogonal version of the VLLOT, which is called the *VLLBT*, hereafter. The transform matrices of the VLLOT $\boldsymbol{A}$ for forward and $\boldsymbol{B}$ for inverse transformations are given as

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_0 \\ \boldsymbol{A}_1 \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{A}}_0 & \bar{\boldsymbol{A}}_0 \\ \hat{\boldsymbol{A}}_1 & \boldsymbol{0} \end{bmatrix}, \tag{9}$$

and

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_0 \\ \boldsymbol{B}_1 \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{B}}_0 & \bar{\boldsymbol{B}}_0 \\ \hat{\boldsymbol{B}}_1 & \boldsymbol{0} \end{bmatrix}, \tag{10}$$

respectively, where $\hat{\boldsymbol{A}}_0$, $\hat{\boldsymbol{A}}_1$, $\hat{\boldsymbol{B}}_0$, and $\hat{\boldsymbol{B}}_1$ are size of $M \times K$, and $\bar{\boldsymbol{A}}_0$ and $\bar{\boldsymbol{B}}_0$ are size of $M \times (M-K)$.

A factorized form of the VLLBT, which is obtained by biorthogonalization of the factorized VLLOT, can be achieved by relaxing orthogonality of the matrix $\boldsymbol{D}$ in (3). Let $\boldsymbol{H}$ and $\boldsymbol{G}$ be $M \times M$ non-singular matrices such that $\boldsymbol{HG} = \boldsymbol{GH} = \boldsymbol{I}$. Then, forward and inverse transform matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are given as

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{HP} \\ \boldsymbol{H}(\boldsymbol{I}-\boldsymbol{P}) \end{bmatrix} \boldsymbol{Z}, \ \ \boldsymbol{B}^T = \boldsymbol{Z}^{-1} \begin{bmatrix} \boldsymbol{PG} & (\boldsymbol{I}-\boldsymbol{P})\boldsymbol{G} \end{bmatrix}, \tag{11}$$

$\boldsymbol{A}$ and $\boldsymbol{B}$ construct a pair of lapped biorthogonal transform with longer and shorter basis functions (VLLBT). Let $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ be the i-th columns of the transform matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. A pair of vector sets $\{\boldsymbol{a}_i\}_{i=0}^M$ and $\{\boldsymbol{b}_i\}_{i=0}^M$ satisfies both biorthogonality and shift-biorthogonality. In the context of filter banks, $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ are called analysis and synthesis filters of biorthogonal filter banks, respectively.

4. OPTIMAL SHORT FUNCTIONS

In this section, we discuss the design method for short basis functions when the long basis functions are given. Let $\boldsymbol{f}$ be a vector of M consecutive samples of a real wide-sense stationary random process. The well-known Karhunen-Loève transform (KLT) provides the optimal approximation of $\boldsymbol{f}$. Moreover, among all block transforms, the KLT is indeed the best possible transform for minimizing the overall distortion for a given bit allocation. Therefore, the proposed objective function to find short basis functions is also based on a notion of optimal approximation.

4.1. Formulation

The problem we want to solve is the following: Given an oblique projection matrix $\boldsymbol{L}$ whose rank is $K < M$, find the matrix that minimizes a functional

$$J[\boldsymbol{X}] = E_{\boldsymbol{f}} \|\boldsymbol{f} - (\boldsymbol{L}+\boldsymbol{X})\boldsymbol{f}\|^2 \tag{12}$$

under the condition $\mathrm{rank}(\boldsymbol{X}) = N$ for any $N \le M-K$. This criterion leads to the KLT's one if we set $\boldsymbol{L} = \boldsymbol{O}$. Therefore, we will call this $\boldsymbol{X}$ a *subspace Karhunen-Loève (SKL) projector* of rank N.

Fortunately, the analytic solution of the above problem can be derived. Let $\boldsymbol{R} = E_{\boldsymbol{f}}[\boldsymbol{f}\boldsymbol{f}^T]$ be the correlation matrix of the input vector $\boldsymbol{f}$. Assume the rank of $\boldsymbol{R}$ is full. Then, there exist the $M-K$ non-zero eigenvalues of

$$\boldsymbol{Q} = (\boldsymbol{I}-\boldsymbol{L})\boldsymbol{R}(\boldsymbol{I}-\boldsymbol{L})^T \tag{13}$$

such that $\lambda_0 \ge \cdots \ge \lambda_{M-K-1} > 0$, and the corresponding eigenvectors $\boldsymbol{u}_0, \ldots, \boldsymbol{u}_{M-K-1}$.

Theorem 1 *The functional* $J[\boldsymbol{X}]$ *in (12) is minimized by*

$$\boldsymbol{X} = \sum_{n=0}^{N-1} \boldsymbol{u}_n \boldsymbol{u}_n^{*T}, \ \ \text{where } \boldsymbol{u}_n^* = (\boldsymbol{I}-\boldsymbol{P})^T \boldsymbol{u}_n. \tag{14}$$

Proof is omitted here.

Assume that a biorthonormal system $\{\boldsymbol{g}_i\}_{i=0}^{K-1}$, which generates a subspace $\mathcal{S}_1$ in $\mathcal{R}^M$, and its dual $\{\boldsymbol{h}_i\}_{i=0}^{K-1}$ in $\mathcal{R}^M$ are given. Using these functions $\boldsymbol{g}_i$ and $\boldsymbol{h}_i$, we have a projection matrix $\boldsymbol{L} = \sum_{i=0}^{K-1} \boldsymbol{g}_i \boldsymbol{h}_i^T$. If $\boldsymbol{h}_i = \boldsymbol{g}_i$, $\boldsymbol{L}$ gives an orthogonal projection matrix; otherwise, it gives an oblique projection matrix. If we apply the projection matrix $\boldsymbol{L}$ to Theorem 1, we obtain a biorthonormal basis $\{\boldsymbol{u}_i^*\}_{i=0}^{M-K-1}$, which spans a complement $\mathcal{S}_2$ of the subspace $\mathcal{S}_1$, that is, $\mathcal{R}^M = \mathcal{S}_1 \oplus \mathcal{S}_2$, and its dual $\{\boldsymbol{u}_i\}_{i=0}^{M-K-1}$. For $i = 0, \ldots, M-K-1$, vectors $\boldsymbol{u}_i$ and $\boldsymbol{u}_i^*$ derived by using Theorem 1 can be regarded as functions of the given biorthogonal system $\{\boldsymbol{g}_i, \boldsymbol{h}_i\}_{i=0}^{K-1}$. Finally, for $i = K, \ldots, M-1$, setting $\boldsymbol{h}_i = \boldsymbol{u}_{i-K}^*$ and $\boldsymbol{g}_i = \boldsymbol{u}_{i-K}$ gives the biorthonormal basis $\{\boldsymbol{g}_i\}_{i=0}^{M-1}$ and its dual $\{\boldsymbol{h}_i\}_{i=0}^{M-1}$ for $\mathcal{R}^M$. Consequently, the forward and the inverse transform matrices $\boldsymbol{H}$ and $\boldsymbol{G}$ are defined as

$$\boldsymbol{H} = \sum_{i=0}^{M-1} \boldsymbol{h}_i \boldsymbol{e}_i^T, \text{ and } \boldsymbol{G} = \sum_{i=0}^{M-1} \boldsymbol{e}_i \boldsymbol{g}_i^T, \tag{15}$$

respectively. Substituting (15) into (11), we obtain the transform matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. It should be noted that when we find the long basis functions $\{\boldsymbol{a}_i, \boldsymbol{b}_i\}_{i=0}^{K-1}$, we need not determine all columns of $\boldsymbol{H}$ and $\boldsymbol{G}$.

5. DESIGN METHOD

Since the optimal short basis functions with respect to a given set of long basis functions have been found, we only need to determine suitable long basis functions. For application in image coding, we use coding gain as the cost function. Higher coding gain correlates most consistently with higher PSNR. Assume that the signal is the first-order Markov process with the correlation coefficient $\rho = 0.95$, which is widely used in image processing. The correlation matrix is given by $(\boldsymbol{R})_{i,j} = \sigma_{\boldsymbol{f}}^2 \rho^{|i-j|}$, for $i, j = 0, \ldots, 2M-1$. Coding gain for a biorthogonal transform is given by [6]:

$$J_{CG} = 10 \log_{10} \left(\prod_{i=0}^{M-1} \langle \boldsymbol{a}_i, \boldsymbol{R}\boldsymbol{a}_i \rangle \|\boldsymbol{b}_i\|^2 \right)^{-1/M}. \tag{16}$$

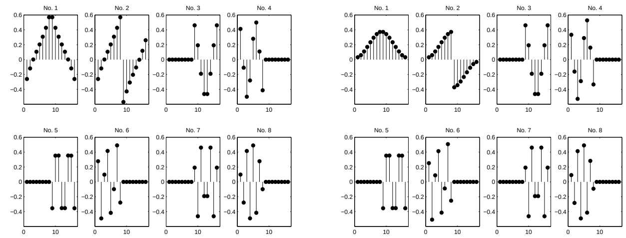
In particular, the number of free parameters can be reduced because of symmetric properties and no DC leakage.

5.1. A Design Example

Using these properties, consider the case $K = 2$ and $M = 8$. Setting $K = 2$ gives the minimum number of the long basis functions because of the existing condition as mentioned previously. In this case, the VLLBT consists of two long basis functions and six short basis functions. Figures 1(a) and 1(b) illustrate the basis functions $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ of the forward and the inverse transforms, respectively. Table 1 shows comparison of coding gain of several transforms.

6. IMAGE CODING APPLICATIONS

An image coding comparison is carried out in order to evaluate the performance of the proposed VLLBT. The transforms to be compared are 1) the DCT, 2) the VLLOT [5], and 3) the proposed VLLBT with no DC leakage.



(a) The basis functions $\boldsymbol{a}_i$ of the forward transform

(b) The basis functions $\boldsymbol{b}_i$ of the inverse transform

Fig. 1. A design example: $M = 8, K = 2$

Table 1. Comparison of coding gain in dB: We choose $M = 8$ for all cases and $K = 2$ for the VLLOT and the VLLBT.

DCT	KLT	LOT [1]	VLLOT [5]	VLLBT
8.826	8.846	9.219	8.954	9.282

For all cases as shown above, we use the same coding method. To be fair, we use a uniform step quantizer with the same step size for each coefficient. In the code assignment step, the same Huffman codebooks as the baseline-JPEG [8] are utilized.

The images used for the experiments are Barbara, Lena, and Pepper, which are standard, well-known 512×512 8-bit grayscale test images. Table 2 contains the peak signal to noise ratios (PSNRs) for decoded images at various bit rates. Moreover, the original and the decoded images for Barbara at 0.25 bpp are shown in Figure 7. Table 2 shows that at 1.00 bpp and 0.50 bpp, comparatively higher bit rates, the coding performance of the DCT based method is superior to those of the other methods. At bit rates lower than 0.50 bpp, the VLLOT and the VLLBT work well, and the proposed VLLBT outperforms the VLLOT in PSNR. It should be noted that the blocking artifacts of the VLLBT-decoded image are much less visible than those of the DCT- and the VLLOT-decoded images. This property is caused by the fact that end values of long basis functions $\boldsymbol{b}_0$ and $\boldsymbol{b}_1$ of the inverse transform are almost zero.

7. CONCLUSIONS

This paper has presented a new framework for a biorthogonal lapped transform that consists of long and short basis functions called the VLLBT. We have shown that when the biorthogonal long basis functions of the VLLBT are given, the optimal short basis functions in the energy compaction sense is derived by solving an eigenvalue problem. Therefore, we can find the short basis functions uniquely without iterative searching, if the long basis functions are determined once.

We have also provided design and image coding examples of the VLLBT. Coding gain of the VLLBT is higher than that of the VLLOT. Moreover, experimental results show that the proposed VLLBT is superior to other conventional transforms in terms of PSNR at high compression ratio. Furthermore, it significantly reduces the annoying blocking artifacts. These results may imply

Table 2. Comparison of PSNR (dB) results for 512×512 "Barbara", 512×512 "Lena", and 512×512 "Pepper" images at different bit rates (bpp)

Bit Rate	1.00	0.50	0.25	0.20
512×512 "Barbara"				
DCT	34.90	29.09	24.49	23.14
VLLOT	33.75	28.67	24.50	23.28
VLLBT	33.61	28.55	24.56	23.42
512×512 "Lena"				
DCT	38.52	34.99	30.73	29.18
VLLOT	38.25	34.74	30.65	29.24
VLLBT	38.12	34.84	30.85	29.47
512×512 "Pepper"				
DCT	36.00	33.88	30.49	28.95
VLLOT	35.58	33.57	30.37	28.89
VLLBT	35.58	33.68	30.85	29.57

that the proposed VLLBT is a promising technique in the field of image coding.

The VLLBT designed by the method demonstrated in this paper includes a potential for image coding using adaptive transforms, which would be constructed by changing the correlation matrix $\boldsymbol{R}$ in (13), adaptively. This problem will be addressed in future.

8. REFERENCES

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(a) The DCT (PSNR = 24.49 dB)

(b) The VLLOT (PSNR = 24.50 dB)

(c) The VLLBT (PSNR = 24.56 dB)

Fig. 2. Comparison of the decoded images