VECTORIAL DPCM CODING AND APPLICATION TO WIDEBAND SPEECH CODING

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ABSTRACT

This paper deals with optimal coding for vectorial signals by means of a decorrelating transform such as DPCM. We show that the optimal causal transform corresponds to a (Lower-Diagonal-Upper) triangular factorization of the autocorrelation matrix of the signal : the transformation matrix is triangular and unit diagonal. Each one of its rows is the optimal prediction filter for the corresponding component of the vector to be coded. We analyze the effect on the coding gain of the perturbation due to backward adaptation (prediction based on the quantized signal), as for DPCM coders. We then show that two previously introduced transformations, in the context of subband coding, appear as special cases of vectorial DPCM coding, and we compare these two transformations when perturbations occur on the reference signal. Finally, whe apply some results of vectorial DPCM coding to wideband speech coding.

1. INTRODUCTION

The transmission of audio signals (bichannel, such as stereo, or multichannel for the MPEG4 standard) naturally suggests the use of a coding technique for vectorial signals. In order to describe the coding technique of a vectorial signal, we first consider, in the second part of this paper, a finite frame of signal, a vector of signal. A linear transform is applied to this vector. The coding operation is then realized by scalar quantization of each component of the vector after transformation. The optimal transform will be such that the distortion generated by the quantization is minimized under the constraint of a finite number of bits. Under the unitarity constraint, the optimal transform is the well-known Karhunen-Loeve transform. The constraint we consider here is not unitarity but causality. The transformation can be considered as a generalization, for the vectorial case, of the classic (A)DPCM coding scheme, where a predicted version of the signal to be coded, based on the past quantized values of the signal, is first subtracted from the signal. We show that the optimal transform in this case corresponds to a LDU factorization of the autocorrelation matrix of the vector to be quantized. Each row in the transform matrix is the optimal prediction filter for the corresponding component of the vector to be coded. An expression for the coding gain is derived. We then inspect what happens when this transformation is backward adapted and analyze the influence of quantization noise (generated by scalar DPCM quantizers) on the coding gain. The third part of the paper is dedicated to the coding technique of vectorial signals. We show how frequential expressions can be obtained for the coding gains. We give in the fourth section two applications of vectorial DPCM coding : we show how two previously introduced transformations, in the context of subband coding (Maison and Vanderdorpe (M&V) [1], and Wong [2]), appear as special cases of the vectorial DPCM coding technique, and we compare these two transformations when the quantization noise on the past values of the signal is taken into account. We finally describe briefly, in the fifth part, an application of our results to wideband coding of speech. For further details about the results exposed in this paper, readers are invited to refer to [3].

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2. VECTORIAL DPCM CODING

2.1. Problem statement

Let us consider the generalization of the classical DPCM coding scheme applied to a vector $X = [x_1 \ldots x_N]^T$, see Figure 1. A matrix transformation L is applied to the vector $X : Y = LX = X - \overline{L}X$, where $\overline{L}X$ is the reference vector. The difference vector $Y = [y_1 \ldots y_N]^T$ is then quantized using a set $\mathbf{Q}$ of quantizers Q_i. The output X^q is $Y^q + \overline{L}X$. Note that the reconstruction error $\tilde{X}$ equals the quantization

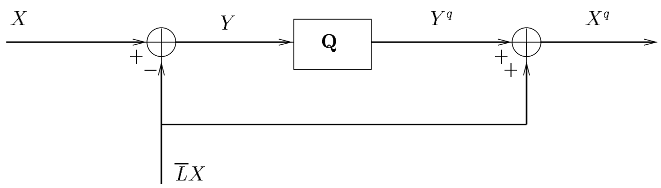

Fig. 1. Vectorial DPCM coding scheme.

error $\tilde{Y}$:

$$\tilde{X} = X - X^q = X - (Y^q + \overline{L}X) = X - \overline{L}X - Y^q = Y - Y^q = \tilde{Y}, \quad (1)$$

as in the unitary case. The constraint imposed on the transformation here is causality, which imposes a lower triangular structure. The unitary aspect of the transform appears in the unicity of the main diagonal ($\overline{L} = I - L$ is hence strictly lower triangular and represents the degrees of freedom of the transformation). The notion of causality could be generalized by working with the permuted components of X and Y, which gives $\mathcal{P}Y = L\ \mathcal{P}\,X$ or $Y = (\mathcal{P}^T\ L\mathcal{P})X$, where $\mathcal{P}$ is a permutation matrix. The coding gain for a transformation L is

$$G_{TC}(L) = \frac{E\|\tilde{X}\|^2_{(I)}}{E\|\tilde{X}\|^2_{(L)}} = \frac{E\|\tilde{X}\|^2_{(I)}}{E\|\tilde{Y}\|^2_{(L)}}, \quad (2)$$

where I is the identity matrix (which corresponds to the absence of transformation), and the notation $\|\tilde{X}\|^2_{(T)}$ denotes the variance of the quantization error on the vector X, obtained for a transformation T. The second equality in (2) follows from the equality (1), as in the unitary case. The SNR for a transformation L is defined as

$$SNR(L) = \frac{E\|X\|^2}{E\|\tilde{X}\|^2_{(L)}} = \frac{E\|X\|^2}{E\|\tilde{Y}\|^2_{(L)}} = \frac{E\|X\|^2}{E\|Y\|^2_{(L)}}\frac{E\|Y\|^2_{(L)}}{E\|\tilde{Y}\|^2_{(L)}} \quad (3)$$

where the first factor represents the gain of the transformation. We now set out to determine the optimal transformation L and bit assignment which maximizes the coding gain. For a given bit assignment, the optimal transformation is

$$L = \arg\max_L G_{TC}(L) = \arg\max_L SNR(L) = \arg\min_L E\|\tilde{X}\|^2_{(L)} \quad (4)$$

2.2. Ideal case

In a first step, we neglect the quantization error on the reference signal, and we suppose an optimal bit assignment. A quantizer Q_i introduces an independent white noise $\tilde{y}_i$ on the component y_i, of variance $\sigma^2_{\tilde{y}_i} = c\,2^{-2R_i}\sigma^2_{y_i}$, where R_i is the number of bits assigned to the quantizer Q_i, and c is a constant depending on the probability density function of the signal to be quantized (one should assume a Gaussian distribution, linear transform invariant).
For a given L, the optimal bit assignment has to minimize $E\|\tilde{Y}\|^2_{(L)} = \sum_{i=1}^N \sigma^2_{y_i} c 2^{-2R_i}$ under the constraint $\sum_{i=1}^N R_i = NR$, where R is the average number of bits assigned to the N quantizers Q_i. Using well-known techniques [4], and making abstraction of the fact that the R_i are integer and non negative, one shows that

$$\sigma^2_{\tilde{y}_i} = c\,2^{-2R_i}\sigma^2_{y_i} = c\,2^{-2R}\left(\prod_{i=1}^N \sigma^2_{y_i}\right)^{\frac{1}{N}} . \quad (5)$$

Note that the optimal quantization error variances $\sigma^2_{\tilde{y}_i}$ are equal (independent of i).
Optimization of L : we should consider $\min_L(\Pi_{i=1}^N \sigma^2_{y_i})^{\frac{1}{N}}$, where the $\sigma^2_{y_i}$ depend on the rows L_i of L : $\sigma^2_{y_i} = \sigma^2_{y_i}(L_{i.})$.
The problem is hence separable, and minimizing $\left(\prod_{i=1}^N \sigma^2_{y_i}\right)^{\frac{1}{N}}$ with respect to L entails minimizing $\sigma^2_{y_i}$ with respect to $L_{i,1:i-1}$. The components y_i appear clearly as the prediction errors of x_i with respect to the past values of X, the $X_{1:i-1}$, and the optimal coefficients $-L_{i,1:i-1}$ are the optimal prediction coefficients. In other words, L is such that

$$LR_{XX}L^T = R_{YY} = D = diag\{\sigma^2_{y_1}, ...\sigma^2_{y_N}\}, \quad (6)$$

where $diag\{...\}$ represents a diagonal matrix whose elements are $\sigma^2_{y_i}$. Since each prediction error y_i is orthogonal to the subspaces generated by the $X_{1:i-1}$, the y_i are orthogonal, and D is diagonal. It follows that

$$R_{XX} = L^{-1}R_{YY}L^{-T}, \quad (7)$$

which represents the LDU factorization of R_{XX}. Referring to (2), the coding gain can be written as

$$G^{(0)}_{TC}(L) = \left(\frac{\det[diag(R_{XX})]}{\det[diag(LR_{XX}L^T)]}\right)^{\frac{1}{N}} \quad (8)$$

where $diag(R)$ denotes here the diagonal matrix that corresponds to the diagonal of the matrix R.

2.3. Quantization effects on the coding gain

Let us now inspect the case where the tranformation is not based on the original signal but on its quantized version. In this case, the output vector becomes

$$Y = X - \overline{L}X^q = X - \overline{L}(X - \tilde{X}) = LX + \overline{L}\tilde{Y} . \quad (9)$$

Y now not only contains the prediction error LX of X, but also the quantization error $\tilde{Y}$ filtered by the optimal predictor $\overline{L}$. In this case again, the optimal bit assignment has to minimize the sum of the $\sigma^2_{\tilde{y}_i}$. It follows that the variances of the quantization noises are $\sigma^2_{\tilde{y}_i} = c2^{-2R}(\prod_{i=1}^N \sigma^2_{y_i})^{\frac{1}{N}} = \sigma^2_{\tilde{y}_1}$, independent of i. The autocorrelation matrix of the noise is hence $R_{\tilde{Y}\tilde{Y}} = \sigma^2_{\tilde{y}_1} I$.
To optimize L, one should consider $\min_L(\det[diag(R_{YY})])$, with this time $R_{YY} = LR_{XX}L^T + \sigma^2_{\tilde{y}_1}\overline{L}\,\overline{L}^T$. One can

show that the resolution of the normal equations leads to the following expression for the coding gain $G_{TC}^{(1)}(L)$, taking into account the perturbations up to first order

$$G_{TC}^{(1)}(L) \approx \left(\frac{\det\left[diag(R_{XX})\right]}{\det\left[diag(LR_{XX}L^T + \sigma_{\tilde{y}_1}^2 \overline{L}\overline{L}^T)\right]} \right)^{\frac{1}{N}} \quad (10)$$

with $LR_{XX}L^T = D$ and $\sigma_{\tilde{y}_1}^2 = c\,2^{-2R}(\det D)^{\frac{1}{N}}$ where D is the diagonal matrix of the non perturbated prediction error variances, and L and $\overline{L}$ are also non perturbated quantities. This expression is established under the high resolution assumption ($\sigma_{\tilde{y}_1}^2 I$ is small in comparison with R_{XX}).

3. DPCM CODING OF VECTORIAL SIGNALS

3.1. Ideal case

Let us now consider the case in which X is composed of a succession of samples of a vectorial signal $\underline{x}_k = [x_{1,k} \cdots x_{M,k}]^T$. $X_k = [\underline{x}_0^T \; \underline{x}_1^T \cdots \underline{x}_k^T]^T$, and also $Y_k = [\underline{y}_0^T \; \underline{y}_1^T \cdots \underline{y}_k^T]^T$ with $\underline{y}_k = [y_{1,k} \cdots y_{M,k}]^T$. For these vectorial signals, it is interesting to consider the limiting case in which the dimension k goes to infinity, for a stationary signal $\underline{x}_k$. In this case, the optimal transform L will lead to a signal $\underline{y}_k$, asymptotically stationary too, since L will become block Toeplitz (with blocks of size $M \times M$). We obtain in this case

$$G_{TC}^{(0)}(L) = \lim_{k\to\infty} \left(\frac{\det\left[diag(R_{X_kX_k})\right]}{\det\left[diag(LR_{X_kX_k}L^T)\right]} \right)^{\frac{1}{Mk}} \quad (11)$$

$$= \left(\frac{\det\left[diag(R_{\underline{x}_k\underline{x}_k})\right]}{\det\left[diag(R_{\underline{y}_k\underline{y}_k})\right]} \right)^{\frac{1}{M}} = \left(\frac{\prod_{i=1}^M \sigma_{x_i}^2}{\prod_{i=1}^M \sigma_{y_i}^2} \right)^{\frac{1}{M}} \quad (12)$$

where $y_{i,k}$ is the optimal prediction error of infinite order of $x_{i,k}$, based on $\left\{\underline{x}_{-\infty:k-1}, x_{1:i-1,k}\right\}$. We shall continue to denote by L_i (now of infinite dimension) the vector of the corresponding prediction coefficients.
There exists a frequency domain expression for $\prod_{i=1}^M \sigma_{y_i}^2$. Writing the prediction operation in the frequency domain, and using the fact that $\underline{y}_k$ is a totally decorrelated signal (its power spectral density can be written as $S_{\underline{yy}}(f) = R_{\underline{yy}} = diag\{\sigma_{y_1}^2, \ldots, \sigma_{y_M}^2\}$), one can show that

$$\prod_{i=1}^M \sigma_{y_i}^2 = e^{\int_{-\frac{1}{2}}^{\frac{1}{2}} \ln[\det(S_{\underline{xx}}(f))]\, df} \quad . \quad (13)$$

3.2. Quantization effects on the coding gain

If we now consider the effects of the quantization in the closed loop, the gain $G_{TC}^{(1)}(L)$ can be expressed as

$$G_{TC}^{(1)}(L) \approx \lim_{k\to\infty} \left(\frac{\det[diag(R_{X_kX_k})]}{\det[diag(LR_{X_kX_k}L^T + \sigma_{\tilde{y}_1}^2 \overline{L}\,\overline{L}^T)]} \right)^{\frac{1}{Mk}} \quad (14)$$

which leads to

$$G_{TC}^{(1)}(L) \approx G_{TC}^{(0)}(L) \left(1 - \sigma_{\tilde{y}_1}^2 \frac{1}{M} \sum_{i=1}^M \frac{||L_i||^2 - 1}{\sigma_{y_i}^2} \right) \quad . \quad (15)$$

As in the ideal case, one can derive an expression for $G_{TC}^{(1)}(L)$ in the frequency domain

$$G_{TC}^{(1)} \approx G_{TC}^{(0)} \left[1 + \frac{\sigma_{\tilde{y}_1}^2}{M} \left(-\int_{-\frac{1}{2}}^{\frac{1}{2}} tr\left(S_{\underline{xx}}^{-1}(f)\right) df + \sum_{i=1}^M \frac{1}{\sigma_{y_i}^2} \right) \right] \quad (16)$$

where, comparing with equation (15), the term $\int_{-\frac{1}{2}}^{\frac{1}{2}} tr\left(S_{\underline{xx}}^{-1}(f)\right) df$ corresponds to $\sum_{i=1}^M \frac{||L_i||^2}{\sigma_{y_i}^2}$.

4. TWO VDPCM APPROACHES COMPARED

In the case of subband coding, in which the components $x_{i,k}$ of the vectorial signal $\underline{x}_k$ correspond to the subband signals, we will now show that two previously introduced transformations for maximizing the coding gain are special cases of a causal unit diagonal transformation. Moreover, the equivalence of these transforms in the ideal case (considering $G_{TC}^{(0)}$) is a consequence of the LDU nature of the optimal transformation.
In [5], Fischer showed the necessity of totally decorrelating the subband signals in order to maximize the coding gain. On one hand, M&V [1] introduced in the classical subband coding scheme a transformation $T(z)$ (matricial filtering). This matrix transforms the vectorial signal $\underline{x}_k = [x_{1,k} \ldots x_{M,k}]^T$, yielding the transformed vectorial signal $\underline{y}_k = T(q)\,\underline{x}_k$ (where q^{-1} is the unit delay operator). This transform corresponds to the causal MIMO prediction : $T(z) = \sum_{k=0}^{\infty} T_k z^{-k}$, where T_0 is lower triangular and unit diagonal. The MIMO predictor is assumed to be of infinite order. In order to keep the structure causal, each sample of the subband i is predicted by means of the past samples of all subbands, and by means of the present samples of lower index only. In the case $M = 2$, the MIMO predictor is made of 2 intraband scalar predictors and 2 interband scalar predictors. M&V showed that such a transformation leads to an optimal coding gain $G_{TC}^{(0)}$. On the other hand, Wong used the following transform : in the case $M = 2$,

$$T(z) = \begin{bmatrix} 1 & 0 \\ 0 & T_{22}(z) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ W_{21}(z) & 1 \end{bmatrix} \begin{bmatrix} T_{11}(z) & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} T_{11}(z) & 0 \\ T_{22}(z)W_{21}(z)T_{11}(z) & T_{22}(z) \end{bmatrix} \quad . \quad (17)$$

The scalar prediction error filter $T_{11}(z)$ whitens $x_{1,k}$, yielding $y_{1,k}$, $W_{21}(z)$ is a (noncausal) Wiener filter estimating $x_{2,k}$ from $y_{1,k}$, and $T_{22}(z)$ whitens the resulting error signal to yield $y_{2,k}$. This transform hence uses only one interband predictor $W_{21}(z)$. The loss in degrees of freedom occurring with the loss of one interband predictor (T_{12} in M&V's

transformation) is balanced by the non causality of this remaining unique interband predictor. It is shown in [3] that these two transformations can both be expressed as lower triangular unit diagonal transforms, simply by reorganizing the samples in the vector to be coded. The product of the variances of the subband signal is constant, no matter which causal transform we use. The coding gain $G_{TC}^{(0)}$ is hence invariant by permutation. Each permutation leads to another causal decorrelation of the components of one vector. For a stationary vectorial signal, this means that there exists more that one way to decorrelate the scalar signals which compose this signal. The examples of Wong and M&V present in fact (for $M = 2$) two extreme cases of an infinity of variants, which are parametrized by the degree of (non) causality (in the classical sense) of the interband predictor(s).

Let us now compare the approaches of Wong, and M&V in the presence of quantization. The expression (16) shows that in order to maximize the gain $G_{TC}^{(1)}$, one should look for $\max_L \sum_{i=1}^{M} \frac{1}{\sigma_{y_i}^2}$ which leads to maximize the sum of the inverses of the prediction error variances, or in other words make these variances as different as possible. (since $\prod_{i=1}^{M} \sigma_{y_i}^2$ is invariant, whatever the causal transformation involved).

Consider the case $M = 2$: let us assume, without loss of generality, that the variances of the vectorial signals $\sigma_{x_i}^2$ are placed in decreasing order. In this case, one should minimize $\sigma_{y_2}^2$. $\sigma_{y_2}^2$ will be minimized if the greatest number of samples are used to predict $x_{2,k}$. Wong's approach should hence be the best one, since it will lead to a smaller variance for $\sigma_{y_2}^2$. This difference between the two transformations appears only when the prediction is based on a quantized signal, but this is the way in which such decorrelating transforms will be implemented. Another improvement due to Wong's approach appears when the filters are forced to have a finite length. Actually, whereas the correlation of a scalar signal tends to be concentrated around the zero lag, the intercorrelation between two signals may be concentrated around an arbitrary time delay. In M&V's approach, only small time delays will be accounted for. On the other hand, we have the choice in Wong's approach to position the FIR cross prediction filter around the most useful lag.

5. WIDEBAND SPEECH CODING APPLICATION

In the near future, wideband speech coders will be introduced in mobile systems in which the encoded signal band is 7kHz instead of the usual 3.4kHz. One way to construct such a coder is to filter and split the input signal into two subbands, which allows one to use an existing narrowband coder for the lowest subband. In the case of an optimal bit assignment (and since the higher subband has on the average a lower variance than the lower subband), the VDPCM strategy described above should be applied, and Wong's approach should be the best decorrelating predictive transform. Note also that, despite the non causality in the classical sense of this approach, it is well suited for frame based speech coding, which allows a certain degree of non causality. Actually, one can code one frame of signal in the lower subband and then code one frame in the higher subband.

Another special case is when the bit assignment is fixed, and when all the bits are used to code the lower subband. In this case, the quantization noises introduced by the quantization of the signals $y_{1,k}$ and $y_{2,k}$ are $\sigma_{\tilde{y}_1}^2 = c2^{-2R}\sigma_{y_1}^2 = \alpha\sigma_{y_1}^2$, with $\alpha << 1$, and $\sigma_{\tilde{y}_2}^2 = \sigma_{y_2}^2$. The coding gain is

$$G_{TC}(L) = \frac{E\|\tilde{X}\|_{(I)}^2}{\alpha\sigma_{y_1}^2 + \sigma_{y_2}^2} \qquad (18)$$

In this case again, the term $\alpha\sigma_{y_1}^2$ being small compared to $\sigma_{y_2}^2$, one has to minimize $\sigma_{y_2}^2$, and Wong's approach is more efficient. Informal listening tests we performed (using several GSM AMR narrowband codecs) have confirmed the perceptual gain over narrowband coding, introduced by the interband prediction. The prediction of the higher subband is done on the basis of the decoded version of the lower subband. Only the encoded lower subband (and $W_{21}(z)$) gets transmitted ($R_1 = 2R, R_2 = 0$). The decoder produces for the higher subband only its predicted version on the basis of the decoded lower subband. The improvement in perceptual quality is nevertheless significant. Some overhead is required in transmitting the prediction filter $W_{21}(z)$.

6. REFERENCES

[1] B. Maison, L. Vanderdorpe, "About the Asymptotic performance of Multiple-Input/Multiple-Output Linear Prediction of Subband Signals," *IEEE Signal Proc. Letters*, vol. 5, no. 12, December 1998.

[2] P. W. Wong, "Rate Distortion Efficiency of Subband Coding with Crossband Prediction," *IEEE Trans. on Inf. Theory*, Januar 1997.

[3] D. Mary, Dirk T.M. Slock, "Codage DPCM Vectoriel et Application au Codage de la Parole en Bande Élargie," in *CORESA 2000*, Poitiers, France, October 2000.

[4] N.S. Jayant, P. Noll, *Digital Coding of Waveforms*, Signal Processing. Prenctice Hall, 1984.

[5] T. R. Fischer, "On the Rate-Distortion Efficiency of Subband Coding," *IEEE trans. on Inf. Theory*, vol. 38, no. 2, March 1992.